



KEMENTERIAN PENDIDIKAN MALAYSIA
Jabatan Pendidikan Negeri Sabah

MODULE

ADDITIONAL MATHEMATICS

FORM 4
2021

(ENGLISH VERSION)

JABATAN PENDIDIKAN NEGERI SABAH

PREFACE

Understanding the concept and doing a lot of practice on the concepts learned is a contributing factor to student success in the SPM examination. Most students who are weak in mastering the concept are due to lack of practice. A compact and brief note (One Page Note) that is included in this module is expected to help students to master the concept of the topic. Doing intensive exercises for each topic is also expected to help teachers and students master the topic.



Therefore, this module is hoped to help teachers and students during teaching and learning and during revision exercises before the SPM examination.

MODULE OBJECTIVE

1. Make it easier for students to remember important concepts in the form of more compact graphics.
2. Help students answer revision practice questions before the SPM examination
3. Help teachers overcome the problems of students who do not have any reference sources.

Sincerely from:

Lee Chiong Tee
Coordinator and Head Panels
Additional Mathematics Module 2021

CONTENT

TOPIC 1	FUNCTIONS	1
TOPIC 2	QUADRATICS FUNCTIONS	35
TOPIC 3	SYSTEM OF EQUATIONS	81
TOPIC 4	INDICES, SURDS AND LOGARITHMS	111
TOPIC 5	PROGRESSIONS	172
TOPIC 6	LINEAR LAW	216
TOPIC 7	COORDINATE GEOMETRY	253
TOPIC 8	VECTORS	315
TOPIC 9	SOLUTION OF TRIANGLES	386
TOPIC 10	INDEX NUMBER	430
	WORKSHEET ANSWERS	464

FUNCTIONS

- ONE PAGE NOTE (OPN)

- WORKSHEET

Encik Dennis Chua Ah Thin

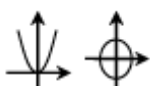
ONE PAGE NOTE (OPN)

types of relation :

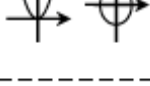
→ one to one



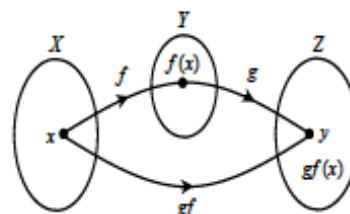
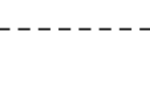
→ one to many



→ many to one



→ many to many

**function :**

→ one to one @ many to one relation

→ each object must mapped with an image only

to whether a graph is a function :

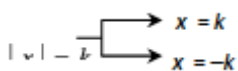
→ draw a vertical line, and it cut the graph at only one point

function $g \circ f$ exists if and only if range of $f \subseteq$ domain of g
 function $f \circ g$ exists if and only if range of $g \subseteq$ domain of f

$$f^n(x) = x \Leftrightarrow f^{kn}(x) = x$$

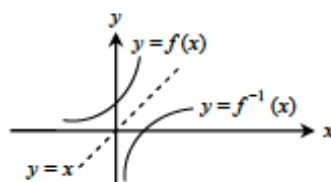
 x is maps onto itself under function h

$$h(x) = x$$



$$f(x) = \frac{ax+b}{cx+d} \begin{cases} f(x) \text{ finite} & cx+d \neq 0 \\ f(x) \text{ infinite} & cx+d = 0 \end{cases}$$

- the graph of $y = f(x)$ and $y = f^{-1}(x)$ are symmetry about the line $y = x$



- domain for $f^{-1}(x)$ = range for $f(x)$ and range for $f^{-1}(x)$ = domain for $f(x)$
- to every **one to one** function $f: x \rightarrow y$, there exist an inverse function $f^{-1}: y \rightarrow x$

$$\left. \begin{array}{l} \sim f(x) = x+k \rightarrow f^{-1}(x) = x-k \\ \sim f(x) = kx \rightarrow f^{-1}(x) = \frac{x}{k} \\ \sim f(x) = ax+b \rightarrow f^{-1}(x) = \frac{x-b}{a} \\ \sim f(x) = \frac{x}{a} + b \rightarrow f^{-1}(x) = a(x-b) \end{array} \right\} \begin{array}{l} \sim f(x) = x^2 \rightarrow f^{-1}(x) = \sqrt{x} \\ \sim f(x) = \frac{ax+b}{cx+d} \rightarrow f^{-1}(x) = \frac{dx-b}{-cx+a} \\ \text{(for checking answer)} \end{array}$$

- if $y = f(x)$ and $y = f^{-1}(x)$ are inverse to each other $\rightarrow ff^{-1}(x) = f^{-1}f(x) = x$

WORKSHEET
TOPIC 1 : FUNCTIONS
[2 – 3 questions → 4 – 9 marks]

Revision [RELATIONS ~ ordered pairs, arrow diagram, cartesian graph]

- 1** Based on the below information, the relation between P and Q is defined by the set of ordered pairs $\{(1, 2), (1, 4), (2, 6), (2, 8)\}$.

$$P = \{ 1, 2, 3 \}$$

$$Q = \{ 2, 4, 6, 8, 10 \}$$

$$(3, 6), (4, 8), \dots$$

$$\text{image of } 3 = 6$$

$$\text{object of } 6 = 3$$

$$\text{function notation, } f(x) = 2x$$

State

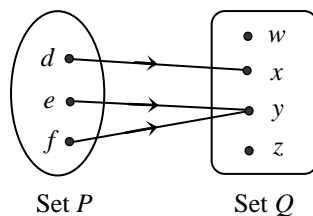
- (a) the image of 1,
 (b) the object of 2.

Answer :

- (a) (b)

[2 marks] [2003, No.1]

- 2** The diagram shows the relation between set P and set Q .



State

- (a) the range of the relation,
 (b) the type of the relation.

Answer :

- (a) (b)

$$\text{range} \subseteq \text{codomain}$$

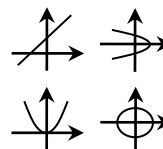
types of relation

:

→ one to one

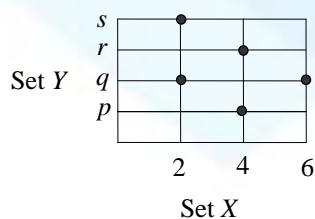
→ one to many

→ many to one



[2 marks] [2004, No.1]

- 3 The diagram shows the relation between set X and set Y in the graph form.



State

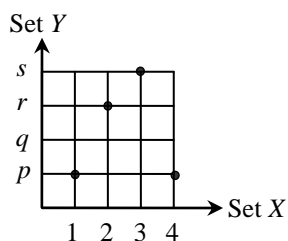
- (a) the objects of q .
 (b) the codomain of the relation.

[2 marks] [2009, No.1]

Answer :

- (a) (b)

- 4 The diagram shows the relation between set X and set Y in the graph form.



State

- (a) the relation in the form of ordered pairs.
 (b) the type of the relation,
 (c) the range of the relation.

[3 marks] [2010, No.1]

Answer :

- (a)
 (b) (c)

- 5 It is given that the relation between set $X = \{0, 1, 4, 9, 16\}$ and set $Y = \{0, 1, 2, 3, 4, 5, 6\}$ is “square of”

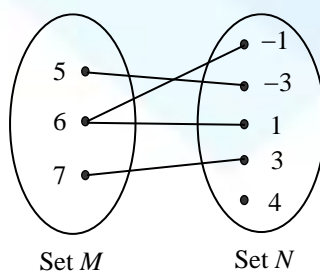
- (a) Find the image of 9.
 (b) Express the relation in the form of ordered pairs.

[3 marks] [2011, No.1]

Answer :

- (a)
 (b)

- 6 The diagram shows the relation between set M and set N .



State

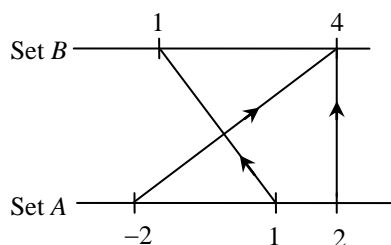
- (a) the object of -1 ,
 (b) the range of the relation.

[2 marks] [2012, No.1]

Answer :

- (a) (b)

- 7 The diagram shows the relation between Set A and Set B in the arrow diagram form.



- (a) Represent the relation in the form of ordered pairs.
 (b) State the domain of the relation.

[2 marks] [2014, No.1]

Answer :

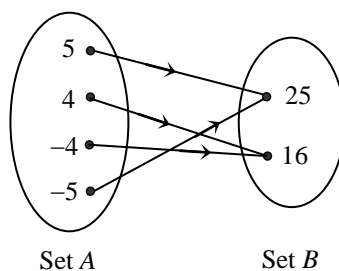
(a)

(b)

1.1 Functions
1.1.1 Explain function using graphical representations and notations.
1.1.2 Determine domain and range of a function.
1.1.3 Determine the image of a function when the object is given and vice versa.

⇒ **functions notations**

- 8** In the diagram, set B shows the images of certain elements of set A .



- (a) State the type of relation between set A and set B .
- (b) Using the function notation, write a relation between set A and set B .

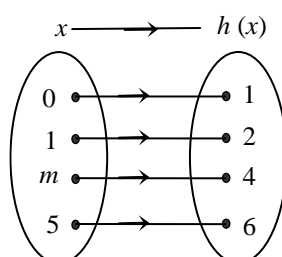
[2 marks] [2006, No.1]

Answer :

(a)

(b)

- 9** The diagram shows the linear function h .



- (a) State the value of m .
- (b) Using the function notation, express h in terms of x .

[2 marks] [2007, No.1]

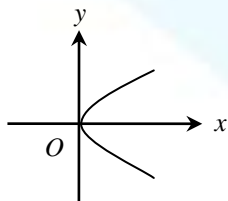
Answer :

(a)

(b)

⇒ **the conditions of existence (the conditions such that a relation is a function)***

- 10 The diagram shows the relation between x and y .



function :

→ one to one @ many to one relation
→ each object must mapped with an image only

to whether a graph is a function :

→ draw a **vertical line**, and it cut the graph at only one point

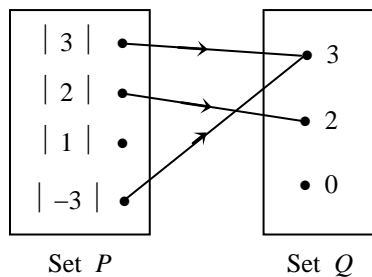
Determine whether the relation is a function. Give reason for your answer.

[2 marks]

[Forecast]

Answer :

- 11 The diagram shows the relation between set P and set Q .



Determine whether the relation is a function. Give reason for your answer.

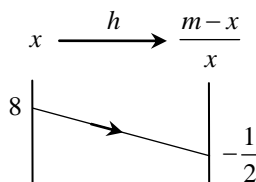
[2 marks]

[Forecast]

Answer :

⇒ **given object & image → find the constant**

- 12 The diagram shows the function $h : x \rightarrow \frac{m-x}{x}$, $x \neq 0$, where m is a constant.



Find the value of m .

(Ans : 4)

[2 marks] [2006, No.2]

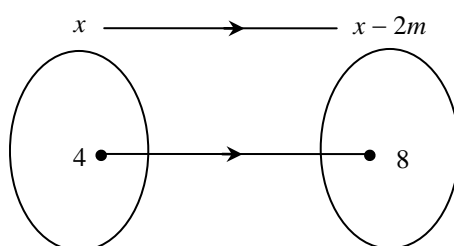
Answer :

- 13 It is given that the function $f(x) = p - 3x$, where p is a constant. Find the value of p such that $f(p) = 4$.

(Ans : -2)
[2 marks] [2013, No.3]

Answer :

- 14 The diagram shows the function $f: x \rightarrow x - 2m$, where m is a constant.



Find the value of m .

(Ans : -2)
[2 marks] [2014, No.2]

Answer :

\Rightarrow **maps onto itself (unchanged under the mapping) / definition***

x is maps onto itself under function h
 \Downarrow
 $h(x) = x$

- 15 Given the function $f: x \rightarrow 3x - 2$, find
- the value of x when $f(x)$ maps onto itself,
 - the value of h such that $f(2 - h) = 4h$.

(Ans : 1)
(Ans : $\frac{4}{7}$)
[4 marks] [2016, No.11]

Answer :

(a)

(b)

- 16 A function f is defined by $f(x) = \frac{x+k}{2x+3}$, $x \neq h$.

(a) Find the value of h .

(b) If $x = 2$ is unchanged under the mapping of function f , find the value of k .

(Ans : 12)

[3 marks] [Forecast]

Answer :

(a)

(b)

$$f(x) = \frac{ax+b}{cx+d} \begin{cases} f(x) \text{ tertakrif} & \rightarrow cx+d \neq 0 \\ f(x) \text{ tak tertakrif} & \rightarrow cx+d = 0 \end{cases}$$

\Rightarrow **absolute functions ~ object, image, domain, codomain, range, graph**

- 17 Given the function $f: x \rightarrow |x - 3|$, find the values of x such that $f(x) = 5$.

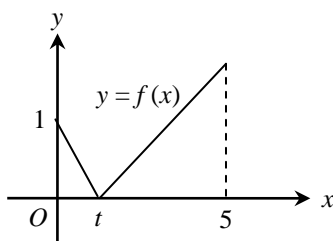
(Ans : -2, 8)

[2 marks] [2007, No.2]

Answer :

$$|x| = k \begin{cases} \rightarrow x = k \\ \rightarrow x = -k \end{cases}$$

- 18 The diagram shows the graph of the function $f(x) = |2x - 1|$, for the domain $0 \leq x \leq 5$.



State

(a) the value of t .

(Ans : $\frac{1}{2}$)

(b) the range of $f(x)$ corresponding to the given domain.

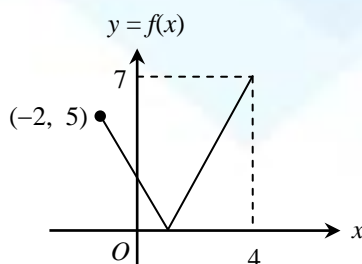
[3 marks] [2008, No.1]

Answer :

(a)

(b)

- 19 The diagram shows the graph of the function $f: x \rightarrow |1 - 2x|$ for the domain $-2 \leq x \leq 4$.



State

- (a) the object of 7,
 (b) the image of 3,
 (c) the domain of $0 \leq f(x) \leq 5$.

(Ans : 5)

[3 marks] [2017, No.9]

Answer :

(a)

(b)

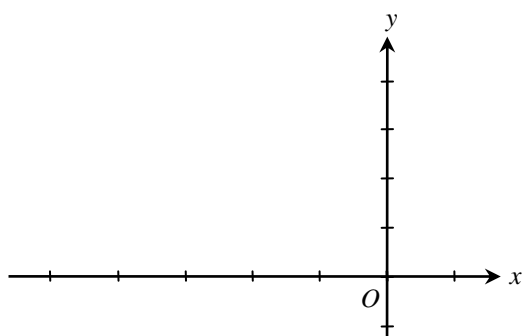
(c)

- 20 (a) Sketch the graph $y = |2x + 6| - 1$ on the given axes for the domain $-5 \leq x \leq 1$. [2 marks]
 (b) Hence, find the corresponding range. [1 mark]

[Forecast]

Answer :

(a)



x			0	
y				0

(b)

1.2 Composite functions

- 1.2.1 Describe the outcome of composition of two functions.
 1.2.2 Determine the composite functions.
 1.2.3 Determine the image of composite functions given the object and vice versa.
 1.2.4 Determine a related function given composite function and another function.
 1.2.5 Solve problems involving composite functions.
-

\Rightarrow given $f(x)$, $g(x) \rightarrow$ find $fg(x)$, $gf(x)$, $f^2(x)$, $g^2(x)$

- 21** The following information is about the function h and the composite function h^2 .

$$h : x \rightarrow ax + b, \text{ where } a \text{ and } b \text{ are constants, and } a > 0$$

$$h^2 : x \rightarrow 36x + 35$$

Find the value of a and of b .

(Ans : $a = 6$, $b = 5$)

[3 marks] [2007, No.3]

Answer :

- 22** Given the functions $f(x) = x - 1$ and $g(x) = kx + 2$, find

(a) $f(5)$,

(b) the value of k such that $gf(5) = 14$.

(Ans : 3)

[3 marks] [2008, No.3]

Answer :

(a)

(b)

- 23** Given the functions $g : x \rightarrow 2x - 3$, and $h : x \rightarrow 4x$, find

(a) $hg(x)$,

(Ans : $8x - 12$)

(b) the value of x if $hg(x) = \frac{1}{2}g(x)$.

(Ans : $\frac{3}{2}$)

[4 marks] [2009, No.2]

Answer :

(a)

(b)

24 Given the functions $g : x \rightarrow x - 8$ and $h : x \rightarrow \frac{x}{3x-2}$.

(a) State condition for the value of x such that function h is defined. **

(b) Find the value of $hg(10)$.

(Ans : $\frac{1}{2}$)

[3 marks] [2010, No.3]

Answer :

(a)

(b)

25 It is given that the functions $g(x) = 4x - 7$ and $h(x) = 2x$. Find the value of find $gh(2)$. (Ans : 9)

[2 marks] [2011, No.2]

Answer :

26 Given the function $f : x \rightarrow 5x + 6$ and $g : x \rightarrow 2x - 1$, find $gf(x)$.

(Ans : $10x + 11$)

[2 marks] [2013, No.2]

Answer :

27 It is given the functions $f(x) = 3x$ and $g(x) = h - kx$, where h and k are constants. Express h in terms of k such that $gf(1) = 4$.

(Ans : $h = 4 + 3k$)

[3 marks] [2015, No.2]

Answer :

- 28 Given the function $m : x \rightarrow px + 1$, $h : x \rightarrow 3x - 5$ and $mh(x) = 3px + q$. Express p in terms of q .
(Ans : $p = \frac{1-q}{5}$)

[3 marks] [2016, No.12]

Answer :

- 29 The function m and n are defined by $m : x \rightarrow 3x + 1$ and $n : x \rightarrow \frac{2x}{4x-1}$.

(a) State the value of x such that n is undefined.

(b) Find the value of k if $mn(k) = \frac{13}{5}$.

(Ans : 4)

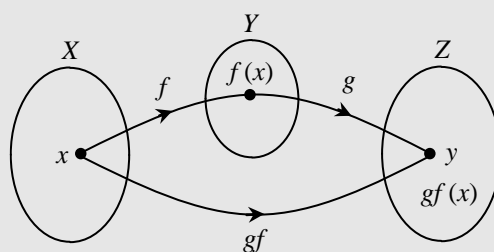
[3 marks] [Forecast]

Answer :

(a)

(b)

MIND think :



function $g \circ f$ exists if and only if range of $f \subseteq$ domain of g

function $f \circ g$ exists if and only if range of $g \subseteq$ domain of f

\Rightarrow given $f(x)$, $fg(x) \rightarrow$ find $g(x)$

- 30** Given the function $h(x) = \frac{6}{x}$, $x \neq 0$ and the composite function $hg(x) = 3x$, find

(a) $g(x)$,

(Ans : $\frac{2}{x}$)

(b) the value of x when $gh(x) = 5$.

(Ans : 15)

[4 marks] [2004, No.3]

Answer :

(a)

(b)

- 31** Given that $f(x) = 3x + 4$ and $fg(x) = 6x + 7$, find

(a) $fg(4)$,

(Ans : 31)

(b) $g(x)$.

(Ans : $2x + 1$)

[4 marks] [2012, No.2]

Answer :

(a)

(b)

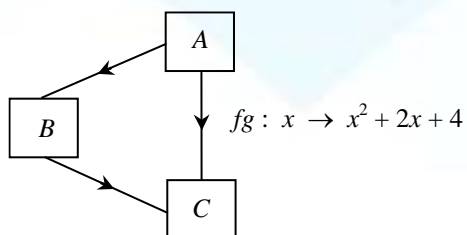
- 32** Given $f(x) = x^2 + 1$, find in terms of x , fungsi $g(x)$ such that $fg(x) = x^2 + 4x + 5$.
(Ans : $x + 2$)

[3 marks] [Forecast]

Answer :

\Rightarrow given $g(x)$, $fg(x) \rightarrow$ find $f(x)$

- 33 The diagram shows the relation between set A , set B and set C .



It is given that set A maps to set B by the function $\frac{x+1}{2}$ and maps to set C by $fg: x \rightarrow x^2 + 2x + 4$.

- (a) Write the function which maps set A to set B by using the function notation.
 (b) Find the function which maps set B to set C .

[Ans : $f(x) = 4x^2 + 3$ @ $f: x \rightarrow 4x^2 + 3$]

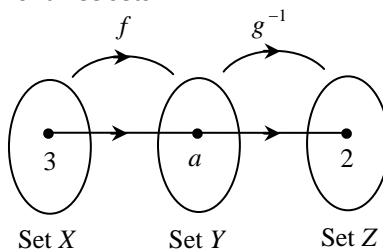
[4 marks] [2018, No.22]

Answer :

(a)

(b)

- 34 The diagram shows the relation of three sets.



It is given that $f: x \rightarrow 2x + 3$ and $g^{-1}f: x \rightarrow \frac{3}{x} + 1, x \neq 0$.

- (a) If a student writes $a = 10$, determine the value is correct or wrong. Give your reason.

(Ans : 9)

- (b) Find $g^{-1}(x)$.

(Ans : $\frac{x+3}{x-3}, x \neq 3$)

[4 marks] [2019, No.8]

Answer :

(a)

(b)

\Rightarrow given $f(x) \rightarrow$ find $f^2(x), f^3(x), f^4(x), \dots, f^n(x) \sim 1$

35 It is given that $f: x \rightarrow \frac{x}{x+1}, x \neq -1$.

(a) Find the iterated function $f^2(x), f^3(x)$ and $f^4(x)$.

[Ans : $f^2(x) = \frac{x}{2x+1}, x \neq -\frac{1}{2} \dots$]

(b) Hence, determine the general rule $f^n(x)$, where n is a positive integer.

[4 marks] [Forecast]

Answer :

(a)

$$f^3(x) = f[f^2(x)] = f^2[f(x)]$$

$$f^4(x) = f[f^3(x)] = f^2[f^2(x)]$$

(b)

36 It is given that $f: x \rightarrow ax + b$ and $f^3(x) = 8x + 7$.

(a) Find the value of a and of b .

(Ans : $a = 2, b = 1$)

(b) Find the expression of $f^4(x)$. Hence, determine the general rule $f^n(x)$, where n is a positive integer.

(Ans : $16x + 15$)

[4 marks] [Forecast]

Answer :

(a)

(b)

\Rightarrow given $f(x) \rightarrow$ find $f^2(x), f^3(x), f^4(x), \dots, f^n(x) \sim 2$

37 It is given that $f: x \rightarrow \frac{3}{x}, x \neq 0$.

(a) Find the iterated function $f^2(x), f^3(x)$ and $f^4(x)$.

[2 marks]

(b) Hence, find the value of

(i) $f^{28}(5)$,

[1 mark]

(ii) $f^{57}(5)$.

[1 mark]

[Forecast]

Answer :

(a)

(b) (i)

$$f^n(x) = x \Leftrightarrow f^{kn}(x) = x$$

(ii)

38 It is given functions $f(x) = p, f^2(x) = q, f^3(x) = r$ dan $f^4(x) = s$. If $f^4(x) = x$,

(a) find in terms of p, q, r , or s for :

(i) $f^{8n}(x)$, where $n = 1, 2, 3, \dots$,

[1 mark]

(ii) $f^{70}(x)$.

[1 mark]

(b) State the value of k , such that $f^k(x) = p$, where $26 \leq k < 33$ and k is an integer.

[1 mark]

[Forecast]

Answer :

(a) (i)

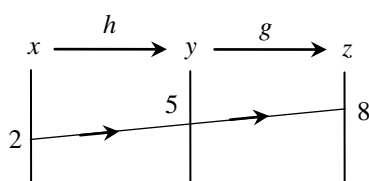
(ii)

(b)

1.3 Inverse functions
1.3.1 Describe inverse of a function.
1.3.2 Make and verify conjectures related to properties of inverse functions.
1.3.3 Determine the inverse functions.

\Rightarrow describe inverse function

- 39 In the diagram shows below, the function h maps x to y and the function g maps y to z .



Determine

(a) $h^{-1}(5)$,

(b) $gh(2)$.

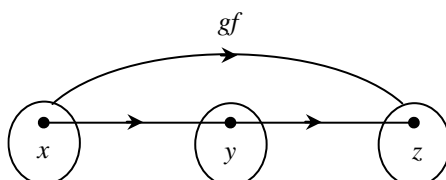
[2 marks] [2005, No.1]

Answer :

(a)

(b)

- 40 The diagram shows the composite function gf that maps x to z .



State

(a) the function that maps x to y .

(b) $g^{-1}(z)$.

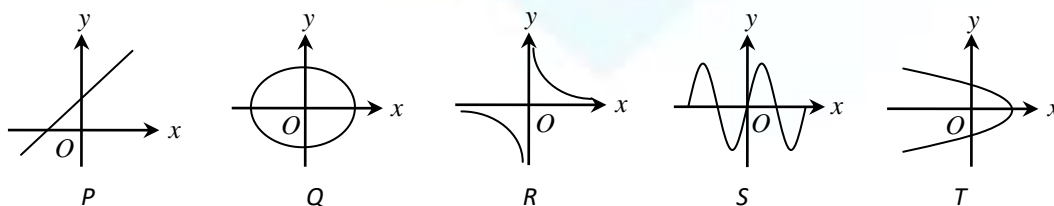
[2 marks] [2015, No.1]

Answer :

(a)

(b)

- 43 The diagram shows five graphs, P , Q , R , S and T .



(a) State the graph (s) which is /are not a function. Give reason for your answer.

(b) State the graph (s) which has / have inverse function. Give reason for your answer.

[4 marks] [Forecast]

Answer :

(a)

(b)

\Rightarrow **properties of inverse function**

44 The diagram in the answer space shows the graph $y = f(x)$.

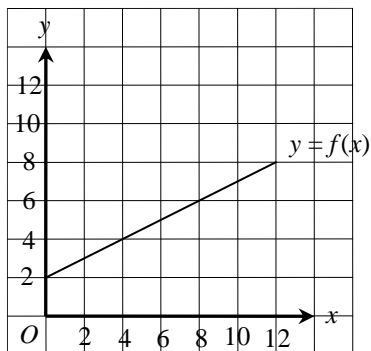
(a) On the diagram in the answer space, draw the inverse function of the graph.

(b) From the graph, state the value of x when $f(x) = f^{-1}(x)$.

[2 marks] [Forecast]

Answer :

(a)



(b)

45 The diagram in the answer space shows the graph $y = h(x)$.

(a) Find $h^{-1}(0)$.

(b) On the diagram in the answer space, draw the graph $y = h^{-1}(x)$.

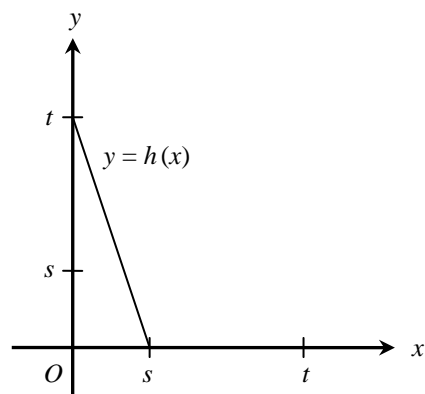
(c) Determine the domain for the graph $y = h^{-1}(x)$.

[3 marks] [Forecast]

Answer :

(a)

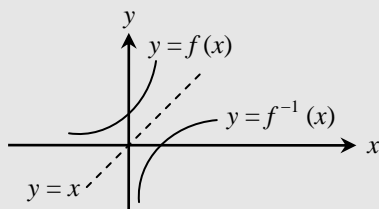
(b)



(c)

MIND think :

- the graph of $y = f(x)$ and $y = f^{-1}(x)$ are symmetry about the line $y = x$



- domain for $f^{-1}(x) = \text{range for } f(x)$
range for $f^{-1}(x) = \text{domain for } f(x)$

\Rightarrow **find inverse function 1a**

- 46** Given the functions $g : x \rightarrow 3x - 1$, find

- (a) $g(2)$,
(b) the value of p when $g^{-1}(p) = 11$.

(Ans : 32)
[3 marks] [2009, No.3]

Answer :

- (a) (b)

- 47** Given that $f : x \rightarrow x + 5$, find

- (a) $f(3)$,
(b) the value of k such that $2f^{-1}(k) = f(3)$.

(Ans : 9)
[3 marks] [2012, No.3]

Answer :

- (a) (b)

⇒ *find inverse function 1b*

- 48 Given that $g : x \rightarrow 5x + 1$ and $h : x \rightarrow x^2 - x + 3$. Find

(a) $g^{-1}(3)$.

(Ans : $\frac{2}{5}$)

(b) $hg(x)$.

(Ans : $25x^2 + 5x + 3$)

[4 marks] [2003, No.2]

Answer :

(a)

(b)

- 49 Given the functions $g : x \rightarrow 5x + 2$, and $h : x \rightarrow x^2 - 4x + 3$, , find

(a) $g^{-1}(6)$,

(Ans : $\frac{4}{5}$)

(b) $hg(x)$,

(Ans : $25x^2 - 1$)

[4 marks] [2008, No.2]

Answer :

(a)

(b)

⇒ *find inverse function 1c*

- 50 Given the functions $h^{-1} : x \rightarrow 4x + m$ and $h : x \rightarrow 2kx + \frac{5}{8}$, where m and k are constants, find the value of m and the value of k .

(Ans : $k = \frac{1}{8}$, $m = -\frac{5}{2}$)

[3 marks] [2004, No.2]

Answer :

- 51 The following information refer to the functions h and g .

$h : x \rightarrow 2x - 3$ $g : x \rightarrow 4x - 1$

Find $g h^{-1}(x)$.

(Ans : $2x + 5$)

[3 marks] [2005, No.3]

Answer :

52 Given the functions $g : x \rightarrow 2x + 1$ and $h : x \rightarrow 3x + 6$, find

(a) $g^{-1}(x)$,

(b) $hg^{-1}(9)$,

(Ans : 18)

[3 marks] [2010, No.2]

Answer :

(a)

(b)

53 Given the function $g : x \rightarrow 2x - 8$, find

(a) $g^{-1}(x)$,

(b) the value of p such that $g^2\left(\frac{3p}{2}\right) = 30$.

(Ans : 9)

[4 marks] [2017, No.10]

Answer :

(a)

(b)

\Rightarrow find inverse function 2

54 The function w is defined as $w(x) = \frac{5}{2-x}$, $x \neq 2$. Find

(a) $w^{-1}(x)$,

(Ans : $\frac{2x-5}{x}$)

(b) $w^{-1}(4)$.

[3 marks] [2005, No.2]

Answer :

(a)

(b)

- 55 The inverse function h^{-1} is defined by $h^{-1} : x \rightarrow \frac{2}{3-x}$, $x \neq 3$. Find

(a) $h(x)$.

(Ans : $\frac{3x-2}{x}$)

(b) the value of x such that $h(x) = -5$.

(Ans : $\frac{1}{4}$)

[4 marks] [2011, No.3]

Answer :

(a)

(b)

\Rightarrow using concept inverse / given $f(x)$, $fg(x) \rightarrow$ find $g(x)$

- 56 Given the functions $f : x \rightarrow 2x - 1$ and $f^{-1}g : x \rightarrow \frac{x-2}{2x-6}$, $x \neq 3$. Find the value of $gf(3)$. (Ans : $\frac{1}{2}$)

[4 marks] [Forecast]

Answer :

$$\begin{array}{c} f^{-1}(y) = x \\ \updownarrow \\ f(x) = y \end{array}$$

\Rightarrow using $ff^{-1}(x) = x$ @ $f^{-1}f(x) = x$ / given $g(x)$, $fg(x) \rightarrow$ find $f(x)$

- 57 Given the function $g : x \rightarrow 3x + 1$ and $fg : x \rightarrow 9x^2 + 6x - 4$, find

(a) $g^{-1}(x)$,

(b) $f(x)$.

(Ans : $x^2 - 5$)

[3 marks] [2016, No.13]

Answer :

(a)

(b)

$$\begin{array}{c} f[f^{-1}(x)] = x \\ @ \\ f^{-1}[f(x)] = x \end{array}$$

- 58 Given $f(x) = \frac{3}{x-p}$, $x \neq p$ and a function g such that $g^{-1}g(p-3) = f(p-4)$. Find the value of p .
(Ans : $\frac{9}{4}$)

[3 marks] [Forecast]

Answer :

- 59 Given $f(x) = \frac{1}{x-2}$, $x \neq 2$ and $g(x) = \frac{1}{x} + 2$.

(a) Find $fg(x)$. (Ans : x)

(b) State the relation between functions $f(x)$ and $g(x)$.

[3 marks] [Forecast]

Answer :

(a)

(b)

 \Rightarrow others

- 60 Given that $f: x \rightarrow \sqrt{x}$ and $g: x \rightarrow x+4$. Express in terms of f or g or both f and g for :

(a) $x \rightarrow \sqrt{x+4}$,

(b) $x \rightarrow x^2 + 4$,

(c) $x \rightarrow x+8$

[3 marks] [Forecast]

Answer :

(a)

(b)

(c)

PAPER 2 **\Rightarrow Part A \rightarrow 6 – 7 marks**

- 61 Given that $f(x) \rightarrow 3x - 2$ and $g(x) = \frac{x}{5} + 1$, find

(a) $f^{-1}(x)$,

[1 mark]

(b) $f^{-1}g(x)$,

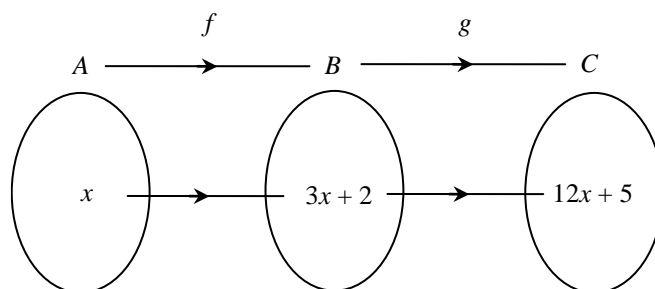
(Ans : $\frac{x+15}{15}$) [2 marks]

(c) $h(x)$ such that $hg(x) = 2x + 6$.

(Ans : $10x - 4$) [3 marks]
[2006, No.2]

Answer :

- 62 In the diagram, the function f maps set A to set B and the function g maps set B to set C .



Find

- (a) in terms of x , function

(i) which maps set B to set A .

(ii) $g(x)$.

(Ans : $4x - 3$)

[5 marks]

- (b) the value of x such that $fg(x) = 8x + 1$.

(Ans : 2) [2 marks]

[2014, No.3]

Answer :

63 It is given that $f: x \rightarrow 2x - 3$ dan $g: x \rightarrow 1 - 3x$. Find

(a) (i) $g(5)$,

(ii) the value of m if $f(m+2) = \frac{1}{7}g(5)$, (Ans : $-\frac{3}{2}$)

(iii) $gf(x)$. (Ans : $-6x + 10$)

[5 marks]

(b) Hence, sketch the graph of $y = |gf(x)|$ for $-1 \leq x \leq 3$. State the range of y .

(Ans : $0 \leq y \leq 16$) [3 marks]

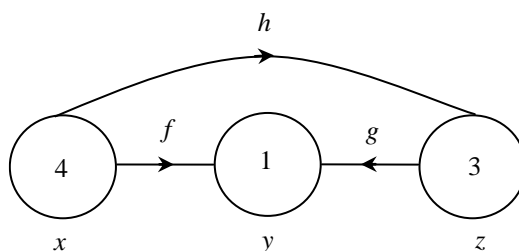
[2018, No.2]

Answer :

FORECAST

64 The diagram shows the mapping from x to y under $f: x \rightarrow px + q$ and the mapping from z to y under

$$g: z \rightarrow \frac{8}{3z + q}, \quad z \neq -\frac{q}{3}.$$



Find

(a) the value of p and the value of q ,

(Ans : $p = \frac{1}{2}$, $q = -1$) [3 marks]

(b) the function which map y to z ,

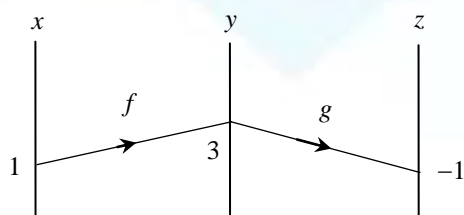
(Ans : $\frac{y+8}{3y}$) [2 marks]

(c) the function which map x to z .

(Ans : $\frac{x+14}{3x-6}$) [2 marks]

Answer :

- 65 In the diagram, the function $f(x) = ax + b$ maps x to y and the function $g(y) = \frac{a}{1-y}$, $y \neq 1$ maps y to z .



Find

- the values of a and b ,
- the function which map y to x ,
- the function that map z to x .

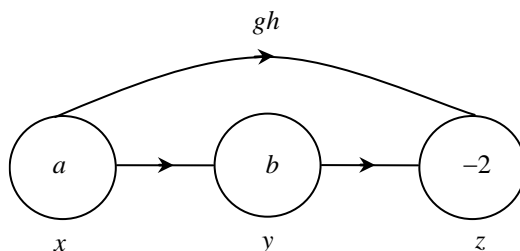
(Ans : $a = 2$, $b = 1$) [3 marks]

(Ans : $\frac{y-1}{2}$) [2 marks]

(Ans : $-\frac{1}{z}$) [2 marks]

Answer :

- 66 In the diagram, the function f maps x to y and the function g maps y to z .



Given that $h^{-1}(x) = x + 5$ and $gh(x) = 3x - 11$. Find

- in terms of x , function

- $h(x)$,

[1 mark]

- $g(x)$,

(Ans : $3x + 4$) [2 marks]

- the value of a and of b .

(Ans : $a = 3$, $b = -2$) [3 marks]

Answer :

67 Given that $f: x \rightarrow 3 + 2x$ and $g: x \rightarrow \frac{8}{x+1}$, $x \neq -1$. Express the following in the same form :

(a) f^{-1} , [1 mark]

(b) g^{-1} , (Ans : $\frac{8-x}{x}$) [2 marks]

(c) $g^{-1}f^{-1}$, (Ans : $\frac{19-x}{x-3}$) [2 marks]

(d) $(fg)^{-1}$, [1 mark]

Answer :

68 Given that $f: x \rightarrow \frac{1+x}{1-x}$, $x \neq 1$. Express the following in the same form :

(a) f^2 , (Ans : $-\frac{1}{x}$) [2 marks]

(b) f^3 , (Ans : $\frac{x-1}{x+1}$) [2 marks]

(c) f^4 , (Ans : x) [2 marks]

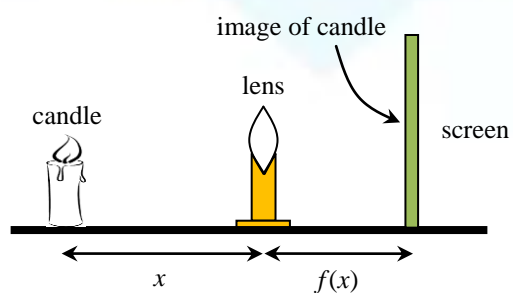
(d) f^{37} , [1 mark]

(e) f^{95} . [1 mark]

Answer :

$$(fg)^{-1}(x) = g^{-1}f^{-1}(x)$$

- 69 Chelsea runs a physics experiment about the image generated by convex lenses as shown in the following figure.

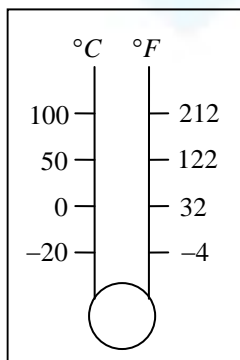


It is found that the image of candle that produce is given by $f(x) = \frac{10x}{x-10}$, $x \neq k$. Find

- the value of k , [1 mark]
- the distance of the image of the candle, if the distance of the candle from the lens is 14 cm, (Ans : 35) [2 marks]
- the distance of the candle to the lens if the lens is located in the middle between the candle and its image. (Ans : 20) [3 marks]

Answer :

- 70** Jackley runs a chemistry experiment for preparation of the Science Practical Examination. The diagram shows the scale for temperatures in Celsius ($^{\circ}\text{C}$) and Fahrenheit ($^{\circ}\text{F}$) on a thermometer. The relation between temperature in $x^{\circ}\text{C}$ and $y^{\circ}\text{F}$ is given by $y = 1.8x + k$, where k is a constant.



- (a) Find the value of k . (Ans : 32) [2 marks]
- (b) If the temperature of today is 32°C , what is its temperature in $^{\circ}\text{F}$ (Ans : 89.6) [2 marks]
- (c) Form a function that allows us to change the temperature in the unit of Fahrenheit ($^{\circ}\text{F}$) to degrees Celsius ($^{\circ}\text{C}$). (Ans : $x = \frac{y-32}{1.8}$) [2 marks]

Answer :

CONTINUOUS EXERCISES

71 The inverse function $f^{-1}(x)$ is defined by $f^{-1}: x \rightarrow \frac{2}{5}(x^2 - 10), x \geq 0$. Find

(a) $f(x)$, [1 mark]

(b) the domain of $f(x)$. (Answer : $x \geq -4$) [1 mark]

Answer :

(a)

(b)

72 A function h is defined by $h : x \rightarrow 2 - \frac{a}{x}$, where $x \neq 0$, and a is a constant. Given

$\frac{1}{2}h^2(2) + h^{-1}(-1) = -1$, find the possible values of a . (Ans : -8, 3)

[4 marks]

Answer :

- 73** The handbags produced by the manufacturer will be sent to retailer before selling them to customers. During a sales promotion, the manufacturer offer a RM75 rebate from RM x to the retailer. While retailer offers a 5% discount from RM x to the customers.

- (a) Find the price that customer has to pay for a handbag in term of x . [Ans : $g(x) = 0.95(x + 75)$] [3 marks]
- (b) If a customer buys a new handbag at the price of RM499.90.
- (i) find the price from the manufacturer, (Ans : 526.21) [2 marks]
- (ii) calculate the profit gained by the retailer. (Ans : 48.69) [2 marks]

Answer :

- 74** It is given that $x = \frac{7 + h(x)}{3 - h(x)}$, where $h(x) \neq 0$, find $h^{-1}(2)$. (Ans : 9)

[3 marks]

Answer :

- 75** The function f and g are defined as $f : x \rightarrow x - 3$ and $g : x \rightarrow x^2$ respectively. Find

- (a) $gf(x)$, .
- (b) $h(x)$ such that $hgff(x) = x^2 - 6x + 3$. [Ans : $h(x) = x - 6$] [3 marks]

Answer :

- (a) (b)

QUADRATIC EQUATIONS AND QUADRATIC FUNCTIONS

- ONE PAGE NOTE (OPN) - WORKSHEET

**Encik AK Sapri bin Yaakob
Puan Tan Woon Shin**

ONE PAGE NOTES

“QUADRATIC FUNCTIONS”

Quadratic Equations
 $y = ax^2 + bx + c$
 $a > 0$ $a < 0$
 $c > 0$ $c < 0$
 $(a=3)$ $(a=-1)$

Quadratic Functions
 $f(x) = ax^2 + bx + c$
 $a > 0$ $a < 0$

find the roots of the quadratic equation (the value of x that satisfies the equation)

using calculator
 (CASIO fx-570MS, CANON F-570SG)
 mode mode mode
 1 1 2
 $a = b = c =$
 $\rightarrow x_1 = ? \quad x_2 = ?$
NOTE : if appear x only
 $\rightarrow x_1 = x_2$

using formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Form Quadratic Equations from given Roots

roots = α, β
 $(x - \alpha)(x - \beta) = 0$
SOR = sum of roots
POR = product of roots
 $x^2 - (\text{SOR})x + (\text{POR}) = 0 \sim \text{SOR} = \alpha + \beta \quad \& \quad \text{POR} = \alpha\beta$
 $\text{SOR} = -\left(\frac{b}{a}\right)$ and $\text{POR} = \frac{c}{a}$

completing the square
 $x^2 + bx + c = 0$
 $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = 0$
 $x^2 - bx + c = 0$
 $\left(x - \frac{b}{2}\right)^2 - \left(-\frac{b}{2}\right)^2 + c = 0$
 $ax^2 + bx + c = 0$
 $a \left[x^2 + \frac{b}{a}x \right] + c = 0$
 $a \left[\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 \right] + c = 0$
 $a \left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c = 0$

$y = ax^2 + bx + c \rightarrow c = 0$
 $x = -\frac{b}{2a}$
 symmetry $(-)$ \rightarrow symmetry $(+)$
 $a < 0$ $a > 0$
 $b < 0$ $b > 0$
 If a and b are same sign \rightarrow symmetry to the **left** of y -axis
 If a and b are different sign \rightarrow symmetry to the **right** of y -axis

Quadratic Inequalities
 $y = f(x) \rightarrow a > 0$
 $f(x) < 0 \rightarrow \alpha < x < \beta$
 $f(x) > 0 \rightarrow x < \alpha, x > \beta$
 $f(x) \text{ intersect } x\text{-axis at 2 distinct points}$
 $[2 \text{ different / distinct real roots}]$
 $b^2 - 4ac > 0$ (positive discriminant)

Vertex Form
 $f(x) = a(x - h)^2 + k$
 • is reflected at x -axis
 $f(x) = -a(x - h)^2 - k$
 • is reflected at y -axis
 $f(x) = a(x + h)^2 - k$
 $a > 0$ parabola
 [equation of tangent]
 $y = k$
 (h, k) [minimum point]
 $x = h$
 [equation of axis of symmetry]

General Form
 $f(x) = ax^2 + bx + c$
 $a < 0$, maximum point
 $\left[-\frac{b}{2a}, f\left(-\frac{b}{2a}\right) \right]$
 $f\left(-\frac{b}{2a}\right) = \frac{4ac - b^2}{4a}$
Intercept Form
 $f(x) = a(x - p)(x - q)$
 Turning point / vertex
 $\left[\frac{p+q}{2}, f\left(\frac{p+q}{2}\right) \right]$
 $k = f\left(-\frac{b}{2a}\right) = \frac{4ac - b^2}{4a}$
 $f\left(\frac{p+q}{2}\right) = f\left(-\frac{b}{2a}\right)$

domain : $-\infty \leq x \leq \infty$
 relation \sim many to one
 domain : $x \leq h$ @ $x \geq h$
 relation \sim one to one
 $h = -\left(\frac{b}{2a}\right) = \frac{p+q}{2}$

$y = ax^2 + bx + c$
 $a > 0$ and $c > 0$
 $a > 0$ and $c < 0$
 If a and c are same sign \rightarrow roots are same sign
 If a and c are different sign \rightarrow roots are different sign

$f(x) = (x - \lambda)^2$
 $f(x) = -(x - \lambda)^2$
 $f(x) \text{ touches } x\text{-axis at only 1 point}$
 $x\text{-axis is the tangent to } f(x)$
 $[2 \text{ equal real roots}]$
 $b^2 - 4ac = 0$

$f(x) \text{ does not intersect } x\text{-axis}$
 $f(x) \text{ is always above @ below } x\text{-axis}$
 $f(x) \text{ is always positive @ negative}$
 $[\text{no real roots / imaginary roots}]$
 $b^2 - 4ac < 0$ (negative)
REMARKS :
 real roots
 $b^2 - 4ac \geq 0$

Sketch the graph of a quadratic function
 (i) find the minimum @ maximum point
 (ii) find the y -intercept
 \rightarrow value of c @ substitute $x = 0, f(0)$
 (iii) find the x -intercept
 \rightarrow substitute $y = 0 \sim$ calculator (EQN)
 (iv) find the value of $f(m)$ and $f(n)$ for the given domain : $m \leq x \leq n$.

WORKSHEET
TOPIC 2 : QUADRATIC FUNCTIONS
[2 – 4 questions → 4 – 13 marks]

Revision

\Rightarrow roots of a quadratic equation \sim values of x (variable / unknown) that satisfy the equation

- 1** It is given that -1 is one of the roots of the quadratic equation $x^2 - 4x - p = 0$. Find the value of p . (Ans : 5)

[2 marks] [2008, No.4]

Answer :

- 2** It is given -7 is one of the roots of the quadratic equation $(x + k)^2 = 16$, where k is a constant. Find the values of k . (Ans : 3, 11)

[2 marks] [2015, No.5]

Answer :

- 3** If $-\frac{3}{2}$ is one root of the equation $2x^2 - 7x = 18 + px$, where p is constants. Find

(a) the value of p , (Ans : 2)

(b) the other root of the equation. (Ans : 6)

[4 marks] [Forecast]

Answer :

(a)

(b)

MIND think :

- The roots of a quadratic equation :

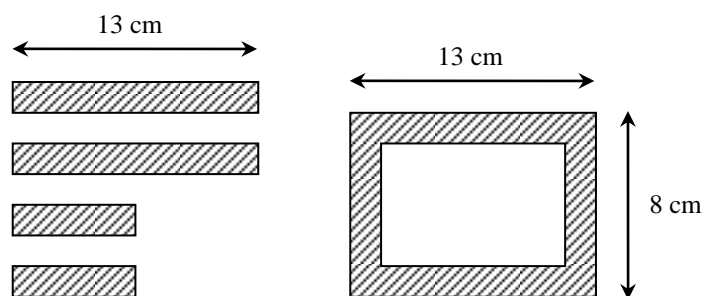
\rightarrow the values of the variable / unknown (x) that a quadratic equation.

- 4 If $p - 1$ and $q + 2$ are the roots of the quadratic equation $x^2 + 5x + 4 = 0$. Find the values of p and q .
 (Ans : $p = -3, q = -3$ and $p = 0, q = -6$)
 [4 marks] [Forecast]

Answer :

⇒ **problems involving quadratic equations**

- 5 The diagram shows the front view of four pieces of wood with the same width. The total front area of the four pieces of wood is 20 cm^2 . The four pieces of wood are used to produce a rectangular photo frame as shown in Diagram.



Calculate the width, in cm, of the wood.

(Ans : $0.5 / \frac{1}{2}$)

[3 marks] [2016, No.25]

Answer :

MIND think :

- factorization using calculator : **CASIO** fx-570MS, **CANON** F-570SG, **OLYMPIA** ES-570MS . . .

Example : $3x^2 - 5x - 12 = 0 \rightarrow$ [mode] [mode] [mode] [1] [▶] [2]

$a ? 3 = b ? -5 = c ? -12 \Rightarrow x_1 = 3$

$= \rightarrow x_2 = -1.33 \dots$ [shift] [a b/c] $-\frac{4}{3}$

NOTE : if appear x only $\rightarrow x_1 = x_2$ (2 equal roots)
 $(x - \text{root})^2 = 0$

\downarrow
 $(x - 3)(3x + 4) = 0$

- 6 The age of Gloria is five times of her son. Four years ago, the product of their ages is 52. Find their ages now.
(Ans : 30, 6)
[4 marks] [Forecast]

Answer :

- 7 Annabella bought a certain number of pens for RM60. If each pen had a discount of 20 cents, she could have bought 10 more pens for the same amount of money.

Find the number of pens that Annabella bought.

(Ans : 50)
[4 marks] [Forecast]

Answer :

- 8 During the training session for the Olympics, Kenney cycled 60 km from Kota Kinabalu to Kota Belut at a constant speed. If he increases the speed by 10 kmh^{-1} , he will arrive Kota Belut half an hour earlier.

(a) Form a quadratic equation in terms of x , which shows the movement of Kenney.
(Ans : $x^2 + 10x - 1200 = 0$) [2 marks]

(b) Calculate the original speed of Kenney.
(Ans : 30) [2 marks]
[Forecast]

Answer :

- 9 Boeoon wants to fence his vegetable garden using fencing wire. The area of his vegetable garden is 1787.5 m^2 . The length of the garden is 10 m less than twice its width. The bulk price of a roll of 15 m long fencing wire is RM150, while the retail price of 1 m is RM12.50. Determine the lowest cost, in RM, needed to buy the fencing wire.

(Ans : 1775)

[4 marks] [Forecast]

Answer :

2.1 Quadratic equations and inequalities

2.1.1 Solve quadratic equations using the method of completing the square and formula.

\Rightarrow **determine the roots of a quadratic equation ~ formula**

- 10 Solve the quadratic equation $2x(x - 4) = (1 - x)(x + 2)$. Give your answer correct to four significant figures.

(Ans : -0.2573, 2.591)

[3 marks] [2003, No.3]

Answer :

- 11 Solve the quadratic equation $x(2x - 5) = 2x - 1$. Give your answer correct to three decimal places.

(Ans : 0.149, 3.351)

[3 marks] [2005, No.5]

Answer :

- 12 Eva thrown a ball from a with the height of ball, h m from the ground after t second is given by $h = -4.9t^2 + 18t + 1.5$. Determine whether the ball can reach a height of 15 m from the ground. Give reason for your answer.

(Ans : 2.62, 1.05)

[3 marks] [Forecast]

Answer :

MIND think :

- solve by using formula : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example : $3x^2 - 5x - 12 = 0 \rightarrow x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-12)}}{2(3)} \rightarrow x = \frac{5 \pm \sqrt{169}}{6} \rightarrow x = 3, -\frac{4}{3}$

\Rightarrow **determine the roots of a quadratic equation ~ completing the square**

- 13** Solve the quadratic equation $2x^2 - 6x = x(x + 3) - 4$ by using completing the square. Give your answer correct to three decimal places.
(Ans : 8.531, 0.469)
[3 marks] [Forecast]

Answer :

- 14** Solve the quadratic equation $2x(x - 3) = (x + 4)(1 - x)$ by using completing the square. Give your answer correct to three significant figures.
(Ans : -0.758, 1.76)
[3 marks] [Forecast]

Answer :

MIND think :**completing the square**

$$x^2 + bx + c = 0$$

$$(x + \frac{b}{2})^2 - (\frac{b}{2})^2 + c = 0$$

$$x^2 - bx + c = 0$$

$$(x - \frac{b}{2})^2 - (-\frac{b}{2})^2 + c = 0$$

$$ax^2 + bx + c = 0$$

$$a[x^2 + \frac{b}{a}x] + c = 0$$

$$a[(x + \frac{b}{2a})^2 - (\frac{b}{2a})^2] + c = 0$$

$$a(x + \frac{b}{2a})^2 - \frac{b^2}{4a} + c = 0$$

2.1.2 Form quadratic equations from given roots.

\Rightarrow form a quadratic equation from given roots ~ 1

- 15** Form the quadratic equation which has the roots -3 and $\frac{1}{2}$. Give your answer in the form $ax^2 + bx + c = 0$, where a , b , and c are constants. (Ans : $2x^2 + 5x - 3 = 0$)
[2 marks] [2004, No.4]

Answer :

- 16** Form the quadratic equation which has the root $\frac{1}{2}$. Give your answer in the form $ax^2 + bx + c = 0$, where a , b , and c are constants. (Ans : $4x^2 - 4x + 1 = 0$)
[2 marks]
[Forecast]

Answer :

- 17** Given that one of root of a quadratic equation is reciprocal to the other. If 5 is one root of the equation, form the quadratic equation. Give your answer in the form $ax^2 + bx + c = 0$, where a , b , and c are constants. (Ans : $5x^2 - 26x + 5 = 0$)
[2 marks]
[Forecast]

Answer :

MIND think :

- A quadratic equation, $ax^2 + bx + c = 0$, with roots, α and β .
 $\sim (x - \alpha)(x - \beta) = 0$
 $\sim x^2 - (\alpha + \beta)x + \alpha\beta = 0$ @ $x^2 - (\text{SOR})x + \text{POR} = 0 \Rightarrow \text{SOR} = \text{sum of roots, POR} = \text{product of roots}$
 $\sim \text{SOR} = \alpha + \beta = -\frac{b}{a}, \text{POR} = \alpha\beta = \frac{c}{a}$

⇒ **form a quadratic equation from given roots ~ 2**

- 18** It is given that the quadratic equation $3x^2 + 8x + 7 = 0$ has roots α and β . Form a quadratic equation with roots 3α and 3β .
(Ans : $x^2 + 8x + 21 = 0$)
[3 marks] [2016, No.17]

Answer :

- 19** It is given that m and n are the roots of the quadratic equation $2x^2 - 5x - 3 = 0$. Form quadratic equation with roots $m - 1$ and $n - 1$.
(Ans : $2x^2 - x - 6 = 0$)
[3 marks] [Forecast]

Answer :

- 20** It is given that p and q are the roots of the quadratic equation $2x^2 = 4 - x$. Form quadratic equation with roots p^2 and q^2 . [Note : $a^2 + b^2 = (a + b)^2 - 2ab$]
(Ans : $4x^2 - 17x + 16 = 0$)
[3 marks] [Forecast]

Answer :

- 21** Given α and β are the roots of quadratic equation $x^2 + 6x + 3 = 0$. Find the value of :

(a) $\alpha + \beta$,

(b) $\frac{\alpha^2 + \beta^2}{\alpha\beta}$.

(Ans : 10)

[3 marks] [Forecast]

Answer :

(a)

(b)

- 22 The roots of the equation $x^2 + 3x - 4 = 0$ are α and β , and the roots of the equation $x^2 + 6x + p = 0$ are $\frac{k}{\alpha}$ and $\frac{k}{\beta}$, find the values of k and p . (Ans : $k = -8$, $p = -16$)

[4 marks] [clon 2015, K2, 5(b)]

Answer :

\Rightarrow form a quadratic equation from given roots ~ 3

- 23 It is given that 3 and $m + 4$ are the roots of the quadratic equation $x^2 + (n - 1)x + 6 = 0$, where m and n are constant. Find the value of m and of n . (Ans : $m = -2$, $n = -4$)

[3 marks] [2012, No.4]

Answer :

- 24 Given the quadratic equation $2x^2 + mx - 5 = 0$, where m is a constant, find the value of m if

(a) one of the roots of the equation is 2, (Ans : $-\frac{3}{2}$)

(b) the sum of roots of the equation is -4 . (Ans : 8)

[4 marks] [2014, No.5]

Answer :

(a)

(b)

- 25 It is given that the quadratic equation $hx^2 - 3x + k = 0$, where h and k are constants, has roots β and 2β . Express h in terms of k . (Ans : $\frac{2}{k}$)

[3 marks] [2018, No.21]

Answer :

- 26 One of the root of the equation $x\left(6 - \frac{n}{x}\right) = 2x^2 + 9$ is one third the other. Find the value of n .
(Ans : $-\frac{45}{8}$)

[3 marks] [Forecast]

Answer :

- 27 Given α and β are the roots of the equation $(1 - 2x)(x + 5) = k$ and $\alpha = 2\beta$. Find the value of k .
(Ans : 14)

[3 marks] [Forecast]

Answer :

- 28 Given $\frac{\alpha}{2}$ and $\frac{\beta}{2}$ are the roots of the equation $kx(x - 1) = 2m - x$. If $\alpha + \beta = 6$ and $\alpha\beta = 3$, find the value of k and of m .
(Ans : $k = -\frac{1}{2}$, $m = \frac{3}{16}$)

[3 marks] [Forecast]

Answer :

2.1.3 Solve quadratic inequalities.

\Rightarrow **quadratic inequalities**

- 29 Find the range of values of x for which $x(x - 4) \leq 12$.

(Ans : $-2 \leq x \leq 6$)

[3 marks] [2004, No.5]

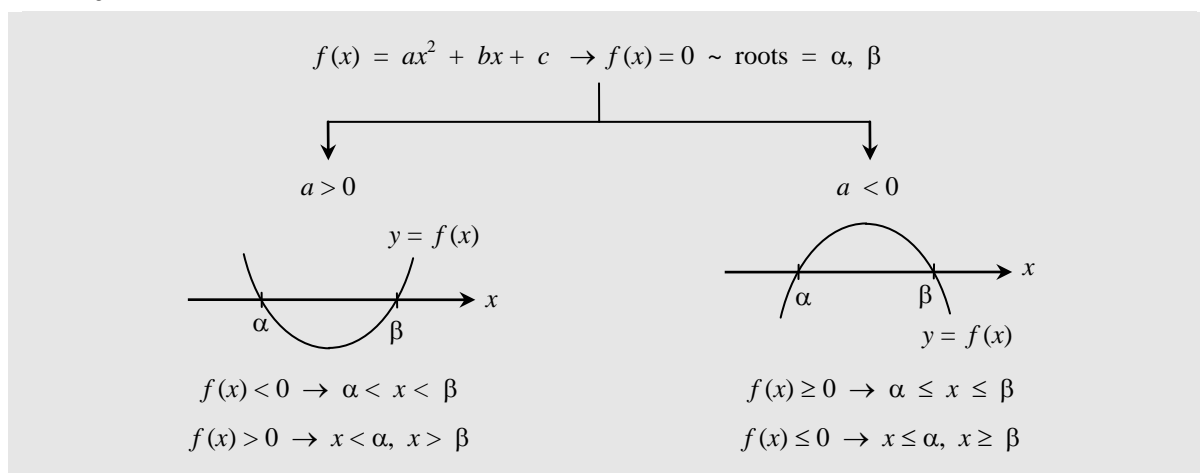
Answer :

- 30 Find the range of the values of x for $(2x - 1)(x + 4) > 4 + x$.

(Ans : $x < -4, x > 1$)
[2 marks] [2006, No.5]

Answer :

MIND think :



- 31 Find the range of the values of x for $(x - 3)^2 < 5 - x$.

(Ans : $1 < x < 4$)
[3 marks] [2008, No.6]

Answer :

- 32 Find the range of value of x for $3x^2 - 5x - 16 \leq x(2x + 1)$.

(Ans : $-2 \leq x \leq 8$)
[3 marks] [2011, No.6]

Answer :

- 33 Given that $f(x) = -3x^2 + 2x + 13$, find the range of values of x for $f(x) \leq 5$.

(Ans : $x \leq -\frac{4}{3}, x \geq 2$)
[3 marks] [2012, No.6]

Answer :

- 34** Find the range of values of x such that the quadratic function $f(x) = 6 + 5x - x^2$ is negative.
 (Ans : $x < -1, x > 6$)
 [3 marks] [2017, No.12]

Answer :

\Rightarrow problems involving quadratic inequalities

- 35** Firdaus has a rectangular plywood with dimension $3x$ metre in length and $2x$ metre in width. He cut part of the plywood into square shape with sides of x metre to make a table surface. Find the range of values of x if the remaining area of the plywood is at least $(x^2 + 4)$ metre².
 (Ans : $x \geq 1$)
 [3 marks] [2018, No.19]

Answer :

2.2.1 Relate types of roots of quadratic equations to the discriminant value.

2.2.2 Solve problems involving types of roots of quadratic equations.

\Rightarrow types of roots of the quadratic equation ~ 1

36 It is given that quadratic equation $x(x - 5) = 4$

- (a) Express the equation in the form $ax^2 + bx + c = 0$.
- (b) State the sum of roots of the equation.
- (c) Determine the type of roots of the equation.

[3 marks] [2013, No.4]

Answer :

(a) (c)

(b)

37 In the answer space, match the type of roots for each of the given quadratic equation.

[3 marks]

[Forecast]

Answer:

$$x^2 - \frac{4}{3}x + \frac{4}{9} = 0 \quad \bullet$$

• 2 different real roots

$$x(1 - 3x) = 2 \quad \bullet$$

• 2 equal real roots

$$3x^2 = 2x + 7 \quad \bullet$$

• no real roots / imaginary roots

MIND think :

Types of Roots of Quadratic Equations → by determine the value of discriminant, $b^2 - 4ac$

two different roots

@
intersects at 2 distinct
points

↓
 $b^2 - 4ac > 0$

two equal roots @
tangent

↓
 $b^2 - 4ac = 0$

no roots /
imaginary roots
@ does not
intersect

↓
 $b^2 - 4ac < 0$

real roots

↓
 $b^2 - 4ac \geq 0$

⇒ **types of roots of the quadratic equation ~ 2**

- 38** The quadratic equation $x(x + 1) = px - 4$ is satisfied by two distinct values of x . Find the range of values of p .
(Ans : $p < -3$, $p > 5$)
3 marks] [2003, No.4]

Answer :

- 39** A quadratic equation $x^2 + px + 9 = 2x$ has two equal roots. Find the possible values of p .
(Ans : -4 , 8)

[4 marks] [2006, No.3]

Answer :

- 40** (a) Solve the following quadratic equation :

$$3x^2 + 5x - 2 = 0$$

(Ans : $\frac{1}{3}$, -2)

- (b) The quadratic equation $hx^2 + kx + 3 = 0$, where h and k are constants, has two equal roots.
Express h in terms of k .

(Ans : $h = \frac{k^2}{12}$)

[4 marks] [2007, No.4]

Answer :

(a)

(b)

- 41** The quadratic equation $x^2 + x = 2px - p^2$, where p is a constant, has two different roots. Find the range of values of p .
(Ans : $p < \frac{1}{4}$)

[3 marks] [2009, No.4]

Answer :

- 42** The quadratic equation $(1 - p)x^2 - 6x + 10 = 0$, where p is a constant, has two different roots. Find the range of values of p .
(Ans : $p > \frac{1}{10}$)

[3 marks] [2010, No.5]

Answer :

- 43** The quadratic equation $mx^2 + (1 + 2m)x + m - 1 = 0$ has two equal roots. Find the value of m .
(Ans : $-\frac{1}{8}$)

[3 marks] [2011, No.4]

Answer :

- 44** A quadratic equation $x(x - 4) = h - 2k$, where h and k are constants, has two equal roots. Express h in terms of k .
(Ans : $h = 2k - 4$)

[3 marks] [2012, No.5]

Answer :

- 45** Given the quadratic equation $(1 - a)x^2 - 2x + 5 = 0$ has no roots, find the range of values of a .
(Ans : $a < \frac{4}{5}$)

[2 marks] [2014, No.3]

Answer :

- 46 (a) It is given that one of the roots of the quadratic equation $x^2 + (p + 3)x - p^2 = 0$, where p is a constant, is negative of the other. Find the value of the product of roots.
(Ans : -9) [2 marks]

- (b) It is given that the quadratic equation $mx^2 - 5nx + 4m = 0$, where m and n are constants, has two equal roots. Find $m : n$.
(Ans : 5 : 4) [2 marks]
[2017, No.13]

Answer :

- (a) (b)

- 47 It is given that quadratic equation $(px)^2 + 5qx + 4 = 0$ has two equal roots while the quadratic equation $hx^2 - x + p = 0$ has no roots, where p , q and h are constants. Express the range of q in terms of h .
(Ans : $q > \frac{1}{5h}$, $q < -\frac{1}{5h}$)
[3 marks] [2019, No.9]

Answer :

\Rightarrow types of roots of the quadratic equation ~ 3

- 48 The straight line $y = 5x - 1$ does not intersect the curve $y = 2x^2 + x + p$. Find the range of values of p .
(Ans : $p > 1$)
[4 marks] [2005, No.4]

Answer :

- 49 It is given that the curve $y = (p - 2)x^2 - x + 7$, where p is a constant, intersects with the straight line $y = 3x + 5$ at two points. Find the range of values of p .
(Ans : $p < 4$)
[3 marks] [2018, No.20]

Answer :

- 50 Show that the straight line $y = x + k$ intersects the curve $x^2 + y^2 - 5x = 4$ at two distinct points if $4k^2 + 20k - 57 < 0$

[3 marks] [Forecast]

Answer :

- 51 Given that the straight line $y = mx + m - 2$ is the tangent to the curve $y = x^2 + 3x + 1$. Find the possible values of m .

(Ans : -1, 3)

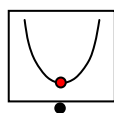
[3 marks] [Forecast]

Answer :

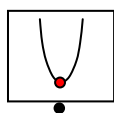
2.3 Quadratic Functions

2.3.1 Analyse and make generalisation about the effects of changes of a , b and c and in $f(x) = ax^2 + bx + c$ towards the shape and position of the graph.

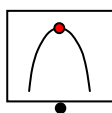
- 52 The diagram shows the graphs, $f(x) = ax^2 + bx + c$. Match the following graphs with the possible values of a .



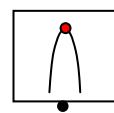
$$a = -5$$



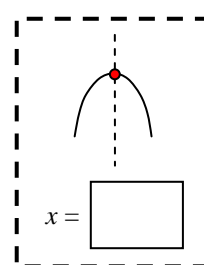
$$a = -1$$



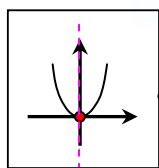
$$a = \frac{1}{2}$$



$$a = 3$$

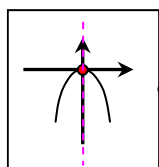
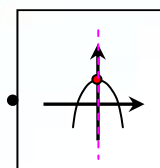


- 53 Match the following quadratic functions with the correct graph.



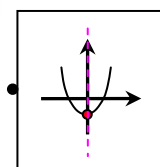
$$f(x) = ax^2 \sim a > 0$$

$$f(x) = ax^2 \sim a < 0$$

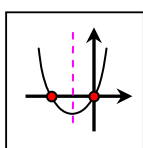


$$f(x) = ax^2 + c \sim a > 0, c < 0$$

$$f(x) = ax^2 + c \sim a < 0, c > 0$$

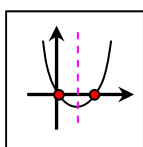
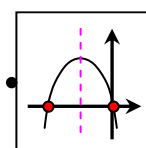


- 54 Match the following quadratic functions with the correct graph.



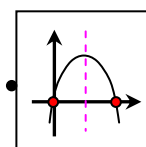
$$f(x) = ax^2 + bx \sim a > 0, b > 0$$

$$f(x) = ax^2 + bx \sim a > 0, b < 0$$

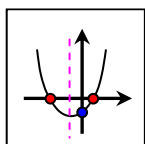
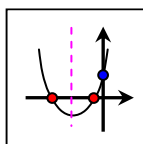


$$f(x) = ax^2 + bx \sim a < 0, b > 0$$

$$f(x) = ax^2 + bx \sim a < 0, b < 0$$

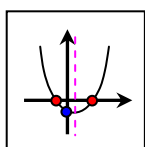
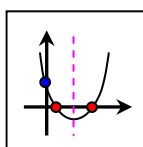
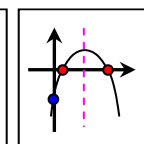
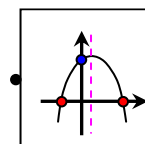


- 55 Match the following quadratic functions with the correct graph.



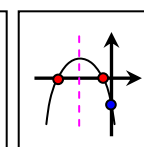
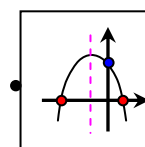
$$f(x) = ax^2 + bx + c \sim a > 0, b > 0$$

$$f(x) = ax^2 + bx + c \sim a > 0, b < 0$$



$$f(x) = ax^2 + bx + c \sim a < 0, b > 0$$

$$f(x) = ax^2 + bx + c \sim a < 0, b < 0$$



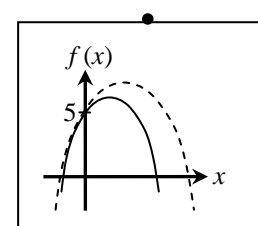
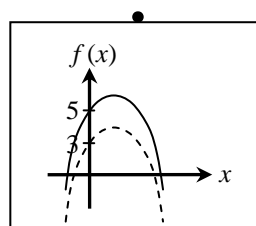
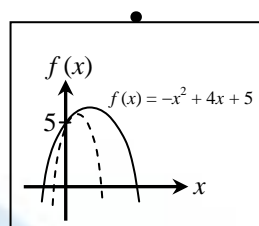
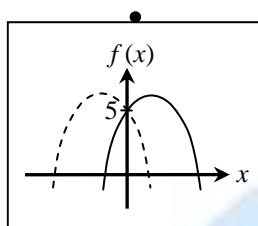
- 56 The diagram shows the graph for $f(x) = -x^2 + 4x + 5$, where $a = -1$, $b = 4$ and $c = 5$. Match the graph of $f(x)$ formed for the change of values in a , b and c .

$$a \rightarrow -4$$

$$a \rightarrow -0.25$$

$$b \rightarrow -4$$

$$c \text{ decrease by } 2$$



2.3.2 Relate the position of the graph of quadratic functions with type of roots.

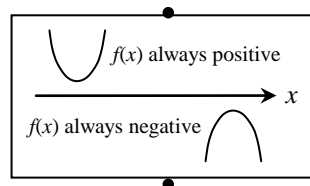
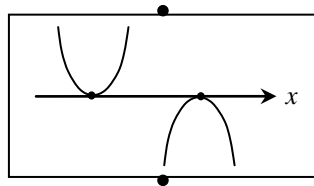
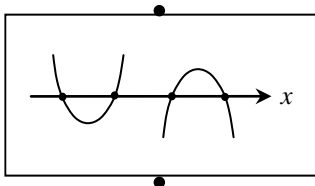
⇒ relate the position of quadratic function graphs with types of roots for $f(x) = 0$ ~ 1

57 Match the position of the graph of quadratic functions with the correct value of discriminant, $b^2 - 4ac$.

~ the graph does not intersect the x -axis.
~ no real roots.
(imaginary roots)

~ the graph intersects x -axis at two different points.
~ two real and different roots.

~ the graph intersects x -axis at one point only. (x -axis tangent of the graph)
~ two real and equal roots.



$$b^2 - 4ac = 0$$

$$b^2 - 4ac < 0$$

$$b^2 - 4ac > 0$$

58 The graph of a quadratic function $f(x) = px^2 - 2x - 3$, where p is a constants, does not intersect the x -axis. Find the range of values of p .
(Ans : $p < -\frac{1}{3}$)

[3 marks] [2013, No.5]

Answer :

59 The graph of a quadratic function $f(x) = px^2 - 8x + q$, where p and q are constants, has a maximum point.

(a) Given p is an integer such that $-2 < p < 2$, state the value of p .

(b) Using the answer from (a), find the value of q when the graph touches the x -axis at one point
(Ans : -16)

[3 marks] [2015, No.3]

Answer :

(a)

(b)

- 60 Given the quadratic function $f(x) = x^2 + 2wx + 3w - 2$, where w is a constant, is always positive (lies completely above the x -axis) when $p < w < q$. Find the value of p and of q .
(Ans : $p = 1$, $q = 2$)

[3 marks] [2016, No.18]

Answer :

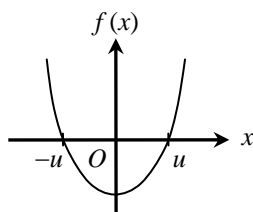
- 61 Show that the graph of a quadratic function $f(x) = p^2x^2 + 3px + 2$ is always intersect the x -axis for all values of p .

[3 marks] [Forecast]

Answer :

\Rightarrow relate the position of quadratic function graphs with types of roots for $f(x) = 0$ ~ 2

- 62 The diagram shows the graph of a quadratic function $f(x) = \frac{p}{x^n} + qx + r$ such that p , q , r , n and u are constants.



- (a) State the value of n .
(b) If $f(x) = 0$ and the product of roots is r , state the value of
(i) q ,
(ii) p .

(Ans : 1)

[3 marks] [2019, No.2]

Answer :

- (a) (b) (i)

(ii)

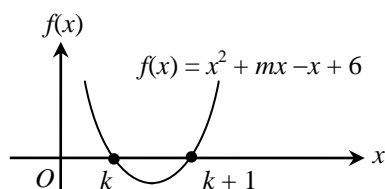
- 63 Given function $f : x \rightarrow x^2 + px + q$. If $f(x) > 0$ for $x < 2$ and $x > 3$. Find the value of p and of q .
(Ans : $p = -5$, $q = 6$)
[3 marks] [Forecast]

Answer :

- 64 Given that $f(x) = 2x^2 + px + 16$ and that $f(x)$ is only negative when $2 < x < k$. Find the values of p and k .
(Ans : $p = -12$, $k = 4$)
[3 marks] [Forecast]

Answer :

- 65 The diagram show the graph of a quadratic function $f(x) = x^2 + mx - x + 6$, where n is a constant.



Find

- (a) the roots of the quadratic equation $x^2 + mx - x + 6 = 0$.
(b) the value of m .

(Ans : 2, 3)

(Ans : -4)

[3 marks] [Forecast]

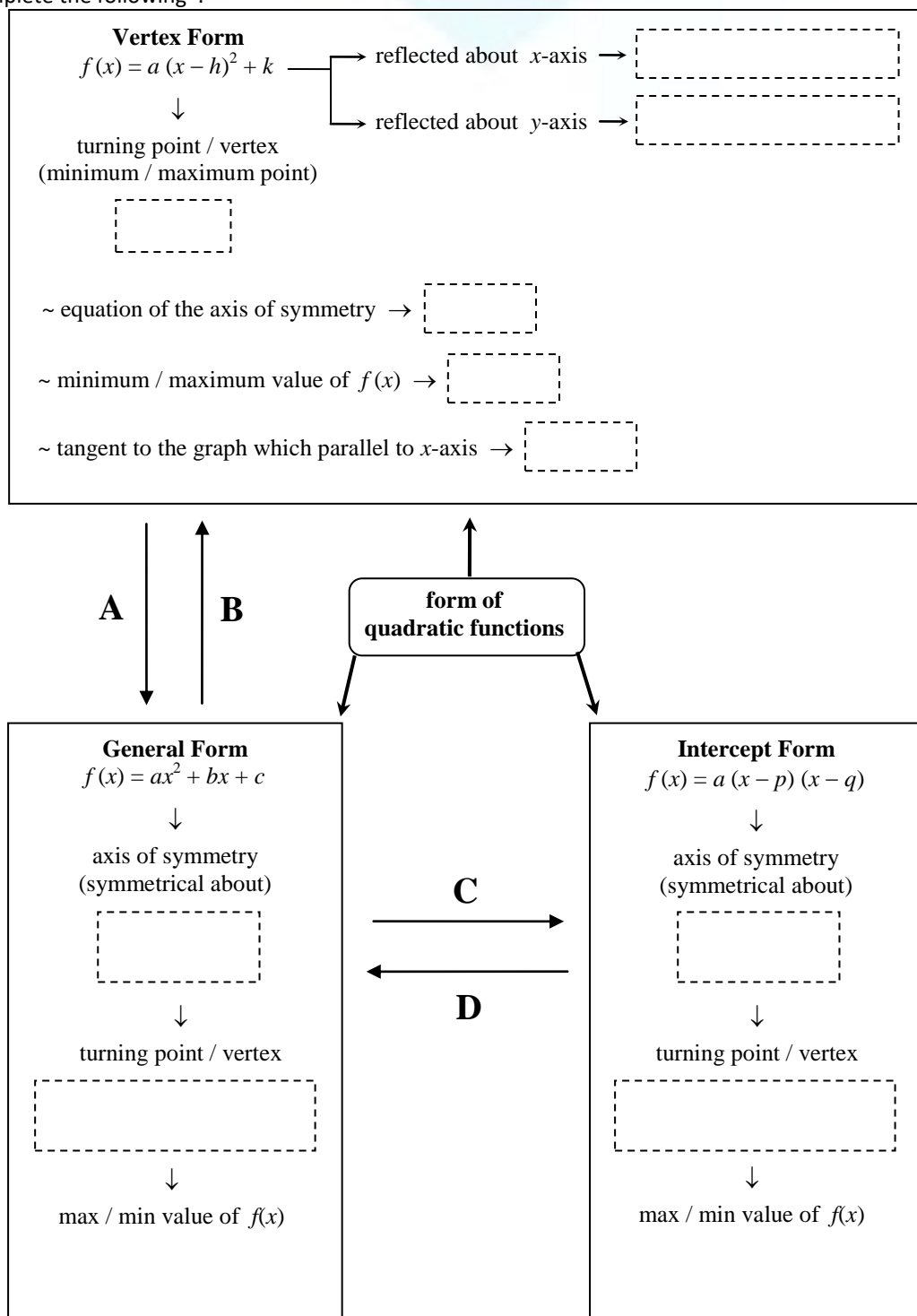
Answer :

(a)

(b)

2.3.3 Relate the vertex form of quadratic functions, $f(x) = a(x - h)^2 + k$ with other forms of quadratic functions.

66 Complete the following :



A =

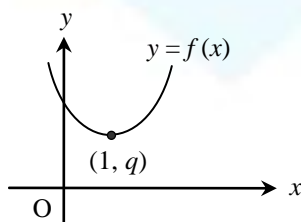
C =

B =

D =

\Rightarrow **vertex form (turning point - minimum @ maximum point) ~ 1**

- 67** The diagram shows the graph of a quadratic function $f(x) = 3(x + p)^2 + 2$, where p is a constant.



The curve $y = f(x)$ has a minimum point $(1, q)$, where q is a constant. State

- (a) the value of p ,
- (b) the value of q ,
- (c) the equation of the axis of symmetry.

[3 marks] [2005, No.6]

Answer :

- (a)
- (b)
- (c)

- 68** The quadratic function $f(x) = p(x + q)^2 + r$, where p , q and r are constants, has a minimum value of -4 . The equation of the axis of symmetry is $x = 3$. State

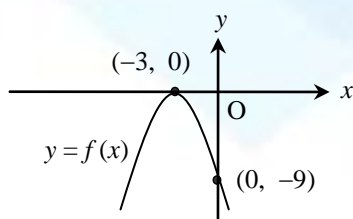
- (a) the range of values of p ,
- (b) the value of q ,
- (c) the value of r .

[3 marks] [2008, No.5]

Answer :

- (a)
- (b)
- (c)

- 69 The diagram shows the graph of quadratic function $f(x) = -(x+p)^2 + q$, where p and q are constants.



State

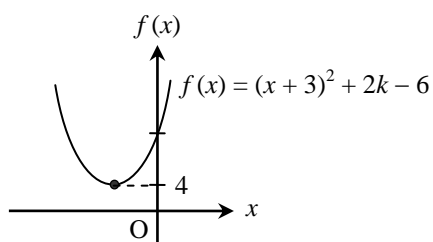
- (a) the value of p ,
- (b) the equation of the axis of symmetry.

[2 marks] [2009, No.5]

Answer :

- (a)
- (b)

- 70 The diagram shows the graph of quadratic function $f(x) = (x+3)^2 + 2k - 6$, where k is a constant.



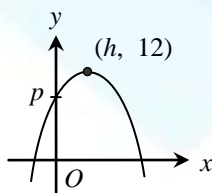
- (a) State the equation of axis of symmetry of the curve.
- (b) Given that the minimum value of the function is 4, find the value of k . (Ans : 5)

[3 marks] [2011, No.5]

Answer :

- (a)
- (b)

- 71** The diagram shows the graph of a quadratic function $f(x) = -(x - 2)^2 + 3k$, where k is a constant.



Given $(h, 12)$ is the maximum point of the graph,

- (a) state the value of h and of k ,
 (b) find the value of p .

(Ans : $h = 2, k = 4$)

(Ans : 8)

[3 marks] [2013, No.6]

Answer :

- (a) (b)

- 72** The graph of a quadratic function $f(x) = 3[2h - (x - 1)^2]$, where h is a constant, has maximum point $(1, h - 10)$.

- (a) State the value of h ,
 (b) State the type of roots for $f(x) = 0$. Justify your answer.

(Ans : -2)

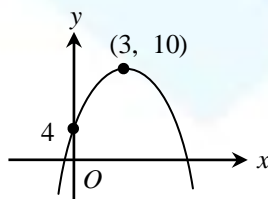
(Ans : no roots)

[3 marks] [2019, No.3]

Answer :

- (a) (b)

- 73 In the diagram, $(3, 10)$ is the maximum point of a graph with an equation in the form of $y = a(x + b)^2 + c$.



Find

- (a) the value of a , (Ans : $a = -\frac{2}{3}$)
 (b) the equation of the curve when the graph is reflected about the y -axis .

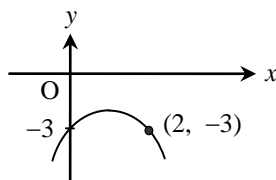
[3 marks] [Forecast]

Answer :

- (a) (b)

⇒ **vertex form (turning point - minimum @ maximum point) ~ 2**

- 74 The diagram shows the graph of the function $y = -(x - k)^2 - 2$, where k is constant.



Find

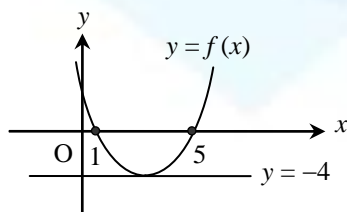
- (a) the value of k ,
 (b) the equation of the axis of symmetry,
 (c) the coordinates of the maximum point.

[3 marks] [2004, No.6]

Answer :

- (a) (b) (c)

- 75 The diagram shows the graph of a quadratic function $y = f(x)$. The straight line $y = -4$ is a tangent to the curve $y = f(x)$.



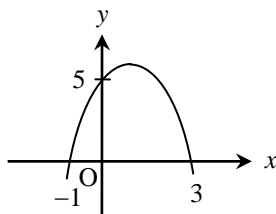
- (a) Write the equation of the axis of symmetry of the curve.
 (b) Express $f(x)$ in the form of $f(x) = (x + b)^2 + c$, where b and c are constant.

[3 marks] [2006, No.4]

Answer :

- (a) (b)

- 76 The diagram shows the graph of a quadratic function $y = f(x)$.



State

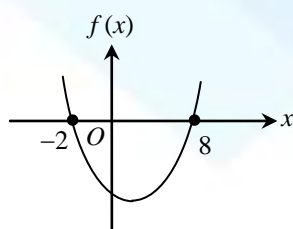
- (a) the roots of the equation $f(x) = 0$,
 (b) the equation of the axis of symmetry of the curve.

[3 marks] [2010, No.4]

Answer :

- (a) (b)

- 77 The diagram shows the graph of the quadratic function $f(x) = (x - k)^2 - 25$.



State

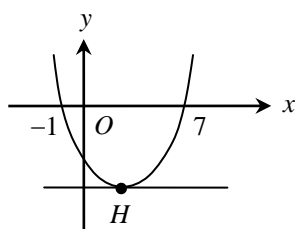
- (a) the value of k ,
- (b) the coordinates of the minimum point of the curve,
- (c) the range of values of x when $f(x)$ is negative.

[3 marks] [2014, No.4]

Answer :

- (a)
- (b)
- (c)

- 78 The diagram shows the graph $y = a(x - p)^2 + q$, where a , p and q are constants. The straight line $y = -8$ is a tangent to the curve at point H .



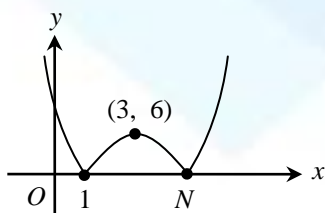
- (a) State the coordinates of H .
- (b) Find the value of a . (Ans : $\frac{1}{2}$)
- (c) If the graph is reflected about the x -axis, write the equation of the curve. **

[3 marks] [2018, No.18]

Answer :

- (a)
- (b)
- (c)

- 79 The diagram show the graph of a quadratic function $f(x) = |a(x+p)^2 + q|$, where $a > 0$.



- (a) State the coordinates of point N.
 (b) Find the function of the graph.

(Ans : $a = \frac{3}{2}$)

[3 marks] [Forecast]

Answer :

(a)

(b)

⇒ **general form → vertex form**

- 80 The quadratic function $f(x) = x^2 + 2x - 4$ can be expressed in the form $f(x) = (x + m)^2 - n$, where m and n are constants. Find the value of m and n .

(Ans : $m = 1, n = 5$)

[3 marks] [2007, No.6]

Answer :

- 81 The quadratic function $f(x) = -x^2 + 4x + a^2$, where a is a constant, has maximum value 8. Find the values of a .

(Ans : ± 2)

[3 marks] [2009,

No.6]

Answer :

82 The quadratic function f is defined by $f(x) = x^2 + 4x + h$, where h is a constant.

(a) Express $f(x)$ in the form $(x + m)^2 + n$, where m and n are constants.

(b) Given the minimum value of $f(x)$ is 8, find the value of h .

(Ans : 12)

[4 marks] [2017, No.11]

Answer :

(a)

(b)

83 Prove that $y = x^2 + 2x + 7$ is always positive for all values of x . Hence state the smallest value of y .

(Ans : 6)

[3 marks] [Forecast]

Answer :

84 Find the smallest integer of p such that $f(x) = x^2 - 4x + p$, where p is a constant, is always greater than 5.

(Ans : 10)

[3 marks] [Forecast]

Answer :

\Rightarrow **vertex form \rightarrow general form \rightarrow intercept form**

85 The quadratic function $f(x) = -3\left(x - \frac{1}{3}\right)^2 + \frac{4}{3}$ can be expressed in the form $f(x) = a(x - p)(x - q)$, where a , p and q are constants and $p > q$. Find the value of p and q .

(Ans : $p = 1$, $q = -\frac{1}{3}$)

[3 marks] [Forecast]

Answer :

2.3.4 Analyse and make generalisation about the effects of changes of a , h dan k in quadratic functions $f(x) = a(x - h)^2 + k$ towards the shape and position of the graphs.

- 86** The diagram shows the graph for $f(x) = -2(x - 3)^2 + 5$, where $a = -2$, $h = 3$ and $k = 5$. Match the graph of $f(x)$ formed for the change of values in a , h and k .

$a \rightarrow -7$	$a \rightarrow -1$	h increase by 3	k decrease by 3

- 87** A quadratic function $f(x) = (x + 3)^2 + 2k$, where k is a constant, has a minimum value of -6 . Find
- the value of k ,
 - the equation of the axis of symmetry when the graph moves 2 units to the left,
 - minimum value of $f(x)$ when the graph moves 10 units upwards.

[4 marks] [Forecat]

Answer :

- (a) _____ (b) _____
- (c) _____

2.3.5 Sketch graphs of quadratic functions.

- 88** The quadratic function $f(x) = -x^2 + 4x - 3$ can be expressed in the form of $f(x) = -(x - 2)^2 + k$, where k is a constant.

(a) Find the value of k .

(Ans : 1)

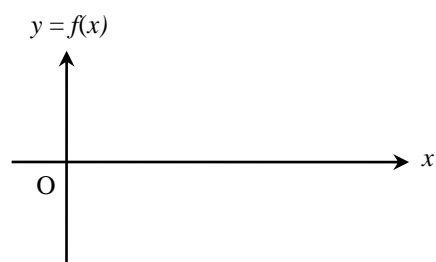
(b) Sketch the graph of the function $f(x)$ on the given axes.

[4 marks] [2010, No.6]

Answer :

(a)

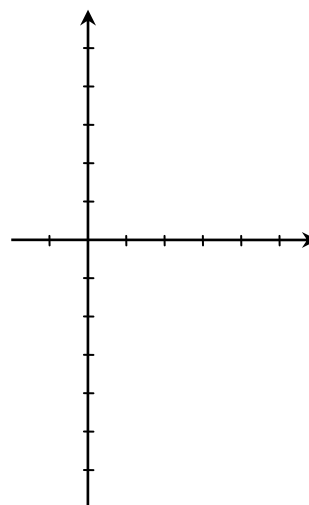
(b)



- 89** Sketch the graph $f(x) = -3x^2 + 12x - 16$ for the domain $-1 \leq x \leq 5$. Hence, state the axis of symmetry of the graph.

[4 marks] [Forecast]

Answer :



MIND think :

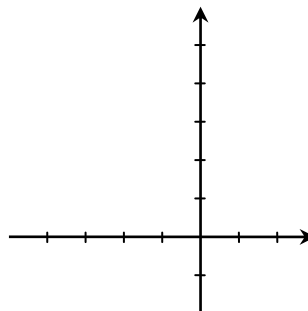
- **steps to sketch the graph of a quadratic function**

- (i) find the maximum / minimum point (turning point) by completing the square
- (ii) find the y -intercept \rightarrow the value of c @ substitute $x = 0, f(0)$
- (iii) find the x -intercept \rightarrow substitute $y = 0$
- (iv) find the corresponding range, y for the given domain, x

- 90 Sketch the graph $f(x) = 2\left(x + \frac{1}{2}\right)(x+3)$ for the domain $-4 \leq x \leq 2$. Hence, find the quadratic function that will be formed if the graph $f(x)$ is reflected about the y -axis.
 [Ans : $f(x) = 2\left(x - \frac{7}{4}\right)^2 - \frac{25}{8}$]

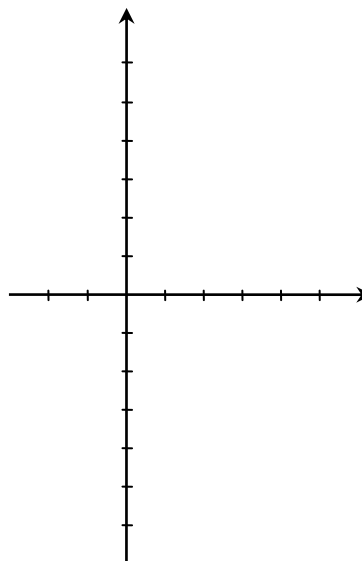
[4 marks] [Forecast]

Answer :



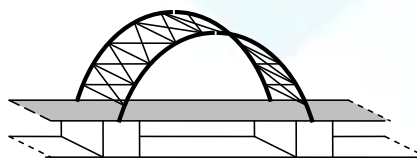
- 91 Sketch the graph of $f(x) = |4 - (x - 1)^2|$ for the domain $-2 \leq x \leq 5$. Hence, find the range of $f(x)$ corresponding to the given domain.
 (Ans : $0 \leq f(x) \leq 12$)
 [4 marks] [Forecast]

Answer :



2.3.6 Solve problems involving quadratic functions.

- 92 The diagram shows a bridge supported by parabolic construction.



Given that the equation of the construction is given by $y = -\frac{1}{1000}x^2 + c$ metres. The distance between the two ends of the curve over the bridge is 400 meters, and the height of the bridge from the ground is 6 meters. Find the maximum height of the construction.

(Ans : 46)

[3 marks] [Forecast]

Answer :

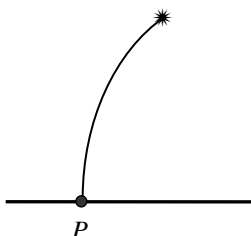
- 93 The number of food packets prepared by a food stall in a day is given by function $f(x) = 3x^2 - 24x + 2k - 1$, where k is a constant and x is the number of workers in the stall. Find the smallest value of integer k if the number of food packets provided must exceed 515 packets a day, and the number of workers at that moment.

(Ans : $k = 283$)

[3 marks] [Forecast]

Answer :

- 94 The function $h(t) = -4t^2 + 32t$, represents the height of the fireworks, in meters, after t seconds launched from point P , as shown in the diagram.



Point P is a origin and the fireworks exploded at the highest point. Find the height, in cm, the fireworks explode.

(Ans : 64)

[3 marks] [Forecast]

Answer :

- 95 A football player kicks a ball 5m from the left side of the football middle field. The locus of the ball is represented by $f(x) = kx^2 + \frac{7}{18}x + 3p$, where k and p are constants. The ball reaches a maximum height of 7 m and touches the surface of the field 60 m from the ball kicked. Find the value of k and of p .
 (Ans : $k = -\frac{7}{900}$, $p = \frac{77}{108}$)
 [4 marks] [Forecast]

Answer :

PAPER 2

⇒ Quadratic Equations ~ Part A → 7 – 8 marks

- 96 The quadratic equation $x^2 - 5x + 6 = 0$ has roots h and k , where $h > k$.
- (a) Find
- (i) the value of h and of k , (Ans : $h = 3$, $k = 2$)
 - (ii) the range of x if $x^2 - 5x + 6 > 0$. (Ans : $x < 2$, $x > 3$)
- [5 marks]
- (b) Using the values of h and of k from (a) (i), form the quadratic equation which has roots $h + 2$ and $3k - 2$.
 (Ans : $x^2 - 9x + 20 = 0$) [2 marks]
 [2009, No.2]

Answer :

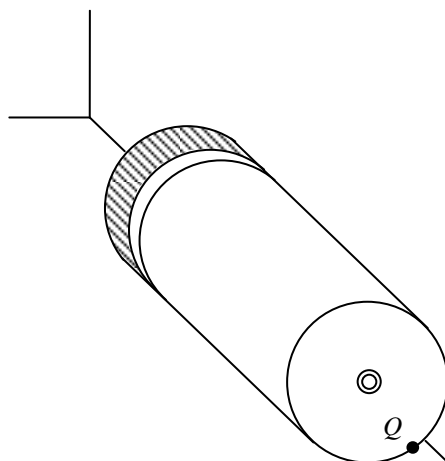
- 97 A quadratic equation $x^2 + 4(3x + k) = 0$, where k is a constant has roots p and $2p$, $p \neq 0$
- (a) Find the value of p and of k . (Ans : $p = -4$, $k = 8$) [5 marks]
- (b) Hence, form the quadratic equation which has the roots $p - 1$ and $p + 6$.
 (Ans : $x^2 + 3x - 10 = 0$) [3 marks]
 [2012, No.2]

Answer :

- 98 It is given α and β are the roots of the quadratic equation $x(x-3) = 2h-4$, where h is a constant.
- (a) Find the range of values of h if $\alpha \neq \beta$. (Ans : $h > \frac{7}{8}$) [3 marks]
- (b) Given $\frac{\alpha}{2}$ and $\frac{\beta}{2}$ are the roots of another quadratic equation $2x^2 + kx - 4 = 0$, where k is a constant, find the value of k and of h . (Ans : $k = -3, h = 6$) [4 marks]
[2015, No.5]

Answer :

- 99 The diagram shows a cylindrical container with the length of 20 cm placed on the floor against the wall. Q is a point on the edge of the base of the container. It is given that the distance of point Q is 2 cm from the wall and 1 cm from the floor.



Mira wants to keep the container in a box with a dimension of $21 \text{ cm} \times 7 \text{ cm} \times 7 \text{ cm}$. Determine whether the container can be kept in that box or otherwise. Give a reason for your answer. (Ans : cannot, diameter $10 > 7$)
[6 marks][2017, No.6]
Answer :

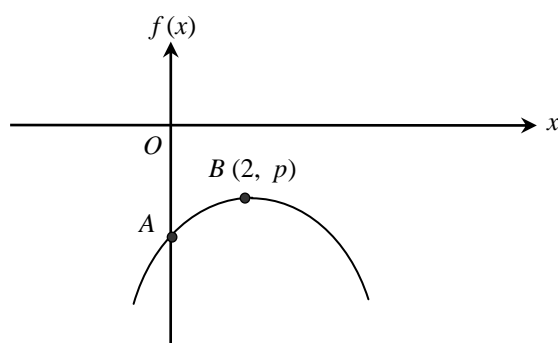
⇒ Quadratic Functions ~ Part A → 6 – 8 marks

100 The function $f(x) = x^2 - 4kx + 5k^2 + 1$ has a minimum value of $r^2 + 2k$, where r and k are constants.

- (a) By using the method of completing the square, show that $r = k - 1$. [4 marks]
- (b) Hence, or otherwise, find the values of k and r if the graph of the function is symmetrical about $x = r^2 - 1$.
(Ans : $k = 0, r = -1$ & $k = 4, r = 3$) [4 marks]
[2003, No.2]

Answer :

101 The diagram shows the curve of a quadratic function $f(x) = -x^2 + kx - 5$. The curve has a maximum point at $B(2, p)$ and intersect the $f(x)$ – axis at point A .



- (a) State the coordinates of A . [1 mark]
- (b) By using the method of completing the square, find the value of k and of p .
(Ans : $k = 4, p = -1$) [4 marks]
- (c) Determine the range of values of x , if $f(x) \geq -5$. (Ans : $0 \leq x \leq 4$) [2 marks]
[2008, No.2]

Answer :

- 102** The curve of a quadratic function $f(x) = 2(x - h)^2 + 2k$ intersects the x -axis at points $(1, 0)$ and $(5, 0)$. The straight line $y = -8$ touches the minimum point of the curve.
- (a) Find the the value of h and of k . (Ans : $h = 3, k = -4$) [2 marks]
- (b) Hence, sketch the graph of $f(x)$ for $0 \leq x \leq 6$. [3 marks]
- (c) If the graph is reflected about the x -axis, write the equation of the curve. [1 mark]
[2016, No.2]

Answer :

⇒ FORECAST

- 103** Cherryna bought some calculators for RM704.
- (a) Cherryna sold 20 of the calculators at a profit of RM6 each. Write down an expression in terms of x , for the selling price of each calculator. [1 mark]
- (b) Cherryna sold the remaining calculators for RM30 each. Write down in terms of x , the total amount of money she received for all the calculators. (Ans : $\frac{14080}{x} + 30x - 480$)
[2 marks]
- (c) If Cherryna received RM920 altogether, shows that $3x^2 - 140x + 1408 = 0$. 2 marks]
- (d) Find the number of calculators Cherryna bought. (Ans : 32) [2 marks]

Answer :

- 104** The diagram shows two types of tiles bought by Melvin, Riverwood and Riverstone, that are in square shape.



RIVERWOOD



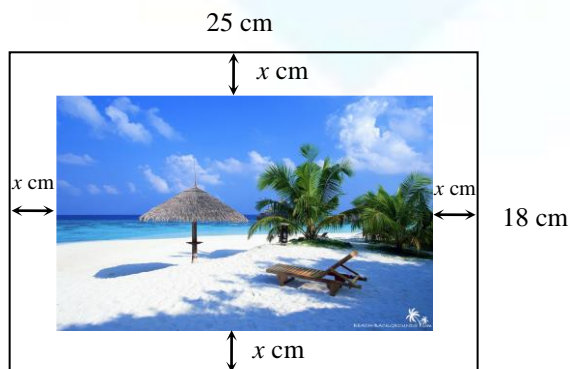
RIVERSTONE

The length of the Riverstone tile is 2 cm more than the length of the Riverwood tile. Given the total area of the two tiles is 340 cm^2 .

- (a) Find the perimeter, in cm, of the Riverstone tile. (Ans : 56) [5 marks]
- (b) What is the minimum number of Riverwood tiles needed to fill a square shape living room with a perimeter of 484 cm ? (Ans : 102) [2 marks]

Answer :

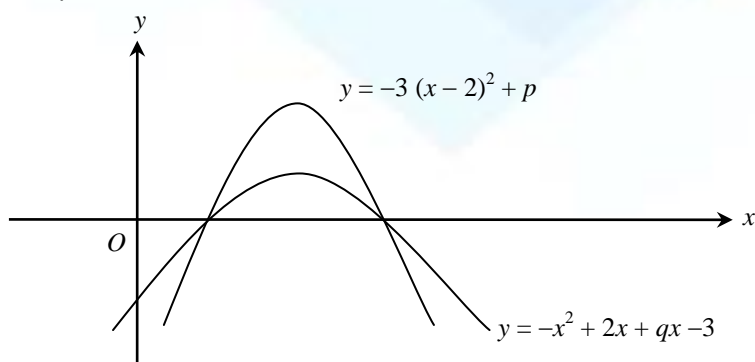
- 105** A page with dimensions $18\text{ cm} \times 25\text{ cm}$ has a border of uniform width $x\text{ cm}$ surrounding the printed part of the page, as shown in the diagram.



- (a) Write a formula for the area L , in terms of x , of the printed part.
 (Ans : $L = 450 - 86x + 4x^2$) [2 marks]
- (b) State the domain dan range of L .
 (Ans : $0 \leq x < 9$, $0 < L \leq 450$) [2 marks]
- (c) Find the margin (width of the border) that should be used to obtain an area of 348.75 cm^2 .
 (Ans : 1.25) [3 marks]

Answer :

- 106** The diagram shows the curve of the functions $y = -x^2 + 2x + qx - 3$ and $y = -3(x - 2)^2 + p$ that intersect at two points at the x -axis.



Find

- (a) the values of p and q . (Ans : $p = 3, q = 2$) [4 marks]
 (b) the maximum point of each curve. [Ans : (2, 3), (2, 1)]
[3 marks]

Answer :

- 107** The quadratic function is defined by $f(x) = 24x - 4x^2 + r$, where r is a constant.

- (a) Express the quadratic function in the form $p(x - q)^2 + 16$. Hence, find the values of p , q and r . (Ans : $p = -4, q = 3, r = -20$) [4 marks]
 (b) Find the turning point of the graph $f(x)$. [Ans : (3, 16)] [1 mark]
 (c) Hence, sketch the graph of the function $f(x)$. [3 marks]

Answer :

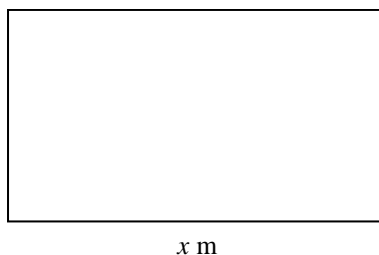
- 108** A bookstore in Telipok town selling Mathematics work books has found out that when books are sold at a price of RM q per book, the profit, RM U , as a function of the price, q is :

$$U(q) = 120q - 10q^2$$

- (a) Sketch the graph of $U(q) = 120q - 10q^2$.
[4 marks]
- (b) Hence, determine the price of a book that should be established in order to maximise profit, and state the maximum profit. (Ans : 6, 360) [2 marks]

Answer :

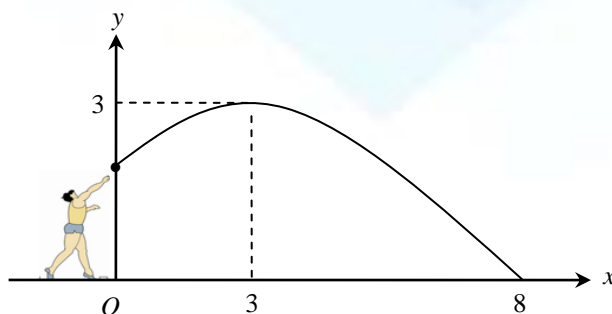
- 109** Emily bought a 100 metres wire to fence her rectangular garden with x metres length, as shown in the diagram.



- (a) Show that the area, $A = 50x - x^2$. [3 marks]
- (b) Express the area in the form $f(x) = a(x + p)^2 + q$. Hence, find the maximum area and the corresponding value of x . (Ans : 625, 25) [3 marks]

Answer :

- 110** The diagram shows Gregory throws shot put. The thrown form a quadratic function $y = f(x)$ where y is the height of the shot put and x is it horizontal distance.



Given that $f(x) = a(x - p)^2 + q$, based on the information in the given diagram.

- (a) Find the values of a , p and q . (Ans : 3, 3, $-\frac{3}{25}$) [4 marks]
- (b) (i) Find the height of the shot put during it released form Gregory's hand.
(Ans : $\frac{48}{25}$) [2 marks]
- (ii) Find the horizontal distance of the shot put when it is at the same height as (b) (i) again.
(Ans : 6) [1 mark]

Answer :

111 Given that $y = p + qx - x^2 = k - (x + h)^2$.

(a) Find the values of h and k in term of p and / or q .

(Ans : $h = -\frac{q}{2}$, $k = \frac{q^2}{4} + p$) [3 marks]

(b) If $q = 2$, state the axis of symmetry of the curve.

(Ans : $x = 1$) [2 marks]

(c) Straight line $y = 3$ touches curve $y = p + qx - x^2$,

(i) state p in term of q ,

(Ans : $p = \frac{12 - q^2}{4}$) [2 marks]

(ii) hence, sketch the graph of the curve.

[3 marks]

Answer :

CONTINUOUS EXERCISES

- 112** The roots of the quadratic equations $\frac{1}{k}x^2 + (k-5)x = -x$, where k is a constant, are of opposite / difference signs. Shows that $0 < k < 5$.

[3 marks] [Forecast]

Answer :

- 113** The function $f: x \rightarrow -x^2 + 6x - 5$ is defined for $x \geq k$, where k is a constant. State the smallest value of k for which $f(x)$ is a one-to-one relation. (Ans : 3)

[2 marks] [Forecast]

Answer :

- 114** The graph of the function $f(x) = ax^2 + bx + c$ has equation of axis of symmetry at $x = 1$ and passes through the points $(0, 1)$ and $(-1, -5)$. Find the value of a , b and c . (Ans : $a = -2$, $b = 4$, $c = 1$)

[4 marks] [Forecast]

Answer :

SYSTEM OF EQUATIONS

- ONE PAGE NOTE (OPN)

- WORKSHEET

Puan Maya Insana Mohd Terang

“ SYSTEM OF EQUATIONS ”

MIND think :

steps in solution : using Gauss elimination

$$ax + by + cz = d$$

$$ex + fy + gz = h$$

$$px + qy + rz = s$$

$$\downarrow$$

$$\begin{pmatrix} a & b & c & d \\ e & f & g & h \\ p & q & r & s \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & X & X & X \\ X & X & X & X \\ X & X & X & X \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & X & X & X \\ 0 & X & X & X \\ 0 & X & X & X \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & X & X & X \\ 0 & 1 & X & X \\ 0 & X & X & X \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & X & X & X \\ 0 & 1 & X & X \\ 0 & 0 & X & X \end{pmatrix}$$

NOTE :

- ☐ $(a + b)^2 = a^2 + 2ab + b^2$
- ☐ $(a - b)^2 = a^2 - 2ab + b^2$
- ☐ $(a + b)(a - b) = a^2 - b^2$

<p>• steps in solution : using elimination method</p> <ol style="list-style-type: none"> rearrange all the equations in the form, $ax + by + cz = d$. do the first elimination. → NOTE : same sign (-), different sign (+) do the second elimination. ~ obtained the value of the first variable. substitute the value of first variable into any equations in (2) to get the value of second variable. substitute the values of first and second variable into any equations in (1) to get the value of third variable. 	<p>• steps in solution : using substitution method</p> <ol style="list-style-type: none"> choose one equation, express one of the variables as subject → example : x in terms of y and z substitute it into the others two equations, and express both equations into same subject. → example : y in terms of z substitute the equations in (2), to each others ~ obtained the value of the first variable. substitute the value of first variable into any equations in (2) to get the value of second variable. substitute the values of first and second variable into equation in (1) to get the value of third variable.
<p>• steps in solving simultaneous equations : using elimination method</p> <ol style="list-style-type: none"> Arranged both equation in general form. Form quadratic equation : <ol style="list-style-type: none"> in terms of x → eliminate y in terms of y → eliminate x Simplify and solve the quadratic equation by using : ~ factorization / calculator - CASIO fx-570MS, CANON F-570SG, OLYMPIA ES-570MS . . . → $\boxed{\text{mode}}$; $\boxed{\text{mode}}$; $\boxed{\text{mode}}$; $\boxed{1}$; $\boxed{\triangleright}$; $\boxed{2}$; → key in value a $\boxed{=}$; key in value b $\boxed{=}$; key in value c $\boxed{=}$; → x_1, x_2 (if x only → $x_1 = x_2$) <p>OR</p> <ol style="list-style-type: none"> Simplify and solve the quadratic equation by using : ~ formula → $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Obtain the values of other unknown by substituting the x_1 and x_2 into linear equation. 	<p>• steps in solving simultaneous equations : using substitution method</p> <ol style="list-style-type: none"> From the linear equation, an unknown is expressed in terms of the other unknown. Substituted (1) into the non-linear equation to form a quadratic equation Arranged the quadratic equation in general form : $ax^2 + bx + c = 0$ Simplify and solve the quadratic equation by using : ~ factorization / calculator - CASIO fx-570MS, CANON F-570SG, OLYMPIA ES-570MS . . . → $\boxed{\text{mode}}$; $\boxed{\text{mode}}$; $\boxed{\text{mode}}$; $\boxed{1}$; $\boxed{\triangleright}$; $\boxed{2}$; → key in value a $\boxed{=}$; key in value b $\boxed{=}$; key in value c $\boxed{=}$; → x_1, x_2 (if x only → $x_1 = x_2$) <p>OR</p> <ol style="list-style-type: none"> Simplify and solve the quadratic equation by using : ~ formula → $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Obtain the values of other unknowns by substituting the x_1 and x_2 into (1).

WORKSHEET
TOPIC 3 : SYSTEMS OF EQUATIONS
[Paper 2, Part A ~ 1 question → 5 – 8 marks]

3.1 Systems of linear equations in three variables

3.1.1 Describe systems of linear equations in three variables.

1 Mark ☐ if the equations are system of linear equations in three variable. If not, mark ☐.

Answer :

$a(5 + 2b) = 1 - c^2$ $2a + c = b$ $a - b + 5c - 7 = 0$	$k + 2m = 8$ $5m = n + 9$ $n - k + 7 = 0$	$3(p + \frac{q}{6}) = q + 4r$ $-r + 6q = 2$ $7q = 2p + 9r - 1$	$3x + 2(y - z) = 7$ $xy = 3y + 5z$ $4x - z + 3y = 5$
<input style="width: 50px; height: 30px;" type="text"/>	<input style="width: 50px; height: 30px;" type="text"/>	<input style="width: 50px; height: 30px;" type="text"/>	<input style="width: 50px; height: 30px;" type="text"/>

MIND think :

- The characteristics of systems of liner equation in three variable :

→ each linear equation has variable.

→ the highest power of each variable is .

3.1.2 Solve systems of linear equations in three variables.

- 2 Using the GeoGebra software, determine the type of solution for the following systems of linear equations in three variable.

Answer :

$\begin{aligned} 4x + 8y + 2z &= 14 \\ 2x - 4y + z &= -2 \\ x - 2y + 0.5z &= 1.5 \end{aligned}$	\bullet	\bullet <div>one solution</div> \bullet	\bullet <div>The planes intersect along a straight line</div> \bullet
$\begin{aligned} x - y - 3z &= -6 \\ 2x + y + z &= 3 \\ -x + 2y + 2z &= 1 \end{aligned}$	\bullet	\bullet <div>infinite solutions</div> \bullet	\bullet <div>The planes do not intersect at any point</div> \bullet
$\begin{aligned} 4x - 7y + 2z &= 6 \\ x - 2y &= 4 \\ 2x - 3y + 2z &= -2 \end{aligned}$	\bullet	\bullet <div>no solution</div> \bullet	\bullet <div>The planes intersect at only one point</div> \bullet

- 3 (a) Solve the system of linear equations using the elimination method :

$$x - 3y + z = 2$$

$$4x - 4y + z = 7$$

$$2x + y - 3z = -4$$

(Ans : $x = 2$, $y = 1$ and $z = 3$)

[6 marks]

[Forecast]

Answer :

MIND think :

- steps in solution : using elimination method

- (1) rearrange all the equations in the form, $ax + by + cz = d$.
- (2) do the first elimination. → **NOTE** : same sign (-), different sign (+)
- (3) do the second elimination. ~ obtained the value of the first variable.
- (4) substitute the value of first variable into any equations in (2) to get the value of second variable.
- (5) substitute the values of first and second variable into any equations in (1) to get the value of third variable.

(b) Solve the system of linear equations using the substitution method :

$$x - 3y + z = 2$$

$$4x - 4y + z = 7$$

$$2x + y - 3z = -4$$

(Ans : $x = 2$, $y = 1$ and $z = 3$)

[6 marks]

[Forecast]

Answer :

MIND think :

- *steps in solution : using substitution method*

(1) choose one equation, express one of the variables as subject → example : x in terms of y and z

(2) substitute it into the others two equations, and express both equations into same subject. → example : y in terms of z

(3) substitute the equations in (2), to each others ~ obtained the value of the first variable.

(4) substitute the value of first variable into any equations in (2) to get the value of second variable.

(5) substitute the values of first and second variable into equation in (1) to get the value of third variable.

(c) Solve the system of linear equations using Gauss elimination :

$$x - 3y + z = 2$$

$$4x - 4y + z = 7$$

$$2x + y - 3z = -4$$

(Ans : $x = 2$, $y = 1$ and $z = 3$)

[6 marks]

[Forecast]

Answer :

MIND think :

steps in solution :
using Gauss elimination

$$ax + by + cz = d$$

$$ex + fy + gz = h$$

$$px + qy + rz = s$$

↓

$$\begin{pmatrix} a & b & c & | & d \\ e & f & g & | & h \\ p & q & r & | & s \end{pmatrix}$$

↓

$$\begin{pmatrix} 1 & X & X & | & X \\ X & X & X & | & X \\ X & X & X & | & X \end{pmatrix}$$

↓

$$\begin{pmatrix} 1 & X & X & | & X \\ 0 & X & X & | & X \\ 0 & X & X & | & X \end{pmatrix}$$

↓

$$\begin{pmatrix} 1 & X & X & | & X \\ 0 & 1 & X & | & X \\ 0 & X & X & | & X \end{pmatrix}$$

↓

$$\begin{pmatrix} 1 & X & X & | & X \\ 0 & 1 & X & | & X \\ 0 & 0 & X & | & X \end{pmatrix}$$

- 4 (a) Solve the system of linear equations using the elimination method :

$$2x + 2y - z = 0$$

$$4y - z = 1$$

$$-x - 2y + z = 2$$

$$(Ans : x = 2, y = \frac{5}{2} \text{ and } z = 9)$$

[6 marks]

[Forecast]

Answer :

- (b) Solve the system of linear equations using the substitution method :

$$2x + 2y - z = 0$$

$$4y - z = 1$$

$$-x - 2y + z = 2$$

$$(Ans : x = 2, y = \frac{5}{2} \text{ and } z = 9)$$

[6 marks]

[Forecast]

Answer :

- (c) Solve the system of linear equations using Gauss elimination :

$$2x + 2y - z = 0$$

$$4y - z = 1$$

$$-x - 2y + z = 2$$

(Ans : $x = 2$, $y = \frac{5}{2}$ and $z = 9$)

[6 marks]

[Forecast]

Answer :

- 5 (a) Solve the system of linear equations using the elimination method :

$$2x + y - 3z = 1$$

$$3x - y - 4z = 7$$

$$5x + 2y - 6z = 5$$

(Ans : $x = 3$, $y = -2$ and $z = 1$)

[6 marks]

[Forecast]

Answer :

(b) Solve the system of linear equations using the substitution method :

$$2x + y - 3z = 1$$

$$3x - y - 4z = 7$$

$$5x + 2y - 6z = 5$$

(Ans : $x = 3$, $y = -2$ and $z = 1$)

[6 marks]
[Forecast]

Answer :

(c) Solve the system of linear equations using Gauss elimination :

$$2x + y - 3z = 1$$

$$3x - y - 4z = 7$$

$$5x + 2y - 6z = 5$$

(Ans : $x = 3$, $y = -2$ and $z = 1$)

[6 marks]
[Forecast]

Answer :

- 6 Solve the system of linear equations :

$$x - y + 2z = 3$$

$$-3x - 2y + z = -6$$

$$4x + z = 11$$

(Ans : $x = 3$, $y = -2$ and $z = -1$)

[6 marks]
[Forecast]

Answer :

- 7 Solve the system of linear equations :

$$3x + y - 2z = -7$$

$$-x - 3y + 5z = 10$$

$$4x - 2y + 3z = 1$$

(Ans : $x = -1$, $y = 2$ and $z = 3$)

[6 marks]
[Forecast]

Answer :

3.1.3 Solve problems involving systems of linear equations in three variables.

- 8 Three students went to a bookshop to buy rulers, markers and pen. The items bought and amount spent are shown in the following table.

<i>Students</i>	<i>Number of rulers</i>	<i>Number of markers</i>	<i>Number of pens</i>	<i>Amount paid (RM)</i>
Lea	2	3	4	11
Melvin	4	3	1	10
Mclarence	1	2	4	8

Find the price, in RM, of a ruler, a marker and a pen.

(Ans : ruler = 0.40, marker = 2.60, pen = 0.60)

[7 marks]

[Forecast]

Answer :

- 9 Product X , Y , Z are assembled from three components A , B , C according to different proportions. Each product X consists of two components of A , four components of B , and one component of C ; each product of Y consists three components of A , three components of B , and two components of C ; each product of Z consists of four components of A , one component of B , and four component of C . A total of 750 components of A , 1000 components of B , and 500 components of C are used.

Find the number of products of X , Y , and Z assembled.

(Ans : $X = 200$, $Y = 50$, $Z = 50$)

[7 marks]

[Forecast]

Answer :

- 10** Two groups of workers have their drinks at a stall. The first group comprising ten workers have five cups of tea, two cups of coffee and three glasses of fruit juice at a total cost of RM11.80. The second group of six workers have three cups of tea, a cup of coffee and two glasses of fruit juice at a total cost of RM7.10. The cost of a cup of tea and three glasses of fruit juice is the same as the cost of four cups of coffee.

Find the cost, in RM, of each drink.

(Ans : tea = 1, coffee = 1.30, fruit juice = 1.40)

[7 marks]

[Forecast]

Answer :

- 11** A factory assembles three types of toys Q , R and S . The total time taken to assemble one unit of R and one unit of S exceeds the time taken to assemble two units of Q by 8 minutes. One unit of Q , two units of R and one unit of S take 31 minutes to assembled. The time taken to assemble two units of Q , one unit of R and three units of S is 48 minutes.

- (a) If x , y and z represent the time, in minutes, taken to assemble each unit of toys Q , R and S respectively, write a system of linear equations to represent the above information.
- (b) Hence, find the time taken to assemble each type of toy.

(Ans : $x = 5$, $y = 8$, $z = 10$)

[7 marks]

[Forecast]

Answer :

- 12** A fruit stall sells three types of fruits, oranges, apples and pineapples. The monthly cost being RM6850 for 2150 of fruits. The cost for an orange, an apple and a pineapples are RM2, RM3 and RM4 respectively. The sale price of an orange, an apple and a pineapples are RM3, RM4.50 and RM5.50 respectively.

If the fruit stall want makes a monthly profit of RM2975, find the minimum number of fruits of each type which has to sell. (Ans : orange = 500, apple = 750, pineapple = 900)

[7 marks]

[Forecast]

Answer :

3.2 Simultaneous equations involving one linear equation and one non-linear equation

3.2.1 Solve simultaneous equations involving one linear equation and one non-linear equation.

- 13** (a) Solve the following simultaneous equations using the substitution method :

$$x - 3y + 4 = 0, \quad x^2 + xy - 40 = 0.$$

(Ans : $x = -6, y = -\frac{2}{3}$ and $x = 5, y = 3$)

[5 marks]

[2008, No.1]

Answer :

NOTE :

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$

MIND think :

- **steps in solving simultaneous equations : using substitution method**

- (1) From the linear equation, an unknown is expressed in terms of the other unknown.
- (2) Substituted (1) into the non-linear equation to form a quadratic equation.
- (3) Arranged the quadratic equation in general form : $ax^2 + bx + c = 0$
- (4) Simplify and solve the quadratic equation by using :
 ~ factorization / calculator - CASIO fx-570MS, CANON F-570SG, OLYMPIA ES-570MS . . .
 → **mode** ; **mode** ; **mode** ; **1** ; **▷** ; **2** ;
 → key in value **a** **=** ; key in value **b** **=** ; key in value **c** **=** ;
 → **x_1, x_2** (if **x** only → **$x_1 = x_2$**)
- (5) Obtain the values of other unknowns by substituting the x_1 and x_2 into (1).

- (b) Solve the following simultaneous equations using the elimination method :

$$x - 3y + 4 = 0 \quad , \quad x^2 + xy - 40 = 0.$$

(Ans : $x = -6, y = -\frac{2}{3}$ and $x = 5, y = 3$)

[5 marks]

[2008, No.1]

Answer :

MIND think :

- **steps in solving simultaneous equations : using elimination method**

- (1) Arranged both equation in general form.
- (2) Form quadratic equation :
 (i) in terms of x → eliminate y
 (ii) in terms of y → eliminate x
- (3) Simplify and solve the quadratic equation by using :
 ~ factorization / calculator - CASIO fx-570MS, CANON F-570SG, OLYMPIA ES-570MS . . .
 → **mode** ; **mode** ; **mode** ; **1** ; **▷** ; **2** ;
 → key in value **a** **=** ; key in value **b** **=** ; key in value **c** **=** ;
 → **x_1, x_2** (if **x** only → **$x_1 = x_2$**)
- (4) Obtain the values of other unknowns by substituting the x_1 and x_2 into linear equation.

(c) Solve the following simultaneous equations using the graphical representation method :

$$x - 3y + 4 = 0 \quad , \quad x^2 + xy - 40 = 0.$$

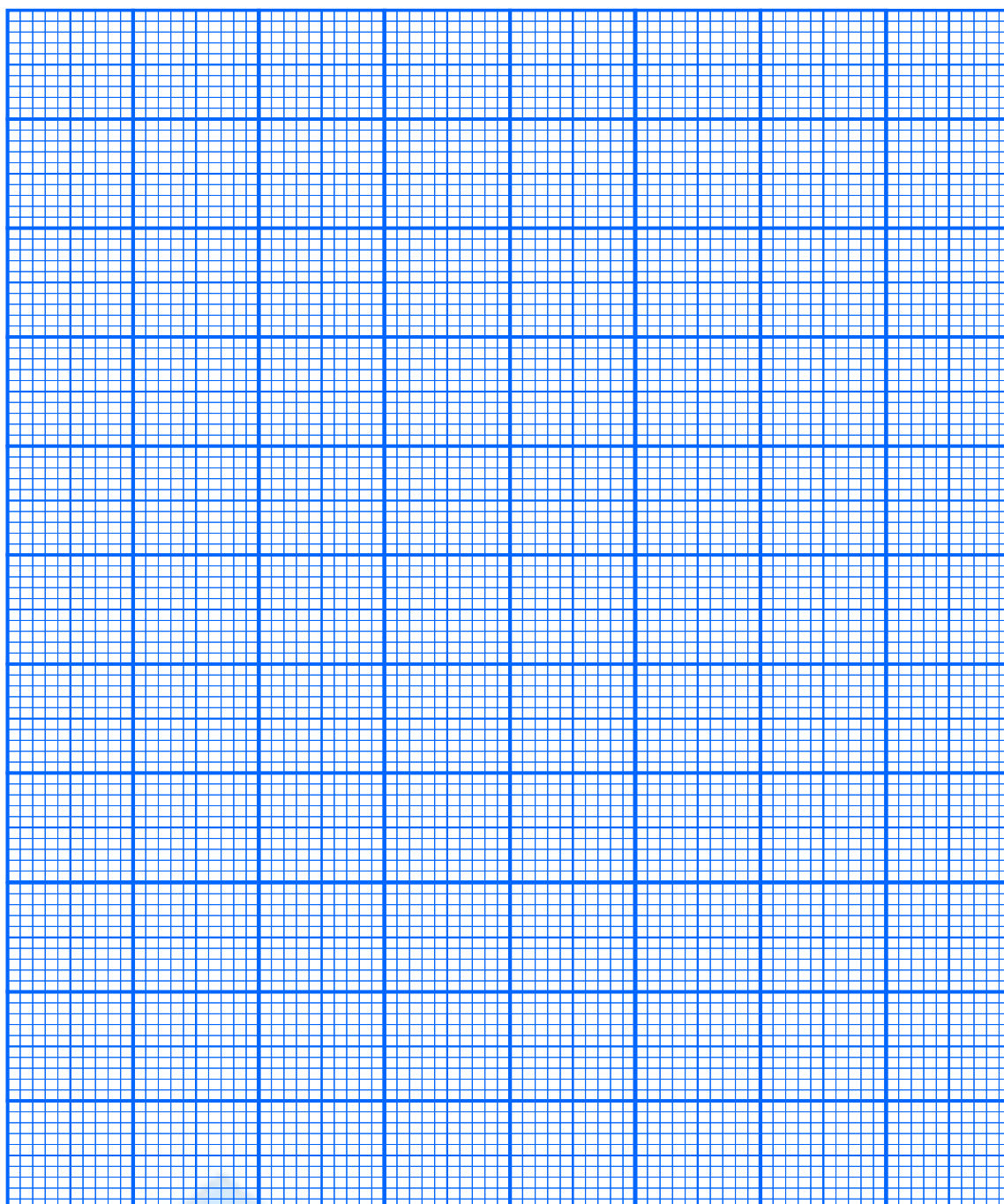
(Ans : $x = -6, y = -\frac{2}{3}$ and $x = 5, y = 3$)

[5 marks]

[2008, No.1]

Answer :

x	-8	-6	-4	-2	0	2	4	6



- 14 (a) Solve the following simultaneous equations using the substitution method :

$$x - 4y = 9 \quad , \quad 3y^2 = 7 - \frac{x}{2} .$$

Give your answers correct to two decimal places.

(Ans : $x = 11.56$, $y = 0.64$ and $x = 3.76$, $y = -1.31$)

[5 marks][Forecast]

Answer :

MIND think :

- **steps in solving simultaneous equations : using substitution method**

- (1) From the linear equation, an unknown is expressed in terms of the other unknown.
- (2) Substituted (1) into the non-linear equation to form a quadratic equation.
- (3) Arranged the quadratic equation in general form : $ax^2 + bx + c = 0$.
- (4) Simplify and solve the quadratic equation by using :

$$\sim \text{formula} \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- (5) Obtain the values of other unknowns by substituting the x_1 and x_2 into (1).

- (b) Solve the following simultaneous equations using the elimination method :

$$x - 4y = 9 \quad , \quad 3y^2 = 7 - \frac{x}{2}.$$

Give your answers correct to two decimal places.

(Ans : $x = 11.56$, $y = 0.64$ and $x = 3.76$, $y = -1.31$)

[5 marks]

[Forecast]

Answer :

MIND think :

- **steps in solving simultaneous equations : using elimination method**

- (1) Arranged both equation in general form.
- (2) Form quadratic equation :
 - (i) in terms of $x \rightarrow$ eliminate y
 - (ii) in terms of $y \rightarrow$ eliminate x
- (3) Simplify and solve the quadratic equation by using :

$$\sim \text{formula} \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- (4) Obtain the values of other unknowns by substituting the x_1 and x_2 into linear equation.

- (c) Solve the following simultaneous equations using the graphical representation method :

$$x - 4y = 9 \quad , \quad 3y^2 = 7 - \frac{x}{2}$$

Give your answers correct to (one / two) decimal places.

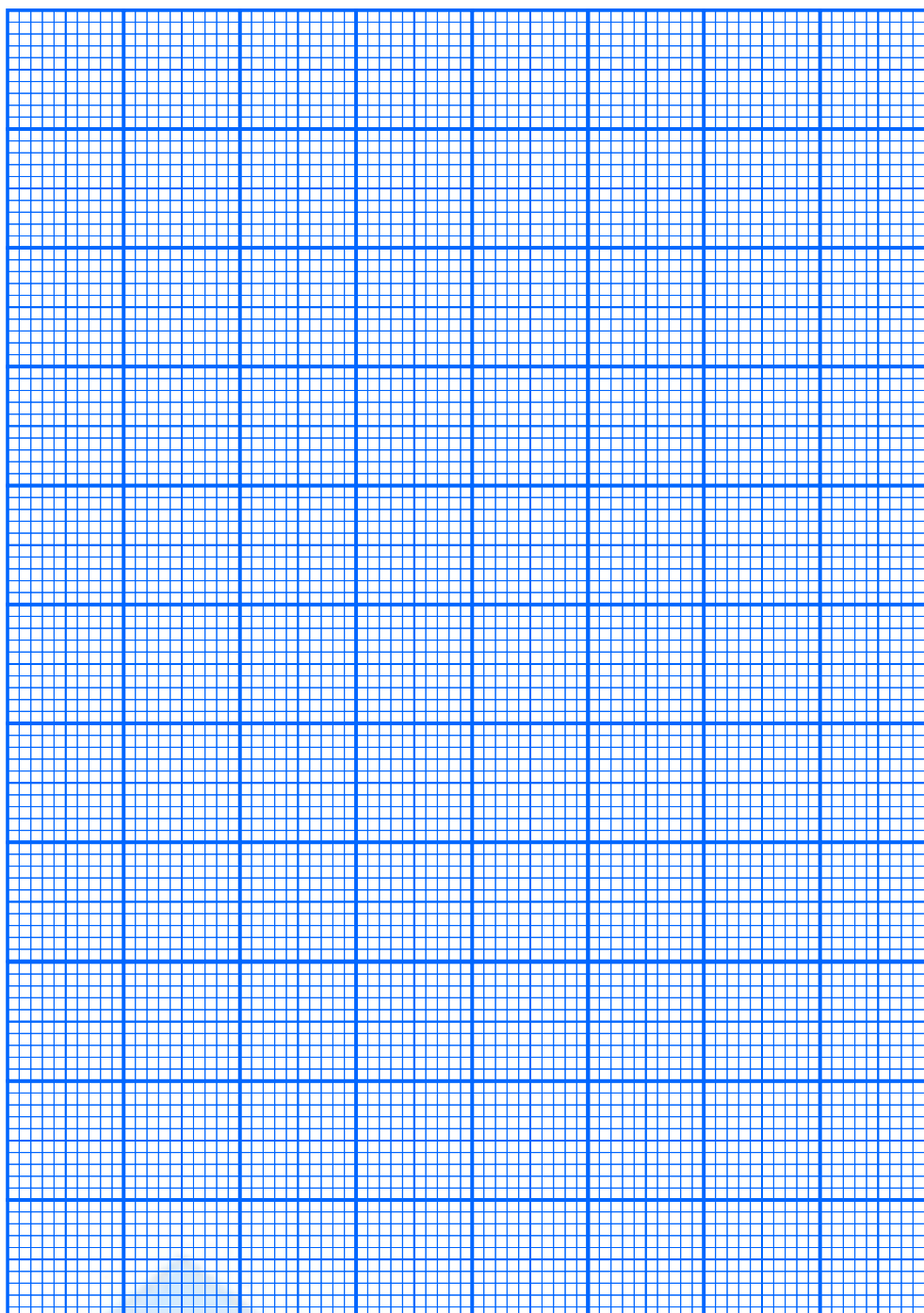
(Ans : $x = 11.56$, $y = 0.64$ and $x = 3.76$, $y = -1.31$)

[5 marks]

[Forecast]

Answer :

y	-1.5	-1.2	-0.8	-0.4	0	0.4	0.8	1.5



⇒ **Part A ~ 1 (using factorisation / calculator)**

- 15** Solve the simultaneous equations $4x + y = -8$ and $x^2 + x - y = 2$.
(Ans : $x = -3$, $y = 4$ and $x = -2$, $y = 0$)

[5 marks]

[2003, No.1]

Answer :

- 16** Solve the simultaneous equations $x + \frac{1}{2}y = 1$ and $y^2 - 10 = 2x$.
(Ans : $x = 3$, $y = -4$ and $x = -\frac{1}{2}$, $y = 3$)

[5 marks]

[2005, No.1]

Answer :

- 17** Solve the following simultaneous equations :

$$2x - y - 3 = 0, \quad 2x^2 - 10x + y + 9 = 0.$$

(Ans : $x = 1$, $y = -1$ and $x = 3$, $y = 3$)

[5 marks]

[2007, No.1]

Answer :

- 18 Solve the simultaneous equations :

$$3x + y = 1, \quad 5x^2 + y^2 + 4xy - 5 = 0$$

(Ans : $x = -1, y = 4$ and $x = 2, y = -5$)

[5 marks]

[2012, No.1]

Answer :

\Rightarrow *forecast*

- 19 Solve the simultaneous equations $4x + y = x^2 + x - y = -3$.

(Ans : $x = -3, y = 9$ and $x = -2, y = 5$)

[5 marks]

Answer :

- 20 Solve the simultaneous equations $\frac{x}{4} - \frac{y}{3} + 2 = 0$ and $\frac{4}{x} + \frac{3}{y} - \frac{4}{3} = 0$.

(Ans : $x = -6, y = \frac{3}{2}$ and $x = 4, y = 9$)

[5 marks]

Answer :

- 21 Given that $(2k, 3p)$ is a solution of the simultaneous equations $x - 2y = 8$ and $\frac{2}{x} + \frac{3}{2y} = \frac{1}{2}$.
Find the value of k and the value of p .

(Ans : $k = 1, p = -1$ and $k = 8, p = \frac{4}{3}$)

[5 marks]

Answer :

\Rightarrow Part A ~ 2

- 22 Solve the simultaneous equations $p - m = 2$ and $p^2 + 2m = 8$.
Give your answers correct to three decimal places.

(Ans : $m = 0.606, p = 2.606$ and $m = -6.606, p = -4.606$)

[5 marks]

[2004, No.1]

Answer :

- 23** Solve the simultaneous equations $2x + y = 1$ and $2x^2 + y^2 + xy = 5$.
Give your answer correct to three decimal places.

(Ans : $x = 1.443$, $y = -1.886$ and $x = -0.693$, $y = 2.386$)

[5 marks]

[2006, No.1]

Answer :

- 24** Solve the simultaneous equations $k - 3p = -1$ and $p + pk - 2k = 0$.
Give your answers correct to three decimal places.

(Ans : $k = 3.731$, $p = 1.577$ and $k = 0.269$, $p = 0.423$). [5 marks]

[2009, No.1]

Answer :

- 25** Solve the simultaneous equations $x - 2y = 7$ and $xy - x = 9y$.
Give your answers correct to two decimal places.

(Ans : $x = 4.76$, $y = -1.12$ and $x = 13.24$, $y = 3.12$)

[5 marks]

[2010, No.1]

Answer :

- 26** Solve the simultaneous equations $y - 2x + 1 = 0$ and $4x^2 + 3y^2 - 2xy = 7$.
Give your answers correct to three decimal places.

(Ans : $x = 1.129$, $y = 1.258$ and $x = -0.295$, $y = -1.590$)

[5 marks]

[2011, No.1]

Answer :

- 27** Solve the simultaneous equation $x + 2y = 1$ and $\frac{3}{x} - \frac{2}{y} = 5$. Give your answer correct to three decimal places.

(Ans : $x = 0.284$, $y = 0.358$ dan $x = 2.116$, $y = -0.558$)

[5 marks]

[2019, No.1]

Answer :

3.1.2 Solve problems involving simultaneous equations ; one linear equation and one non-linear equation.

\Rightarrow problems solving

- 28** Adam planted vegetables on a piece of land. The shape of the land is a right angled triangle. Given the longest side of the land is y metre. The other two sides of the land are x metre and $(2x - 1)$ metre respectively. He fenced the land with 40 metre of barbed wire.

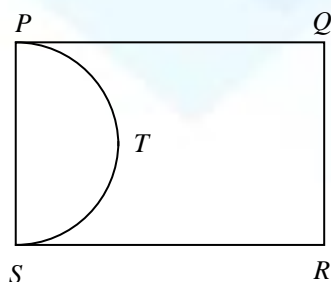
Find the length, in metre, of each side of the land.

(Ans : 8, 15, 17) [7 marks]

[2016, No.3]

Answer :

- 29** The diagram shows the plan of a rectangular garden $PQRS$. The garden consists of a semicircular pond PTS and grassy area $PQRST$.



It is given that $SR = 6y$ metre and $QR = 7x$ metre, $x \neq y$. The area of the rectangular garden $PQRS$ is 168 metre^2 and the perimeter of the grassy area is 60 metre. The pond with uniform depth contains 15.4 metre^3 of water. By using $\pi = \frac{22}{7}$, find the depth, in metre, of water in the pond. (Ans : 0.45). [7 marks]

[2018, No.4]

Answer :

\Rightarrow **forecast**

- 30** The sum of two numbers is 9 and the sum of the squares of the numbers is 53 . Find the product of the numbers. (Ans : 14)

[6 marks]

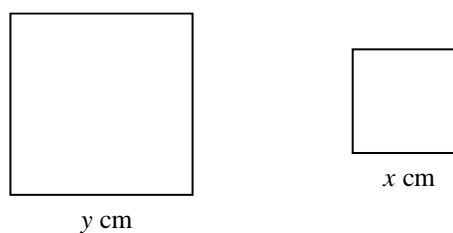
Answer :

- 31** Fernandez bought x number of chickens and y number of ducks for RM208. Given that the total number of chickens and ducks are 20, and the price for each chicken and duck are RM x and RM y respectively, where $y > x$. Find the value of x and the value of y .
(Ans : $x = 8$, $y = 12$)

[6 marks]

Answer :

- 32** A piece of wire, 52 cm in length is cut into two different lengths. Each part of the wire is bent to form a rectangle as shown in the diagram.



If the sum of the area for the both squares is 89 cm^2 , find the values of x and of y .
(Ans : $x = 5$, $y = 8$)

[6 marks]

Answer :

- 33** Given that the different of the circumferences of the two circles is 4π cm and the sum of their areas is 52π cm². Find the radius of each circle. (*Ans* : 4, 6)

[6 marks]

Answer :

- 34** A piece of wire in shape of a circle with radius 14 cm is bent to form a rectangle with sides $(2y + 20)$ cm long $(x + 10)$ cm wide. Given that the area of rectangle is 420 cm², find the values of x and the value of y . (*Ans* : $x=4$, $y=5$ and $x=20$, $y=-3$)

[6 marks]

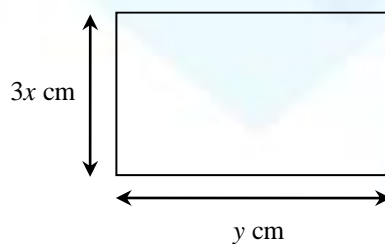
Answer :

- 35** The perimeter of a rectangle is 36 cm and the square of its diagonal is 170 cm². Find the length and width of the rectangle.
(*Ans* : 7, 11)

[6 marks]

Answer :

- 36 The diagram shows a rectangular plank.

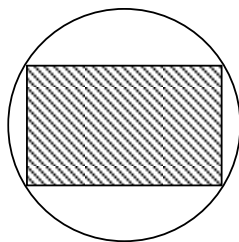


A worker wants to cut the plank into two triangular planks. The perimeter of each triangular plank is 24 cm and the measurement of the longest side of the triangle is $(x + y) \text{ cm}$. Calculate the area, in cm^2 , of the plank.
(Ans : 48)

[6 marks]

Answer :

- 37 The diagram shows a mirror in the shape of a rectangle placed on a table in the shape of a circle.

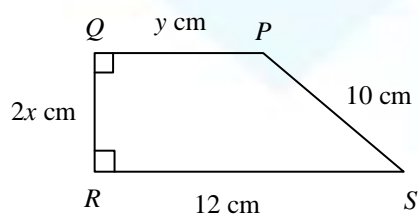


If the perimeter of the rectangle and the diameter of the circle are 44 cm and $\sqrt{340} \text{ cm}$ respectively, find the length and width of the rectangle.
(Ans : length = 18, width = 4)

[6 marks]

Answer :

- 38** A piece of wire, 32 cm in length is bent to form a trapezium $PQRS$ as shown in the diagram, where $\angle PQR = \angle SRQ = 90^\circ$, $PQ = y$ cm, $QR = 2x$ cm, $RS = 12$ cm and $PS = 10$ cm.

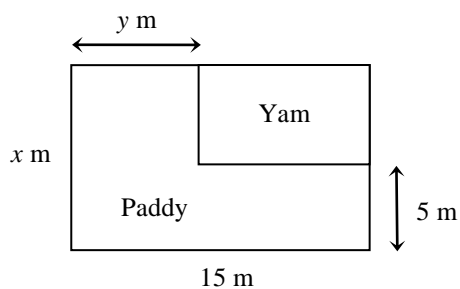


Find the value of x and of y .

(Ans : $x = 3$, $y = 4$)
[6 marks]

Answer :

- 39** Hafizie has a rectangle plot of land. He planted paddy and yam in the area such as the diagram shown below.

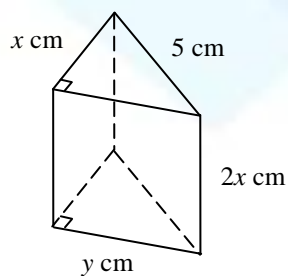


The plantation of yam is in the shape of rectangle. Given that the area of land planted with paddy is 115 m^2 and the perimeter of land planted with yam is 24 m. Find the area of land planted with yam. (Ans : 35 m^2)

[6 marks]

Answer :

- 40 The diagram shows a prism with a uniform cross section in the shape of right-angled triangle.



Given that the height of the prism is $2x$ cm. If the total length of the its sides and the total surface area of the prism are 42 cm and 84 cm^2 respectively, find

- (a) the values of x and y , where both are not whole numbers,

(Ans : $x = 2\frac{4}{5}$, $y = 4\frac{4}{5}$)

- (b) the volume of the prism.

(Ans : $37\frac{79}{125}$)

[7 marks]

Answer :

- 41 A closed rectangular box has a square base. Given that the total length of its sides is 76 cm and the total surfaces area of the box is 238 cm^2 . Find the length of the base and the height of the box.

(Ans : $x = \frac{17}{3}$, $y = \frac{23}{3}$ and $x = 7$, $y = 5$)

6 marks]

Answer :

INDICES, SURDS AND LOGARITHMS

- ONE PAGE NOTE (OPN) - WORKSHEET

**Encik Elbenjoe Wesmin
Puan Nadzrinah binti Ahmad**

NOTE ONE PAGE

“ INDICES, SURDS & LOGARITHMS ”

Indices			Surds	Logarithms
Law of Indices			<ul style="list-style-type: none">surds \rightarrow numbers with radicals $(\sqrt{}, \sqrt[3]{}, \dots, \sqrt[n]{})$, have infinite decimal places and are non-recurring (irrational numbers cannot be converted to fractions) <div>$\sqrt{a} \times \sqrt{a} = a$</div> <div>$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$</div> <div>$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$</div> <div>$(a + \sqrt{b}) \times (a - \sqrt{b}) = a^2 - b$</div> <p>Expression involving surds can be simplified by performing addition, subtraction, and multiplications of surds.</p> <div>$(a+b)^2 = a^2 + b^2 + 2ab$</div> <div>$(a-b)^2 = a^2 + b^2 - 2ab$</div> <div>$(a+b)(a-b) = a^2 - b^2$</div> <div>$\sqrt{a} \times \sqrt{a} = a$</div>	$\log_a N = x \Leftrightarrow N = a^x$ with $a > 0$ and $a \neq 1$
$\underbrace{a \times a \times \dots \times a}_n = a^n$	$a^m \times a^n = a^{m+n}$	$(a^m)^n = a^{m \times n} = (a^n)^m$		Hukum Log
$a^0 = 1$, where $a \neq 0$	$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$	$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$		$\log_a xy = \log_a x + \log_a y$
$\frac{1}{a^n} = a^{-n}$ @	$a^n \times b^n = (ab)^n$	$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$		$\log_a \frac{x}{y} = \log_a x - \log_a y$
$\frac{1}{a^{-n}} = a^n$	$\sqrt[n]{a} = a^{\frac{1}{n}} \Rightarrow \sqrt{a} = a^{\frac{1}{2}} \quad \& \quad \sqrt[3]{a} = a^{\frac{1}{3}}$			$\log_a b^x = x \log_a b$
$\frac{1}{a^{m-n}} = a^{-m+n}$ @	$\frac{k}{a^{-m+n}} = ka^{m-n}$	$\frac{1}{a^m \times a^n} = a^{-(m+n)}$ @ $\frac{1}{a^m \div a^n} = a^{-(m-n)}$		$\log_a b = \frac{\log_c b}{\log_c a}$
$\frac{m}{a^n} = (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m}$ @	$\frac{m}{a^n} = (a^n)^{\frac{1}{m}} = \sqrt[m]{a^n}$		$\log_a b = \frac{1}{\log_b a}$	
If $a^m = a^x$ then $m = n$ or if $a^m = b^x$, then $a = b$ when $a > 0$ and $a \neq 1$			$\log_a 1 = 0$	$\ln x = y \Leftrightarrow x = e^y$

If $a^m = a^n$ then $m = n$ or if $a^m = b^n$, then $a = b$ when $a > 0$ and $a \neq 1$

WORKSHEET

TOPIC 4 : INDICES, SURDS AND LOGARITHMS

[2 – 3 questions → 7 – 10 marks]

Revision → rewrite ; simplify algebraic expressions using laws of indices ; evaluate

⇒ **rewrite**

- 1 The following information is regarding a law of indices.

$$(a^q)^8 = \underbrace{\sqrt{a} \times \sqrt{a} \times \sqrt{a} \times \dots \times \sqrt{a}}_{p \text{ times}}, \text{ where } p \text{ and } q \text{ are constants.}$$

State the value of p and of q .

[2 marks] [2019, No.2]

Answer :

- 2 It is given that $\underbrace{(a \times a \times \dots \times a)}_{(p+q) \text{ times}} \underbrace{(a \times a \times \dots \times a)}_{q \text{ times}} = (\sqrt{a})^n$, where p and q are constants.

Express n in terms of p and q .

(Ans : $n = 2p + 4q$)

[2 marks] [Forecast]

Answer :

MIND think :

LAWS OF INDICES			
$\underbrace{a \times a \times \dots \times a}_{n \text{ times}} = a^n$	$a^m \times a^n = a^{m+n}$	$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$	$\left(a^m\right)^n = a^{m \times n} = \left(a^n\right)^m$
$a^0 = 1$, where $a \neq 0$	$a^n \times b^n = (ab)^n$	$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$	$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
$\frac{1}{a^n} = a^{-n}$ @ $\frac{1}{a^{-n}} = a^n$	$\sqrt[n]{a} = a^{\frac{1}{n}} \Rightarrow \sqrt{a} = a^{\frac{1}{2}} \quad \& \quad \sqrt[3]{a} = a^{\frac{1}{3}}$		
$\frac{1}{a^{m-n}} = a^{-m+n}$ @ $\frac{k}{a^{-m+n}} = ka^{m-n}$	$\frac{1}{a^m \times a^n} = a^{-(m+n)}$ @ $\frac{1}{a^m \div a^n} = a^{-(m-n)}$		
$a^{\frac{m}{n}} = \left(a^m\right)^{\frac{1}{n}} = \sqrt[n]{a^m}$ @ $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m$			

3 (a) Given $(32)^{-\frac{3}{5}} = \frac{1}{\sqrt[m]{32^n}}$, find the value of m and of n . [2 marks]

(b) Given $\frac{1}{5^{\frac{3}{4}}} = \sqrt[n]{5^m}$, find the value of $m - n$. (Ans : -7) [2 marks]

[Mate SPM, J2010, P1, No.24 / clon Mate SPM, J2016, P1, No.20]

Answer :

(a)

(b)

\Rightarrow **simplify 1**

4 Simplify :

(a) $(p^a)^{b+c} \div (p^b)^{a-c} \div (p^c)^{a+b}$ (Ans : 1) [2 marks] [clon UEC, 2013, P1, No.18]

(b) $x^{(a+b)(a-b)} \times x^{(b+c)(b-c)} \times x^{(c-a)(c+a)}$ (Ans : 1) [2 marks] [UEC, 2004, P1, No.18]

Answer :

(a)

(b)

5 (a) Simplify $(\sqrt[3]{x} - \sqrt[3]{y})(\sqrt[3]{x^2} + \sqrt[3]{xy} + \sqrt[3]{y^2})$ (Ans : $x - y$) [3 marks] [UEC, 1990, P2, No.4(b)]

(b) Given $a^{2x} = 2$, find the value of $(a^x + a^{-x})^2$. (Ans : $\frac{9}{2}$) [3 marks] [clon UEC, 2002, P1, No.17]

Answer :

(a)

(b)

⇒ **simplify 2**

6 Simplify : $\frac{(6x^4y^3)^2}{9x^5y}$

(Ans : $4x^3y^5$)

[2 marks] [2014, No.6]

Answer :

7 Simplify :

(a) $\frac{m^{\frac{3}{4}} \times 5\sqrt{m}}{\frac{1}{2}\left(\sqrt[5]{m^2}\right)}$

(Ans : $10m^{\frac{17}{20}}$) [2 marks] [clon Mate SPM, 2012, No.24]

i. $\left[\frac{r^{10} \times (4s^8)^{\frac{1}{2}}}{\sqrt[4]{r^{16}s^{24}}} \right]^3$

(Ans : $\frac{8r^{18}}{s^6}$) [2 marks] [clon Mate SPM, 2017, No.24]

Answer :

(a)

(b)

⇒ **simplify 3**

8 Given $\frac{27h^{10}k^4 \times h^xk^2}{ah^7k} = 3h^5k^y$. Calculate the value of $a + x + y$.

(Ans : 16)

[2 marks] [Forecast]

Answer :

- 9 Given $\frac{2^6 \times 3^x}{2^y \times 3^2} = 18$. Find the value of $x + y$.

(Ans : 9)

[3 marks] [Forecast]

Answer :

 \Rightarrow **evaluate**

- 10 Without using a calculator, find the value of $12^{\frac{3}{2}} \times 2^{-1} \div \sqrt{27}$.

(Ans : 4)

[3 marks] [Forecast]

Answer :

MIND think :

TABLE FOR NUMBERS POWER OF n									
BASE 2	$2^{-3} = \frac{1}{8}$	$2^{-2} = \frac{1}{4}$	$2^{-1} = \frac{1}{2}$	$2^0 = 1$	$2^1 = 2$	$2^2 = 4$	$2^3 = 8$	$2^4 = 16$	$2^5 = 32$
BASE 3	$3^{-3} = \frac{1}{27}$	$3^{-2} = \frac{1}{9}$	$3^{-1} = \frac{1}{3}$	$3^0 = 1$	$3^1 = 3$	$3^2 = 9$	$3^3 = 27$	$3^4 = 81$	$3^5 = 243$
BASE 5	$5^{-3} = \frac{1}{125}$	$5^{-2} = \frac{1}{25}$	$5^{-1} = \frac{1}{5}$	$5^0 = 1$	$5^1 = 5$	$5^2 = 25$	$5^3 = 125$	$5^4 = 625$	
BASE 6	$6^{-3} = \frac{1}{216}$	$6^{-2} = \frac{1}{36}$	$6^{-1} = \frac{1}{6}$	$6^0 = 1$	$6^1 = 6$	$6^2 = 36$	$6^3 = 216$		$2^6 = 64$
BASE 7	$7^{-3} = \frac{1}{343}$	$7^{-2} = \frac{1}{49}$	$7^{-1} = \frac{1}{7}$	$7^0 = 1$	$7^1 = 7$	$7^2 = 49$	$7^3 = 343$		$2^7 = 128$
BASE 10	$10^{-3} = \frac{1}{1000}$	$10^{-2} = \frac{1}{100}$	$10^{-1} = \frac{1}{10}$	$10^0 = 1$	$10^1 = 10$	$10^2 = 100$	$10^3 = 1000$		$2^8 = 256$
$2 = 4^{\frac{1}{2}}$		$3 = 9^{\frac{1}{2}}$		$5 = 25^{\frac{1}{2}}$		$7 = 49^{\frac{1}{2}}$		$2 = 8^{\frac{1}{3}}$	$3 = 27^{\frac{1}{3}}$

4.1 Laws of Indices

4.1.1 Simplify algebraic expressions involving indices using the laws of indices.

\Rightarrow **simplify 1**

11 Simplify : $9^{n+2} \div [3^{1+n} \times 27^{1-n}]$

(Ans : 3^{4n})

[2 marks] [Forecast]

Answer :

12 Simplify : $\frac{16 \times 2^{2n-6}}{4^{2n+1}}$

(Ans : 2^{-4-2n})

[2 marks] [Forecast]

Answer :

13 Simplify : $\frac{49^{2n+1} \times 14^{6-2n}}{56^{2n-5}}$

(Ans : $7^{13} \times 2^{21-8n}$)

[3 marks] [Forecast]

Answer :

\Rightarrow *simplify 2*

- 14 Given that $3^{n+2} - 3^n + 10(3^{n-1}) = h(3^n)$, where h is a constant. Find the value of h . (Ans : $\frac{34}{3}$)
[2 marks] [Forecast]

Answer :

- 15 Show that 9 is a factor of $4^{n+1} + 4^n - 2(4^{n-1})$. [Ans : $\frac{9}{2}(4^n)$]
[2 marks] [Forecast]

Answer :

- 16 Show that $(3^5 + 13)5^{3n} + 2^{8(n+1)}$ is a multiple of 8 for all positive integers of n . [Ans : $256(5^{3n} + 2^{8n})$]
Show that $(3^5 + 13)5^{3n} + 2^{8(n+1)}$ is divisible by 8 for all positive integers of n .
[2 marks] [Forecast]

Answer :

4.1.2 Solve problems involving indices.

\Rightarrow solve equations involving indices 1 ~ with same base 1

17 Solve the equation $32^{4x} = 4^{8x+6}$.

(Ans : 3)
[3 marks] [2004, No.7]

Answer :

18 Solve the equation $8^{2x-3} = \frac{1}{\sqrt{4^{x+2}}}$.

(Ans: 1)
[3 marks] [2006, No.6]

Answer :

19 Given that $9(3^{n-1}) = 27^n$, find the value of n .

(Ans : $\frac{1}{2}$)
[3 marks] [2007, No.8]

Answer :

20 Solve the equation :

$$16^{2x-3} - 8^{4x} = 0$$

(Ans : -3)
[3 marks] [clon 2008, No.7]

Answer :

- 21 Given $3^{n-3} \times 27^n = 243$, find the value of n .

(Ans : 2)

[3 marks] [2009, No.7]

Answer :

- 22 Solve the equation :

$$27(3^{2x+4}) = 1$$

(Ans : $-\frac{7}{2}$)

[3 marks] [2012, No.7]

Answer :

- 23 Given $\frac{25^{h+3}}{125^{p-1}} = 1$, express p in term of h .

(Ans : $p = \frac{2h+9}{3}$)

[3 marks] [2017, No.5]

Answer :

- 24 Solve the equation $\sqrt{16^{x+2}} = \frac{1}{4^x \div 2^{x+1}}$.

(Ans : -1)

[3 marks] [Forecast]

Answer :

- 25 Given $2^{x-3} = (0.125)^{\frac{5}{3}y}$, express y in term of x .

(Ans : $y = \frac{3-x}{5}$)

[3 marks] [clon UEC, 1984, P2, No.2(b)]

Answer :

- 26 Solve the equation $(3^x)^x \times 27^{2x} = 2187$.

(Ans : $-7, 1$)

[3 marks] [Forecast]

Answer :

\Rightarrow solve equations involving indices 2 ~ with same base 2

- 27 Solve the equation $9^x \cdot 2^{2x} = 216$

(Ans : $\frac{3}{2}$)

[3 marks] [Forecast]

Answer :

- 28 Solve the equation $2^x \cdot 5^x = 0.01 (10^{x+1})^4$

(Ans : $-\frac{2}{3}$)

[3 marks] [Forecast]

Answer :

\Rightarrow solve equations involving indices 3 ~ with same base 3

29 Solve the equation $2^{x+4} - 2^{x+3} = 1$.

(Ans : -3)

[3 marks] [2005, No.7]

Answer :

30 Solve the equation $3^{x+2} - 3^x = \frac{8}{9}$.

(Ans : -2)

[3 marks] [2010, No.7]

Answer :

31 Solve the equation : $2^{3x} = 8 + 2^{3x-1}$

(Ans : $\frac{4}{3}$)

[4 marks] [2011, No.7]

Answer :

32 Given $2^p + 2^p = 2^k$, express p in term of k .

(Ans : $p = k - 1$)

[2 marks] [2018, No.16]

Answer :

\Rightarrow solve equations involving indices 4

- 33** Given $3^{2x} = k$, $3^y = h$ and $3^{y+2x} = 7 + 9^x$. Express k in term of h .

(Ans : $k = \frac{7}{h-1}$)

[3 marks] [2015, No.7]

Answer :

- 34** Given $5^{2x} = m$, $5^y = n$ and $5(25^x) - 1 = 5^{2x-y}$. Express m in term of n . (Ans : $m = \frac{n}{5n-1}$)

[3 marks] [clon 2015, No.7]

Answer :

\Rightarrow solve equations involving indices 5 ~ solve by “elimination” or “substitution”

- 35** Solve the simultaneous equations : $3^x \times 9^{y-1} = 243$ and $\frac{2^{3x}}{4^y} = 32$ (Ans : $x = 3, y = 2$)

[4 marks] [Forecast]

Answer :

- 36** Solve the simultaneous equations : $3^x \times 9^{2y} = 1$ and $5^{5x} \times 25^y = \frac{1}{25}$ (Ans : $x = -\frac{4}{9}, y = \frac{1}{9}$)

[4 marks] [Forecast]

Answer :

\Rightarrow solve equations involving indices 6

- 37** Given $3(4^p) = 5(2^q)$ and $9(8^p) = 10(4^q)$, show that $2^{p+1} = 5$.

[3 marks] [Forecast]

Answer :

- 38** Given $3^p = 5^q = 15^r$, express r in terms of p and q .

(Ans : $r = \frac{pq}{p+q}$)

[3 marks] [2016, No.15]

Answer :

- 39** Given $2^a = 5^b = 10^c$, , express a in terms of b and c .

(Ans : $a = \frac{bc}{b-c}$)

[3 marks] [Forecast]

Answer :

\Rightarrow solve equations involving indices 7 ~ let $a^x = y$, form a quadratic equation

- 40** Solve the equation : $3 \times 9^x = 2 \times 3^x + 1$.

(Jwp : 0)

[4 marks] [Forecast]

Answer :

- 41** Solve the equation : $2^{3+2x} + 4 = 33 \times 2^x$.

(Ans : -3, 2)

[4 marks] [Forecast]

Answer :

\Rightarrow solve daily problems

- 42** The number of chairs produced by a furniture factory in January 1990 was 5000 units. It is estimated that the production of chairs will increase with a rate of 20% each year. After t year, the number of chairs that produced is given by $P(1+k)^t$, where P is the number of chairs produced on January 1990 and k is the yearly rate of incensement in production of chairs.

(a) State the value of P and of k .

[1 mark]

(b) Find the number of chairs that produced in January 2002. State your answer in the nearest integer.

(Ans : 44581) [2 marks] [Forecast]

Answer :

(a)

(b)

- 43** The population of a country increases according to the function, $P = 2400000 e^{0.03t}$ where t is the number of years after 1990 and $e = 2.718$.

(a) What is the population of the country in 1990 ?

(b) What is the population of the country in 2010 ?

(Ans : 4372813)

[2 marks]

[Forecast]

Answer :

(a)

(b)

- 44** The temperature of a liquid decreases from 100°C to $x^{\circ}\text{C}$ is according to the equation $x = 100(0.98)^t$ when the liquid is cooled for t seconds.

(a) State the percentage of change in temperature for each second.

[1 mark]

(b) Find the difference in liquid temperature between fifth and sixth second, in nearest Celsius degree.

(Ans : 2) [2 marks]

Answer :

(a)

(b)

- 45** The half life of a radioactive sample is the time taken for its radioactivity to become half its initial value. The half life of the sample is 50 minutes. Its radioactivity, R , counts per minute,

is given by $R = 1024 \left(\frac{1}{2}\right)^{n+1}$, where n is the number of half life experienced

by the sample. The radioactivity of the sample reduces to 4^n counts per minute at t minutes. Find the value of t .

(Ans : 150)

[4 marks]

[Forecast]

Answer :

4.2 Laws of surds
4.2.1 Compare rational numbers and irrational numbers, and hence relate surds to irrational numbers.

\Rightarrow conversion of recurring decimal to fractional form

- 46** Express the recurring decimal 0.969696 as a fraction in its simplest form.

(Ans : $\frac{32}{33}$)

[4 marks] [2004, No.12]

Answer :

- 47** Given $\frac{h}{k} = 0.1\dot{6} \dots$ is a recurring decimal where h and k are positive integers. Find the value of $h + k$.

(Ans : 7)

[4 marks] [Forecast]

Answer :

- 48** Given $p = 1.054\dot{5}6$ is a recurring decimal. Express p as a fraction in its simplest form.

(Ans : $\frac{35117}{33300}$)

[4 marks] [Forecast]

Answer :

\Rightarrow relate surds to irrational numbers ~ a non-recurring decimal

49 Mark (✓) if the term is surd, mark (✗) if the term is not a surd.

(a) $\sqrt{8}$ (b) $\sqrt[3]{120}$ (c) $\sqrt[4]{4096}$ (d) $\sqrt[6]{\frac{64}{512}}$ (e) $\sqrt[7]{\frac{97}{798}}$

☐
☐
☐
☐
☐

MIND think :

- surds \rightarrow numbers with radicals ($\sqrt{\quad}$, $\sqrt[3]{\quad}$, ..., $\sqrt[n]{\quad}$), have *infinite decimal places* and are *non-recurring* (irrational numbers ~ cannot converted to fractions)

4.2.2 Make and verify conjectures on

(i) $\sqrt{a} \times \sqrt{b}$

(ii) $\sqrt{a} \div \sqrt{b}$

and hence make generalisation.

50 State the following value of surd in five decimal places :

$$\sqrt{3} \times \sqrt{5}$$

$$\sqrt{3 \times 5}$$

\therefore Generalisation ~ for $a > 0$, $b > 0 \rightarrow \sqrt{a} \times \sqrt{b} =$

51 State the following value of surd in five decimal places :

$$\sqrt{14} \div \sqrt{2}$$

$$\sqrt{14 \div 2}$$

\therefore Generalisation ~ for $a > 0$, $b > 0 \rightarrow \sqrt{a} \div \sqrt{b} =$

4.2.3 Simplify expressions involving surds.

\Rightarrow **surd in the form of $a\sqrt{b}$, where a and b are integers, and a is a largest value**

52 Solve the following without using a scientific calculator :

(a) If $\sqrt{2} = 1.41$, find the value of $\sqrt{32}$. (Ans : 5.64) [2 marks] [UEC, 1989, P1, No.12]

(b) If $\sqrt{60} = 7.7$, find the value of $\sqrt{540}$. (Ans : 23.1) [2 marks] [UEC, 1997, P1, No.12]

Answer :

(a)

(b)

\Rightarrow **addition and subtraction**

53 Simplify the following expressions :

(a) $\sqrt{12} - \sqrt{3}$ (Ans : $\sqrt{3}$) [2 marks] [UEC, 1982, P1, No.1]

(b) $\sqrt{32} - \sqrt{50}$ (Ans : $-\sqrt{2}$) [2 marks] [UEC, 1992, P1, No.4]

Answer :

(a)

(b)

54 Simplify the following expressions :

(a) $5\sqrt{24} + 4\sqrt{54} + \sqrt{6}$ (Ans : $23\sqrt{6}$) [2 marks] [UEC, 1990, P1, No.11]

(b) $\sqrt{10} - \sqrt{90} - \sqrt{40}$ (Ans : $-4\sqrt{10}$) [2 marks] [UEC, 1994, P1, No.15]

Answer :

(a)

(b)

55 Simplify the following expressions :

(a) $3\sqrt{12} + 2\sqrt{3} - \sqrt{48}$

(Ans : $4\sqrt{3}$) [2 marks] [UEC, 1988, P1, No.24]

(b) $\sqrt{45} + \sqrt{125} - 2\sqrt{20}$

(Ans : $4\sqrt{5}$) [2 marks] [UEC, 2007, P1, No.39]

Answer :

(a)

(b)

56 Simplify : $2\sqrt{18} + 3\sqrt{8} + 3\sqrt{32} - \sqrt{50}$

(Ans : $19\sqrt{2}$) [2 marks] [UEC, 1996, P2, No.3(a)]

Answer :

57 Simplify the following expressions :

(a) $\frac{\sqrt{54}}{3} + \frac{\sqrt{6}}{2} - \frac{\sqrt{24}}{3}$

(Ans : $\frac{5\sqrt{6}}{6}$) [2 marks] [UEC, 1999, P2, No.4(a)]

(b) $\sqrt{48} - \frac{3}{2}\sqrt{27} + \sqrt{243}$

(Ans : $\frac{17\sqrt{3}}{2}$) [2 marks] [UEC, 2008, P1, No.10]

Answer :

(a)

(b)

58 (a) Simplify : $2\sqrt{50} + \sqrt{90} - \frac{\sqrt{40}}{2}$ (Ans : $10\sqrt{2} + 2\sqrt{10}$) [2 marks]

(b) Given that $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, find $\sqrt{48} - \sqrt{6} \times \sqrt{3}$. (Ans : 2.686) [2 marks]

[UEC, 2009, P2, No.5(b) / UEC, 2014, P1, No.11]

Answer :

(a)

(b)

\Rightarrow **multiplication**

59 Without using a scientific calculator, find

(a) $(2\sqrt{3} - \sqrt{2})^2$. (Ans : $14 - 4\sqrt{6}$) [2 marks] [UEC, 1986, P1, No.15]

(b) $\left(1 - \frac{1}{\sqrt{2}}\right)^2$. (Ans : $\frac{3}{2} - \sqrt{2}$) [2 marks] [UEC, 2010, P1, No.12]

Answer :

(a)

(b)

MIND think :

$(a+b)^2 = a^2 + b^2 + 2ab$	$(a-b)^2 = a^2 + b^2 - 2ab$	$(a+b)(a-b) = a^2 - b^2$	$\sqrt{a} \times \sqrt{a} = a$
-----------------------------	-----------------------------	--------------------------	--------------------------------

60 Without using a scientific calculator, find

(a) $(5+2\sqrt{3})(5-2\sqrt{3})$. (Ans : 13) [2 marks] [UEC, 1982, P1, No.2]

(b) $(2\sqrt{2}-3)^2 - (2\sqrt{2}+3)^2$. (Ans : $-24\sqrt{2}$) [2 marks] [UEC, 2010, P2, No.4(a)]

Answer :

(a)

(b)

- 61 Given that $a = 3 + \sqrt{2}$, $b = 3 - \sqrt{2}$, find the value of $a^2b - ab^2$. (Ans : $14\sqrt{2}$)

[3 marks] [UEC, 2014, P1, No.18]

Answer :

4.2.4 Simplify expressions involving surds by rationalising the denominators.

⇒ **division 1**

- 62 Without using a scientific calculator, simplify each of the following :

(a) $\frac{\sqrt{28}}{\sqrt{7}}$ (Ans : 2) [2 marks] [UEC, 1987, P1, No.19]

(b) $\frac{20 + 10\sqrt{2}}{\sqrt{10}}$ (Ans : $2\sqrt{10} + 2\sqrt{5}$) [2 marks] [UEC, 1984, P1, No.9]

Answer :

(a) (b)

- 63 Without using a scientific calculator, simplify : $\sqrt{1\frac{1}{2}} \times 2\sqrt{3}$. (Ans : $3\sqrt{2}$)

[2 marks] [UEC, 2002, P1, No.7]

Answer :

$$\Rightarrow \text{division 2} \quad \sim p\sqrt{a} - q\sqrt{b} \xleftrightarrow{\text{conjugate surd}} p\sqrt{a} + q\sqrt{b}$$

64 Simplify each of the following :

(a) $\frac{1}{\sqrt{5} - \sqrt{2}}$ (Ans : $\frac{1}{3}(\sqrt{5} + \sqrt{2})$) [2 marks] [clon UEC, 1985, P1, No.5]

(b) $\frac{1}{\sqrt{3} + 2}$ (Ans : $2 - \sqrt{3}$) [2 marks] [UEC, 1995, P1, No.15]

Answer :

(a)

(b)

65 Simplify each of the following :

(a) $\frac{2}{\sqrt{6} + 2}$ (Ans : $\sqrt{6} - 2$) [2 marks] [UEC, 2006, P1, No.3(a)]

(b) $\frac{\sqrt{8}}{\sqrt{5} - \sqrt{3}}$ (Ans : $\sqrt{10} + \sqrt{6}$) [2 marks] [UEC, 2011, P1, No.10]

Answer :

(a)

(b)

66 Given that $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, find the value of $\frac{4}{\sqrt{3} - \sqrt{2}}$. (Ans : 12.584)

[3 markah] [UEC, 2010, P1, No.8]

Answer :

⇒ **division 3**

67 Simplify each of the following :

(a) $\frac{\sqrt{2} + 1}{\sqrt{2} - 1}$

(Ans : $3 + 2\sqrt{2}$) [2 marks] [UEC, 1976, P1, No.10]

(b) $\frac{\sqrt{2} - 1}{\sqrt{2} + 1}$

(Ans : $3 - 2\sqrt{2}$) [2 marks] [UEC, 1988, P1, No.25]

Answer :

(a)

(b)

68 Simplify : $\frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$

(Ans : $5 + 2\sqrt{6}$)

[2 marks] [UEC, 1998, P1, No.16]

Answer :

⇒ **division 4**

69 Simplify each of the following :

(a) $\sqrt{8} + \frac{2}{1 - \sqrt{2}}$

(Ans : -2) [3 marks] [UEC, 2001, P2, No.3(b)]

(b) $\frac{\sqrt{6} + \sqrt{3}}{\sqrt{6} - \sqrt{3}} - \sqrt{8}$

(Ans : 3) [3 marks] [UEC, 2013, P2, No.2(a)]

Answer :

(a)

(b)

70 (a) Simplify : $(1 - \sqrt{2})^2 - \frac{1}{\sqrt{2} - 1}$ (Ans : $2 - 3\sqrt{2}$) [3 marks] [UEC, 1995, P2, No.4(a)]

(b) If $\sqrt{3} = 1.732$, find the value of $1 + \frac{\sqrt{3} + 2}{\sqrt{3} - 2}$. (Ans : -12.928) [4 marks] [UEC, 2003, P2, No.5(a)]

Answer :

(a)

(b)

⇒ **division 5**

71 Simplify each of the following :

(a) $\frac{1}{\sqrt{3} - \sqrt{2}} - \frac{1}{\sqrt{3}}$ (Ans : $\frac{3\sqrt{2} + 2\sqrt{3}}{3}$) [3 marks] [UEC, 2000, P2, No.5(a)]

(b) $\frac{1}{\sqrt{6} - 2} - \frac{1}{\sqrt{6} + 2}$ (Ans : 2) [3 marks] [UEC, 2008, P2, No.4(a)]

Answer :

(a)

(b)

72 Simplify each of the following :

(a) $\frac{1}{\sqrt{3} - \sqrt{2}} - \frac{1}{\sqrt{3} + \sqrt{2}}$ (Ans : $2\sqrt{2}$) [3 marks] [UEC, 1993, P1, No.18]

(b) $\frac{1}{\sqrt{5} + \sqrt{3}} - \frac{1}{\sqrt{5} - \sqrt{3}}$ (Ans : $-\sqrt{3}$) [3 marks] [UEC, 2012, P1, No.9]

Answer :

(a)

(b)

⇒ **division 6**

73 (a) Simplify : $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} + \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ (Ans : 8) [3 marks]

(b) Express $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$ in terms of $a\sqrt{5}+b\sqrt{6}$. (Ans : $\sqrt{5}-\sqrt{6}$) [3 marks]

[UEC, 2002, P2, No.5(b) / UEC, 1976, P2, No.3(b)]

Answer :

(a)

(b)

74 Simplify : $\frac{3-\sqrt{2}}{5+\sqrt{2}} + \frac{4+\sqrt{3}}{7-\sqrt{3}}$ (Ans : $\frac{65-16\sqrt{2}+11\sqrt{3}}{46}$)

[2 marks] [Forecast]

Answer :

75 Given that $\sqrt{3} = 1.732$, find the value of $\frac{\sqrt{3}+1}{\sqrt{3}-1} - \frac{1}{\sqrt{3}+1}$ correct to two decimal place. (Ans : 3.37)

[4 marks] [UEC, 2009, P2, No.5(a)]

Answer :

⇒ **division 7**

76 Simplify each of the following

(a) $\frac{6}{\sqrt{12}} + \frac{1}{\sqrt{3} + 2} - 3\sqrt{\frac{1}{3}}$ (Ans : $2 - \sqrt{3}$) [3 marks] [UEC, 2004, P2, No.5(a)]

(b) $\frac{1}{\sqrt{2} - 1} + \frac{2}{\sqrt{5} + \sqrt{3}} - \frac{3}{\sqrt{5} - \sqrt{2}}$ (Ans : $1 - \sqrt{3}$) [3 marks] [UEC, 2007, P2, No.4(a)]

Answer :

(a)

(b)

77 Simplify : $\frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{1 + \sqrt{2}} - \frac{2}{1 + \sqrt{3}}$ (Ans : 0)

[3 marks] [UEC, 2015, P2, No.4(a)]

Answer :

4.2.5 Solve problems involving surds.

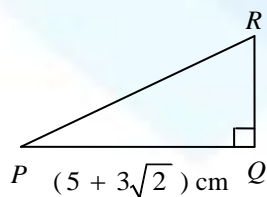
⇒ **solve problems 1 ~ daily problems**

78 The length and width of a rectangle are $(\sqrt{5} + \sqrt{2})$ cm and $(2\sqrt{5} - \sqrt{2})$ cm respectively. Find the area, in cm^2 , of the rectangle in the form of $a + b\sqrt{10}$. (Ans : $8 + \sqrt{10}$)

[2 marks] [Forecast]

Answer :

- 79 The diagram shows a right-angled triangle PQR .



Given that the area of the triangle PQR is $\frac{7}{2} \text{ cm}^2$. Find

- (a) the height, in cm, of the triangle in the form of $a + b\sqrt{2}$,

(Ans : $5 - 3\sqrt{2}$)

- (b) the length, in cm, of PR .

(Ans : $\sqrt{86}$)

[4 marks] [Forecast]

Answer :

(a)

(b)

\Rightarrow solve problems 2 ~ solve equation 1

- 80 Solve the equation $3 + 2\sqrt{y} = 4\sqrt{5} + 1$, express your answer in the form of $a + b\sqrt{5}$, where a and b are rational numbers.

(Ans : $21 - 4\sqrt{5}$)

[3 marks] [Forecast]

Answer :

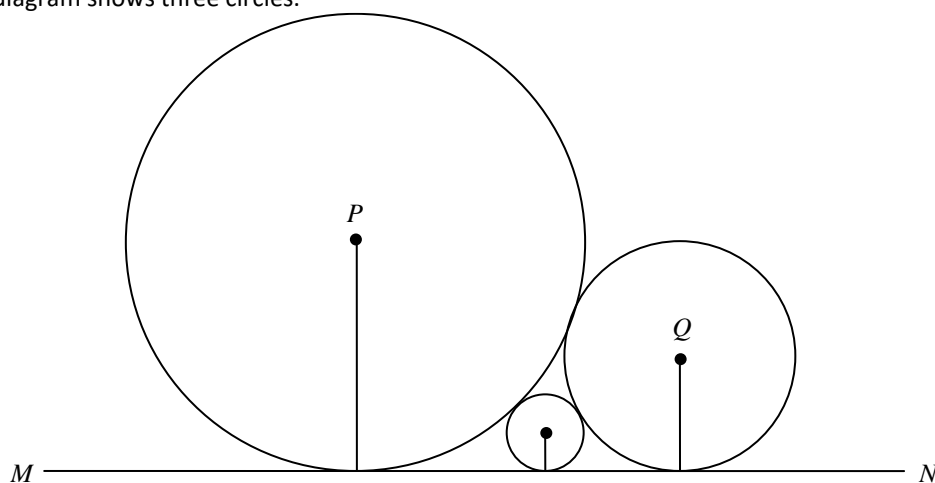
- 81 If $\sqrt{5}x = x + \sqrt{8}$, find the value of x in the form of $\frac{\sqrt{a}}{b}$.

(Ans : $\frac{\sqrt{10} + \sqrt{2}}{2}$)

[3 marks] [Forecast]

Answer :

- 82 The diagram shows three circles.



Circle with centre P has a radius of 2 cm, and circle with centre Q has a radius of 1 cm. MN is a common tangent and all circles touch one another. Find the radius, in cm, of the smaller circle in the form of $a + b\sqrt{2}$.

(Ans : $6 - 4\sqrt{2}$)

[4 marks] [Forecast]

Answer :

\Rightarrow solve problems 3 ~ solve equation 2

83 Solve the equation : $x = 10 - 3\sqrt{x}$

(Answer : 4)

[4 marks] [Forecast]

Answer :

84 Solve the equation : $2x + \sqrt{2 - 7x} = 0$

(Answer : -2)

[4 marks] [Forecast]

Answer :

85 Solve the equation : $3 + \sqrt{5x^2 - 19} = 2x$

(Answer : 2)

[4 marks] [Forecast]

Answer :

86 Solve the equation : $\sqrt{4x + 3} + \sqrt{1 - 4x} = 2$

(Answer : $-\frac{3}{4}, \frac{1}{4}$)

[4 marks] [Forecast]

Answer :

87 Solve the equation : $3\sqrt{1-x} - \sqrt{3x-2} = 1$

(Answer : $\frac{3}{4}$)

[4 marks] [Forecast]

Answer :

88 Solve the equation : $\sqrt{x-3} + \sqrt{x+5} = \sqrt{2x+8}$

(Answer : 4)

[4 marks] [Forecast]

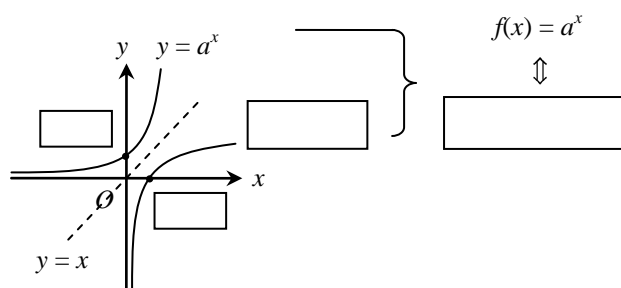
Answer :

4.3 Laws of logarithms

4.3.1 Relate equations in the form of indices and logarithms, and hence determine the logarithm of a number.

89 Complete the following :

$y = a^x$	
\Updownarrow	
$\boxed{} = x$	
$a > \boxed{}$	$\& \ a \neq \boxed{}$
$y > \boxed{}$	



$a^0 = 1 \Leftrightarrow \boxed{}$

$a^1 = a \Leftrightarrow \boxed{}$



$a^{\log_a x} = \boxed{}$

$\log_a a^x = \boxed{}$

$\log_{10} a = \boxed{}$

90 Convert the following to logarithmic form and vice-versa :

Base 2		Base 3		Base 5	
Index	Logarithmic	Index	Logarithmic	Index	Logarithmic
$2^{-4} = \frac{1}{16}$		$3^{-4} = \frac{1}{81}$		$5^{-4} = \frac{1}{625}$	
	$-3 = \log_2 \frac{1}{8}$	$3^{-3} = \frac{1}{27}$			$-3 = \log_5 \frac{1}{125}$
$2^{-2} = \frac{1}{4}$		$3^{-2} = \frac{1}{9}$		$5^{-2} = \frac{1}{25}$	
	$-1 = \log_2 \frac{1}{2}$	$3^{-1} = \frac{1}{3}$			$-1 = \log_5 \frac{1}{5}$
$2^0 = 1$		$3^0 = 1$		$5^0 = 1$	
	$1 = \log_2 2$	$3^1 = 3$			$1 = \log_5 5$
$2^2 = 4$		$3^2 = 9$		$5^2 = 25$	
	$3 = \log_2 8$	$3^3 = 27$			$3 = \log_5 125$
$2^4 = 16$		$3^4 = 81$		$5^4 = 625$	
	$5 = \log_2 32$	$3^5 = 243$			$5 = \log_5 3125$
$2^6 = 64$		$3^6 = 729$			
	$7 = \log_2 128$	$3^7 = 2187$			$3 = \log_x x^3$
$2^8 = 256$		$8 = \log_3 6561$			$2 = \log_x x^2$
	$9 = \log_2 512$				$1 = \log_x x$
$2^{10} = 1024$					
			$-2 = \log_x x^{-2}$		$-1 = \log_x x^{-1}$
					$0 = \log_x x^0$

91 Evaluate each of the following without using a scientific calculator :

(a) $\log_2 \frac{1}{8} \times \log_5 \frac{1}{125}$ (Ans : 9) [2 marks] [UEC, 2015, P1, No.19]

(b) $\log_5 125 + \log_2 \frac{1}{4} - \log_3 \sqrt{3}$ (Ans : $\frac{1}{2}$) [2 marks] [UEC, 1998, P1, No.18]

Answer :

(a)

(b)

92 Evaluate each of the following without using a scientific calculator :

(a) $\log(10\sqrt{10}) - \log_2 0.25$ (Ans : $\frac{7}{2}$) [2 marks] [UEC, 1992, P1, No.5 / UEC, 2001, P1, No.15]

(b) $2\log_{27} 3 - \log_2 \left(\frac{1}{2}\right)^3$ (Ans : $\frac{11}{3}$) [2 marks] [clon UEC, 2011, P1, No.19]

Answer :

(a)

(b)

93 Evaluate each of the following without using a scientific calculator :

(a) $\log_{25} 5 + \log_5 25 - \log_{25} \frac{1}{5}$ (Ans : 3) [2 marks] [UEC, 1993, P1, No.21]

(b) $\frac{1}{2}\log_{81} 9 + 3\log_{16} 2 + \log_9 \frac{1}{9}$ (Ans : 0) [2 marks] [UEC, 2014, P1, No.19]

Answer :

(a)

(b)

94 Solve each of the following :

(a) $\log_{343} x = \frac{1}{3}$ (7) (b) $\lg x = -2$ (0.01) (c) $\log_x 7776 = 5$ (6) (d) $\log_x \frac{1}{81} = -4$ (3)

95 Solve each of the following :

(a) $\log_3 \sqrt{k} = 2$

(Jwp : 81) [2 marks]

(b) $\log_{n^2} 16 = 2$

(Jwp : ± 2) [2 marks]

[Forecast]

Answer :

(a)

(b)

96 Solve each of the following :

(b) $\log_{27} 3y^2 = \frac{1}{3}$

(Ans : ± 1) [3 marks]

(a) $\log_5 (\log_5 x) = \log_5 4$

(Ans : 625) [2 marks]

[Forecast]

Answer :

(a)

(b)

97 Solve each of the following :

(a) $3^{\log_{10} x} = 27$

(Ans : 1000) [3 marks] [UEC, 1987, P1, No.18]

(b) $5^{\log_{1+t} 10} = 10.$

(Ans : 4) [3 marks] [Forecast]

Answer :

(a)

(b)

- 98 Solve the equation $\lg x = 10^{\lg 3}$.
1000)

(Ans :

[2 marks] [Forecast]

Answer :

- 99 (a) Given $\log_{10} a = -2b$, express 100^b in term of a .
(Ans : $\frac{1}{a}$) [3 marks] [UEC, 2012, P1, No.11]

- (b) Given that $x = \log_5 4$. Find the value of $(\sqrt{5})^x$. (Ans : 2) [3 marks] [Forecast]

Answer :

(a)

(b)

- 100 Given that $\log_2 x = 0.66$ and $\log_2 y = 1.68$, find the value of x^2y . (Ans : 8)
[3 marks] [UEC, 2007, P1, No.29]

Answer :

- 101 Given $\log_3 [\log_2 (2x - 1)] = \log_2 4$, find the value of x .

(Ans : $256\frac{1}{2}$)

[3 marks] [Forecast]

Answer :

- 102 Given $\log_3 (m + 3) = n$ and $\log_2 (m - 4) = n - 1$. Show that $m^2 - m - 12 = \frac{1}{2} \times 6^n$.

[3 marks] [Forecast]

Answer :

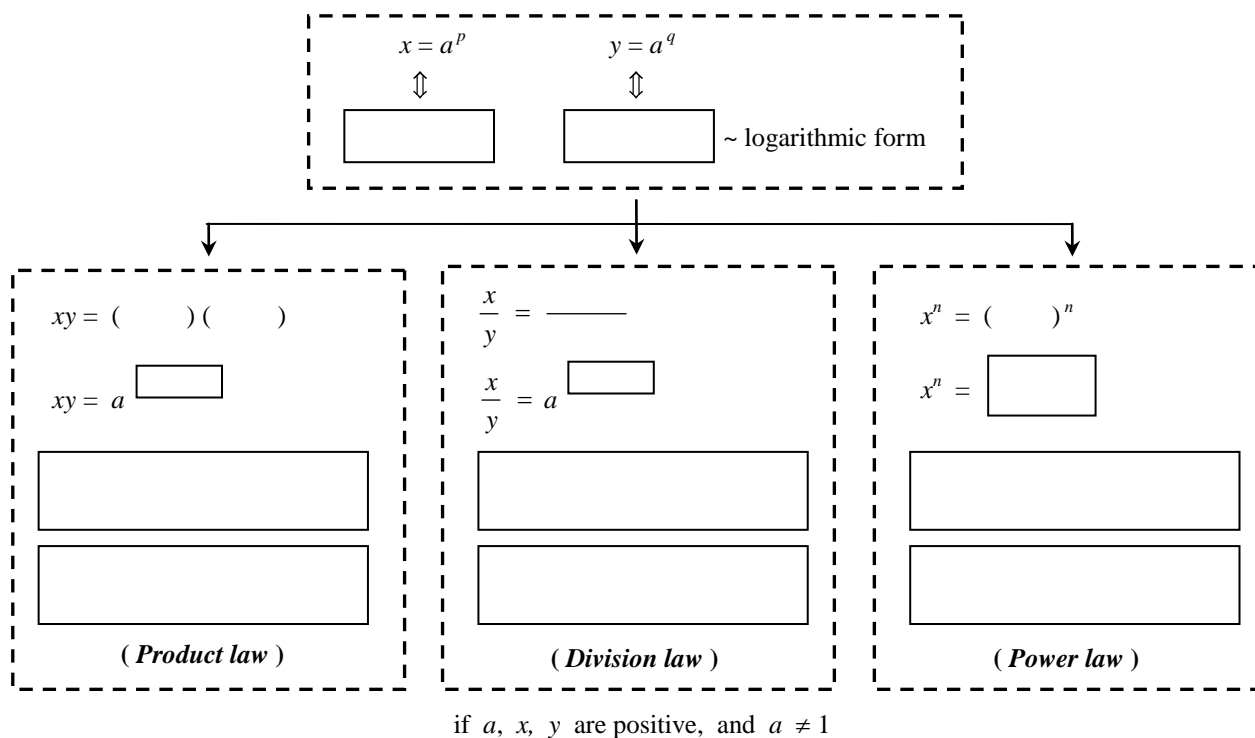
103 By using a scientific calculator, complete each of the following :

$$y = \log_{10} x$$

x	300	3	0.003
y	3.477	1.477	-0.523
		-1.523	

4.3.2 Prove laws of logarithms.

104 Complete the following :



\Rightarrow using law ~ type 1

- 105 (a) Without using a scientific calculator, find the value of $5^{2\log_5 4}$. (Ans : 16) [2 marks]
 (b) Solve the equation $27 = 8^{\log_2 x}$. (Ans : 3) [3 marks]
 [Forecast]

Answer :

(a)

(b)

106 Simplify each of the following :

(a) $\log x^3 \div \log \frac{1}{x}$ (Ans : -3) [2 marks] [UEC, 2004, P1, No.17]

(b) $\frac{\log 5 + \log 125}{2}$ (Ans : $2\log 5$) [2 marks] [UEC, 1988, P1, No.20]

Answer :

(a) (b)

\Rightarrow using law ~ type 2

107 Given $\log 2 = 0.301$ and $\log 7 = 0.845$, find the values of the following logarithms :

(a) $\log 2000$ (Ans : 3.301) [2 marks]

(b) $\log 0.07$ (Ans : -1.155) [2 marks]

[Forecast]

Answer :

(a) (b)

108 Given $\log_5 2 = 0.43$ and $\log_5 7 = 1.21$, find the values of the following logarithms :

(a) $\log_5 70$ (Ans : 2.64) [3 marks]

(b) $\log_5 1.6$ (Ans : 0.29) [3 marks]

[Forecast]

Answer :

(a) (b)

109 Given $\log 2 = 0.3010$ and $\log 3 = 0.4771$, find the values of the following logarithms :

(a) $\log 15$ (Ans : 1.1761) [3 marks]

(b) $\log 0.4444 \dots$ (Ans : -0.3522) [4 marks]

[UEC, 2015, P2, No.3(a) / Forecast]

Answer :

(a) (b)

110 Given $\log 2 = 0.301$ and $\log 3 = 0.477$, find the values of the following logarithms :

(a) $\log 12 - \log 1.5$ (Ans : 0.903) [3 marks] [UEC, 2003, P2, No.5(b)(i)]

(b) $\log \frac{2}{3} + \frac{1}{3} \log 64$ (Ans : 0.426) [4 marks] [UEC, 2012, P2, No.3(b)]

Answer :

(a) (b)

111 Simplify : $\frac{\log 400}{\log_5 25} - (400 \times 25)^0$ (Ans : $\log 2$)

[3 marks] [UEC, 2008, P1, No.15]

Answer :

\Rightarrow using law ~ type 3

112 Without using a scientific calculator, simplify each of the following :

(a) $\frac{\log 9 + \log 25}{\log 15}$ (Ans : 2) [3 marks] [UEC, 2000, P2 No.5(b)]

(b) $\log_3(2\sqrt{7} + 1) + \log_3(2\sqrt{7} - 1)$ (Ans : 3) [3 marks] [UEC, 1996, P1 No.19]

Answer :

(a) (b)

113 Simplify each of the following :

(a) $\frac{\log_2 x^2 + \log_2 y^2}{\log_2 xy}$ (Ans : 2) [3 marks] [UEC, 1997, P1 No.23]

(b) $\frac{\log_m 8 - 6\log_m 3}{\log_m 2 - \log_m 9}$ (Ans : 3) [3 marks] [Forecast]

Answer :

(a) (b)

114 Evaluate each of the following without using a scientific calculator :

(a) $\frac{3}{4}\log_3 81 - \log_3 36^{1.5} + \frac{1}{2}\log_3 64$ (Ans : 0) [4 marks] [UEC, 2008, P2, No.4(c)]

(b) $\log_3 324 - 3\log_3 2 + \frac{1}{2}\log_3 \frac{4}{81}$ (Ans : 2) [3 marks] [UEC, 2010, P2, No.3(b)]

Answer :

(a)

(b)

\Rightarrow using law ~ HOTS

115 Find the value of $(\log 5)^2 + (\log 2)(\log 50)$.

(Ans : 1)

[3 marks] [UEC, 2006, P2, No.1(c)]

Answer :

116 If $a^2 + b^2 = 6ab$, prove that $\log\left(\frac{a-b}{2}\right) = \frac{1}{2}(\log a + \log b)$.

[4 marks] [UEC, 1979, P2, No.6(c)]

Answer :

117 (a) Given $2^{3x} = 9(3^{2x})$. Prove that $x \log_p \frac{8}{9} = \log_p 9$.

[4 marks]

(b) Given $3^m = 12^n$. Show that $\log_2 3 = \frac{2n}{m-n}$.

[4 marks]

[Forecast]

Answer :

(a)

(b)

118 (a) Given $\log_5 3 = k$. If $5^{2\lambda-1} = 15$, express λ in term of k . (Ans : $\lambda = \frac{k+2}{2}$) [3 marks]

(b) Given $\log_3 5 = 1.465$. With using a scientific calculator, solve the equation $5(3^{x-2}) = \sqrt{5}$. (Ans : 1.2675) [3 marks]

[Forecast]

Answer :

(a)

(b)

4.3.3 Simplify algebraic expressions using the laws of logarithms.

\Rightarrow **simplify 1**

119 Simplify each of the following :

(a) $3\log_{10} p - 2$

(Ans : $\log_{10} \left(\frac{p^3}{100} \right)$) [3 marks]

(b) $2\log_2 x - 9\log_2 \sqrt[3]{x} + \log_2 (2x+1)$
marks]

(Ans : $\log_2 \left(\frac{2x+1}{x} \right)$) [3

[Forecast]

Answer :

(a)

(b)

120 Simplify each of the following :

(a) $\log_9 x - 2\log_9 y + \frac{1}{2}$

(Ans : $\log_9 \left(\frac{3x}{y^2} \right)$) [3 marks]

(b) $3 + \log_2 x + \frac{1}{2} \log_2 y - 2 \log_2 (x - y)$

(Ans : $\log_2 \left(\frac{8x\sqrt{y}}{(x-y)^2} \right)$) [3 marks]

[Forecast]

Answer :

(a)

(b)

\Rightarrow **simplify 2**

121 (a) If $\log_{10} 2 = a$, express $\log_{10} 25$ in term of a . (Ans : $2(1 - a)$) [3 marks] [UEC, 1987, P1 No.20]

(b) Given $\log_{10} 25 = p$, express $\log_{10} 50$ in term of p . (Ans : $\frac{1}{2} p + 1$) [3 marks]
[Forecast]

Answer :

(a)

(b)

122 Given $\log_5 2 = m$ and $\log_5 7 = p$, express $\log_5 4.9$ in terms of m and p . (Ans : $2p - m - 1$)
[4 marks] [2004, No.8]

Answer :

- 123** Given that $\log_m 2 = p$ and $\log_m 3 = r$, express $\log_m \left(\frac{27m}{4} \right)$ in terms of p and r .

(Ans : $3r - 2p + 1$) [4 marks] [2005, No.9]

Answer :

- 124** Given that $\log_2 x = h$ and $\log_2 y = k$, express $\log_2 \frac{x^3}{y}$ in terms of h and k .

(Ans : $3h - k$) [3 marks] [2011, No.8]

Answer :

- 125** Given that $\log 2 = a$ and $\log 3 = b$, express $\log_{10} \left(2 - \frac{1}{3} \right)$ in terms of a and b .

(Ans : $1 - a - b$) [3 marks] [UEC, 1999, P1 No.19]

Answer :

- 126** Given that $\log 2 = a$ and $\log 3 = b$, express $\log_{10} \frac{3}{2} + \log_{10} \frac{5}{4} + \log_{10} \frac{9}{8}$ in terms of a and b .

(Ans : $1 - 7a + 3b$)

[4 marks] [UEC, 1997, P2 No.?)

Answer :

4.3.4 Prove $\log_a b = \frac{\log_c b}{\log_c a}$ and use the relationship to determine the logarithm of a number.

127 Complete the following :

$x = \log_a b$

\Updownarrow

$a^x = b$

 ~ index form

$\log_c a^x = \log_c b$ ~ take \log_c on both sides

\Downarrow

if $c = b$ \rightarrow

HINT

$\log_a b = k \rightarrow \log_b a =$

$\log_{a^n} b =$ $=$

if a, b, c are positive, and $a \neq 1, c \neq 1$

128 Convert each of the following to “bases 10” and “natural logarithm”. Hence, determine the values.

logarithm	$\log_9 7$	$\log_{2.1} 4000$	$\log_2 \frac{1}{12}$	$\log_{\frac{16}{3}} 36$	$\log_{\frac{20}{3}} \frac{4}{3}$
base 10					
natural logarithm					
value					

NOTE : natural logarithm $\rightarrow \log_e @ \ln$

\Rightarrow changing base 1

129 Given $a = \frac{1}{x^3}$, find

(a) $\log_x a$,

(Ans : -3)

(b) $2\log_a x$.

(Ans : $-\frac{2}{3}$)

[3 marks] [2013, No.7]

Answer :

(a)

(b)

130 Given $\log_k 9 = 2$, find the value of

(a) k ,

(Ans : 3)

(b) $\log_9 \left(\frac{1}{k} \right)$.

(Ans : $-\frac{1}{2}$)

[3 marks] [2014, No.7]

Answer :

(a)

(b)

131 Given $\log_8 h = k$, express in terms of k :

(a) $\log_2 h$,

(Ans : $3k$) [2 marks]

(b) $\log_h 512$.

(Ans : $\frac{3}{k}$) [2 marks]

[Forecast]

Answer :

(a)

(b)

132 (a) Given that $m^t = 8$. Express $\log_2 m$ in terms of t .

(Ans : $\frac{3}{t}$) [3 marks]

(b) Given $p = \log_5 x$, express $\log_x 25$ in term of p .

(Ans : $\frac{2}{p}$) [3 marks]

[Forecast]

Answer :

(a)

(b)

133 (a) If $\log_b x = k$, express $\log_{\frac{1}{b}} x$ in term of k . (Ans : $-k$) [3 marks]

(b) If $\log_{\sqrt{b}} 7 = v$, express $\log_7 b$ in term of v . (Ans : $\frac{2}{v}$) [3 marks]

[Forecast]

Answer :

(a)

(b)

134 Given that $\log_9 15 = m$, express $\log_5 9$ in term of m . (Ans : $\frac{2}{2m-1}$) [3 marks]

[Forecast]

Answer :

\Rightarrow changing base 2

135 Without using a scientific calculator, simplify each of the following :

(a) $\log_9 \sqrt{3} - \log_{1.4} \frac{49}{25}$ (Ans : $-\frac{7}{4}$) [3 marks]

[Forecast]

(b) $\log_{\sqrt{2}} 8 + \log_4 \sqrt{2}$ (Ans : $6\frac{1}{4}$) [3 marks] [UEC, 2007, P2 No.3(b)]

Answer :

(a)

(b)

136 Simplify each of the following :

(a) $\frac{\log_a 16}{\log_a 4} + \frac{\log_5 a}{\log_{125} a}$

(Ans : 5) [3 marks]

(b) $\log_4 128 + \log_{m+1} \frac{1}{\sqrt[3]{m+1}}$

(Ans : $\frac{19}{6}$) [3 marks]

[Forecast]

Answer :

(a)

(b)

137 (a) Simplify : $\log_4 a \times \log_{25} 8 \times \log_{\sqrt[3]{a}} 125$

(Ans : $\frac{27}{4}$) [3 marks]

(b) Given $\log_3 q \times \log_p 81 \times \log_q p^{3q} = 9$, find the value of q .
(Ans : $\frac{3}{4}$) [3 marks]

[Forecast]

Answer :

(a)

(b)

\Rightarrow **changing base 3**

138 Given that $\log_2 b = x$ and $\log_2 c = y$, express $\log_4 \left(\frac{8b}{c} \right)$ in terms of x and y .

[Ans : $\frac{1}{2} (3 + x - y)$] [4 marks] [2007, No.7]

Answer :

- 139** Given that $\log_2 3 = a$ and $\log_2 5 = b$, express $\log_8 45$ in terms of a and b .

[Ans : $\frac{1}{3}(2a + b)$] [3 marks] [2010, No.8]

Answer :

- 140** Given $\log_m C = x$, express in term of x

(a) $\log_m \left(\frac{1}{C} \right),$

(b) $\log_{\sqrt{m}} Cm^3.$

(Ans : $2x + 6$)

[4 marks] [2019, No.10]

Answer :

(a)

(b)

- 141** (a) Given $r = 2^m$ and $t = 2^n$, express $\log_8 \left(\frac{32}{rt^4} \right)$ in terms of m and n .

[Ans : $\frac{1}{3}(5 - m - 4n)$] [4 marks]

- (b) Given $3^p = 5$ and $9^q = 2$, express $\log_3 50$ in terms of p and q .

(Ans : $2p + 2q$) [4 marks]

[Forecast]

Answer :

(a)

(b)

- 142** Given $\log_2 3 = p$ and $\log_3 7 = q$. Express $\log_3 31.5$ in terms of p and q .
(Ans : $2 + q - \frac{1}{p}$) [4 marks] [Forecast]

Answer :

- 143** Given that $\log_9 p = x$ and $\log_3 q = y$, express $\log_3 \left(\frac{81p}{q} \right)$ in terms of x and y .
(Ans : $4 + 2x - y$) [4 marks] [Forecast]

Answer :

\Rightarrow changing base 4

- 144** Given $\log_p 2 = x$ and $\log_p 5 = y$, express $\log_5 8p^2$ in terms of x and y .
(Ans : $\frac{3x+2}{y}$) [3 marks] [2015, No.6]

Answer :

- 145** Given $\log_a 7 = r$, express in term of r :

(a) $\log_a 49$,

(Ans : $2r$)

(b) $\log_7 343a^2$.

(Ans : $3 + \frac{2}{r}$ @ $\frac{3r+2}{r}$)

[4 marks] [2016, No.14]

Answer :

(a)

(b)

- 146** Given that $\log_a 5 = p$ and $\log_a 7 = q$, express $\log_{35} a^3$ in terms of p and q .

(Ans : $\frac{3}{p+q}$) [3 marks] [Forecast]

Answer :

- 147** Given $\log_c 3 = h$ and $\log_c 2 = k$, express $\log_2 72$ in terms of h and k .

(Ans : $\frac{2h+3k}{k}$) [4 marks] [Forecast]

Answer :

- 148** Given $\log_x 5 = m$ and $\log_x 7 = n$. Express $\log_{\sqrt{7}} 25x^2$ in terms of m and / or n .

(Ans : $\frac{4m+4}{n}$) [4 marks] [Forecast]

Answer :

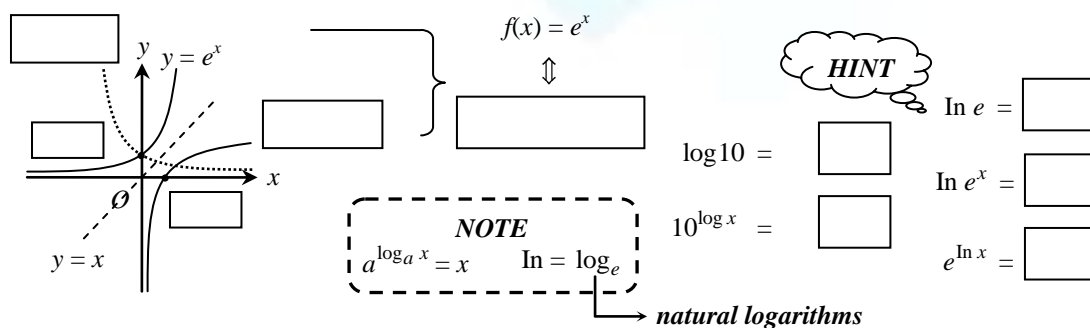
- 149** Given $\log_2 3 = p$ and $\log_3 7 = q$. Express $\log_{42} 63$ in terms of p and q .

(Ans : $\frac{p(2+q)}{1+p+pq}$) [4 marks] [Forecast]

Answer :

4.3.5 Solve problems involving the laws of logarithms.

150 Complete the following :



\Rightarrow solve index equation ~ involving logarithm

151 Solve each of the following :

(a) $9^{\frac{2m}{3}} = 20$

(Ans : 2.045) [3 marks]

(b) $2^{3x-1} - 7 = 0$

(Ans : 1.269) [3 marks]

[Forecast]

Answer :

(a)

(b)

152 Solve the equation $4^{2x-1} = 7^x$.

(Ans: 1.677)
[4 marks] [2003, No.6]

Answer :

153 Solve each of the following :

(a) $\left(\frac{4}{3}\right)^{1-x} = 5^x$

(Ans : 0.152) [3 marks]

(b) $4^{2x-1} = 3^{x+2}$

(Jwp : 2.141) [3 marks]
[Forecast]

Answer :

(a)

(b)

154 Solve each of the following :

(a) $3^{2x} \cdot 4^x = 7^{x+1}$.

(Ans : 1.188) [3 marks]

(b) $3^{3x} \cdot 2^{x-2} = 5^{3-2x}$.

(Ans : 0.862) [3 marks]

[Forecast]

Answer :

(a)

(b)

155 Solve the equation $4^{2x} + 4^{2x-1} = 4$.

(Ans : 0.420)

[3 marks] [Forecast]

Answer :

156 Solve the equation : $2^{x+1} + 3(2^{1-x}) = 7$.

(Ans : 0.585, 1)

[4 marks] [Forecast]

Answer :

\Rightarrow solve natural logarithmic equation

157 Solve each of the following :

(a) $\ln(3x-5) = \frac{5}{2}$.

(Ans : 5.727) [2 marks]

(b) $\ln(x+3)^2 = 5$

(Ans : 9.182, -15.182) [2 marks]

[Forecast]

Answer :

(a)

(b)

158 Solve each of the following :

(a) $e^{x+2} = 10$

(Ans : 0.303) [2 marks]

(b) $3e^{2x} = 9.6$

(Ans : 0.582) [2 marks]

[Forecast]

Answer :

(a)

(b)

MIND think :

$$\ln x = y \iff x = e^y$$

159 Solve the equation : $2e^{2x} = 7e^x - 3$

(Ans : 1.099, -0.693)

[4 marks] [Forecast]

Answer :

160 Solve each of the following :

(a) $e^{x+1} - 2e^{3x} = 0$

(Ans : 0.153) [4 marks]

(b) $e^{2x+1} - 6e^{x-1} = 0$

(Ans : -0.2.0) [4 marks]

[Forecast]

Answer :

(a)

(b)

\Rightarrow solve logarithmic equation ~ use antilog

161 Solve each of the following :

(a) $6^{\log_2 x} = 5$

(Ans : 1.864) [3 marks]

(b) $\log_n 3 = \log_2 5$

(Ans : 1.605) [3 marks]

[Forecast]

Answer :

(a)

(b)

\Rightarrow solve logarithmic equation ~ same base

162 Solve the equation $\log_3 4x - \log_3 (2x - 1) = 1$.

(Ans : $\frac{3}{2}$)

[3 marks] [2005, No.8]

Answer :

163 Given that $\log_2 xy = 2 + 3 \log_2 x - \log_2 y$, express y in terms of x .

(Ans : $y = 2x$)

[4 marks] [2006, No.7]

Answer :

164 Solve the equation $2 + \log_3 (x - 1) = \log_3 x$.

(Ans : $\frac{9}{8}$)

[3 marks] [2006, No.8]

Answer :

165 Solve the equation :

$$1 + \log_2(x - 2) = \log_2 x$$

(Ans : 4)

[3 marks] [2012, No.8]

Answer :

166 Solve the equation :

$$\log_3 2 + \log_3(x - 4) = 1$$

(Ans : $\frac{11}{2}$)

[3 marks] [2013, No.8]

Answer :

\Rightarrow solve logarithm equation ~ change base 1

167 Given that $\log_2 T - \log_4 V = 3$, express T in terms of V .

(Ans : $T = 8\sqrt{V}$)

[4 marks] [2003, No.5]

Answer :

168 Given that $\log_4 x = \log_2 3$, find the value of x .

(Ans : 9)

[3 marks] [2008, No.8]

Answer :

- 169 Given that $\log_8 p - \log_2 q = 0$, express p in terms of q .

(Ans : $p = q^3$)

[3 marks] [2009, No.8]

Answer :

- 170 Solve the equation :

$$\log_m 324 - \log_{\sqrt{m}} 2m = 2$$

(Ans : 3)

[4 marks] [2017, No.6]

Answer :

\Rightarrow solve logarithm equation ~ change base 2

- 171 (a) Given $P = \log_a Q$, state the conditions of a .

[1 mark]

- (b) Given $\log_3 y = \frac{2}{\log_{xy} 3}$, express y in terms of x .

(Ans : $y = \frac{1}{x^2}$) [3 marks]

[2018, No.17]

Answer :

(a)

(b)

- 172 Solve each of the following :

(a) $\log_4(4x+5) = \frac{1}{2} + \frac{1}{\log_{7x} 4}$.

(Ans : $\frac{1}{2}$) [4 marks]

(b) $\log_3 x = 5 - 6\log_x 3$

(Ans : 9, 27) [4 marks]

[Forecast]

Answer :

(a)

(b)

173 Solve the equation : $\log_3 x + 2 = \log_x 27$

(Ans : $\frac{1}{27}, 3$)

[4 marks]

[Forecast]

Answer :

4.4 Applications of indices, surds and logarithms

4.4.1 Solve problems involving indices, surds and logarithms.

174 In a culture experiment, a scientist took some bacteria. The function that shows the number of bacteria after t hour is given by $P(t) = 50(10)^{0.3t}$.

(a) How many bacteria are taken at the beginning of the experiment ?

(b) When will the number of bacteria reach one million ?

(Ans : 14.34)

[4 marks]

[Forecast]

Answer :

(a)

(b)

175 The population in Pekan Telipok is expected to increase at an annual rate of 3%. The increase in population follows the equation $P = P_0(R)^t$, where P_0 is the original population and P is the new population after t years.

(a) State the value of R .

(b) Find the value of t such that the population doubles the original population.

(Ans : 23.45)

[4 marks]

[Forecast]

Answer :

(a)

(b)

176 A liquid cools from its original temperature of 100°C to $x^{\circ}\text{C}$ in t seconds. Given that $x = 100(0.98)^t$, find

(a) the temperature of the liquid after cooled for 8.5 seconds,

(Ans : 84.22)

(b) the time, in second, for the temperature of the liquid dropped by 60°C .

(Ans : 45.35)

[4 marks] [Forecast]

Answer :

(a)

(b)

177 (a) Mclarance invested RM50000 in a bank and did not withdraw any money from his account. After n years, his savings becomes $50000\left(\frac{6}{5}\right)^n$. Calculate the minimum number of years for his saving exceeded one million ringgit for the first time.

(Ans : 17) [3 marks]

(b) The population of a country can be estimated with the growth model, $P = 2400000 e^{0.03t}$ where t is the number of years after 1990. When the population of the country will exceed 4.42 millions for the first time ?

(Ans : 2011) [3 marks]

[Forecast]

Answer :

(a)

(b)

- 178 (a) The price of a computer can be determine with the equation,

$$n \log_{10} \left(1 - \frac{2}{m} \right) = \log_{10} x - \log_{10} y.$$

In this equation, the computer with m years of usage and the price RM y will drop to RM x after being used for n years. A computer is bought at RM10000 has 5 years of usage. If the price of the computer drops to RM2000, find the years of usage for that computer. *Ans : 3.15* [3 marks]

- (b) In astronomy, the diameter of a planet, d km, is calculated by using the formula $10^{\log_{10} d} = 5001 - 10^{0.2g}$, where g is the absolute magnitude of the planet. Find the diameter of the planet when given the absolute magnitude is 18.48 km. *(Ans : 35.08)* [3 marks]

[Forecast]

Answer :

- (a) (b)

PAPER 2

⇒ Part A → 6 – 8 marks

- 179 (a) Simplify : $\log_2(2x+1) - 5\log_4 x^2 + 4\log_2 x$ *(Ans : $\log_2 \left(\frac{2x+1}{x} \right)$)* [4 marks]
- (b) Hence, solve the equation : $\log_2(2x+1) - 5\log_4 x^2 + 4\log_2 x = 3$
(Ans : $\frac{1}{6}$) [2 marks]

[2011, No.2]

Answer :

- (a) (b)

180 It is given that $p = 2^x$ and $q = 2^y$.

(a) Express $\frac{8^{x+y}}{4^x}$ in terms of p and q .

(Ans : pq^3) [3 marks]

(b) Find $\log_4 \frac{4p^2}{q}$ in terms of x and y .

[Ans : $\frac{1}{2}(2 + 2x - y)$] [5 marks]

[2014, No.4]

Answer :

(a)

(b)

181 Express $2^{n+2} - 2^{n+1} + 2^{n-1}$ in the form $p(2^{n-1})$, where p is a constant.

Hence, solve the equation $8(2^{n+2} - 2^{n+1} + 2^{n-1}) = 5(2^{n^2})$.

(Ans : $p = 5$; $n = -1, 2$)

[6 marks]

[2019, No.2]

Answer :

⇒ **FORECAST**

- 182 Two experiments are carried out by a scientist and obtain the following two equations:

$$\log_x (y + 7) = 2 - \log_x 3$$

$$9^{x+1} = \frac{81}{3^y}$$

Find the value of x and of y that satisfy both experiments.

(Ans : $x = 3$, $y = -4$)

[6 marks]

Answer :

- 183 Given $\log_x 40 = p$ and $\log_x 50 = q$.

Express $\log_x 2$ and $\log_x 5$ in terms of p and q .

$$(Ans : \log_x 2 = \frac{2p-q}{5}, \log_x 5 = \frac{3q-p}{5})$$

[6 marks]

Answer :

- 184 Given $\log_4 x = a$ and $\log_2 y = b$.

If $xy = 128$ and $\frac{x}{y} = 4$. Find the value of a and of b .

$$(Ans : a = 2\frac{1}{4}, b = 2\frac{1}{2})$$

[6 marks]

Answer :

- 185 Given $\log_b xy^3 = m$ and $\log_b x^3 y^2 = n$.

Find $\log_b \sqrt{xy}$ in terms of m and n .

(Ans : $\frac{m+2n}{14}$)

[6 marks]

Answer :

CONTINUOUS EXERCISES

- 186 Find the value of x which satisfy the equation $\frac{2}{\log_x xy} + \frac{2}{\log_y xy} + 8 = 5x$.

(Ans : 2)

[4 marks] [clon SBP 2020, No.15]

Answer :

- 187 Solve the equation $\log_{x^2} 64 = 2 + \log_2 x$.

(Ans : 2, $\frac{1}{8}$)

[4 marks] [clon Johor 2020, No.17]

Answer :

- 188 Given $\log_5 4 = x$ and $\log_8 80 = y$. Express x in terms of y .

(Ans : $x = \frac{2}{3y-4}$)

[4 marks] [clon YIK 2020, No.10]

Answer :

PROGRESSION

- ONE PAGE NOTE (OPN)

- WORKSHEET

Encik Adry Colodius

ONE PAGE NOTES “ PROGRESSION ”

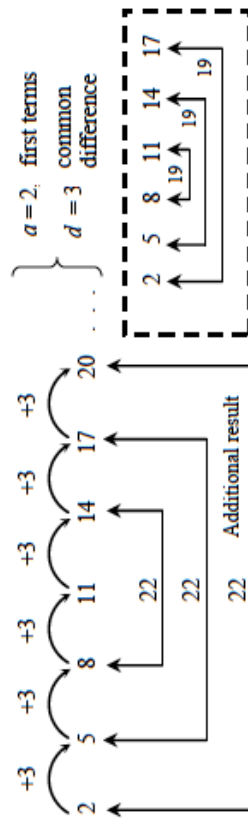
FOR: ARITHMETIC PROGRESSION & GEOMETRIC PROGRESSION

$$\underbrace{T_1 + T_2 + T_3 + T_4 + T_5 + T_6 + T_7 + T_8 + T_9 + T_{10} + T_{11} + T_{12} + T_{13} + T_{14} + T_{15}}_{S_{15}} \quad S_3$$

$$\begin{aligned} T_1 &= S_1 = a \\ T_2 &= S_2 - S_1 \\ T_3 &= S_3 - S_2 \\ &\vdots \\ T_n &= S_n - S_{n-1} \end{aligned}$$

ARITHMETIC PROGRESSION

(i) features



$$\therefore d = T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = \dots = T_n - T_{n-1} \rightarrow d \neq 0; n > 1 @ n \geq 2$$

(ii) The n^{th} term, T_n

$$\left. \begin{aligned} T_1 &= a \\ T_2 &= a + d \\ T_3 &= a + 2d \\ T_4 &= a + 3d \\ &\vdots \end{aligned} \right\} T_n = a + (n-1)d$$

Positive integer

$$\text{Min Arithmetic} \\ T_n = \frac{T_{n-1} + T_{n+1}}{2}$$

(iii) Sum of the first n term, S_n

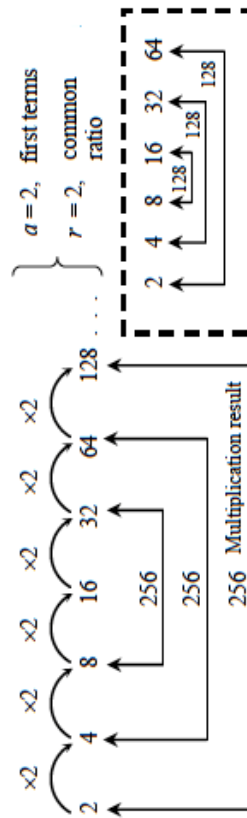
$$\left. \begin{aligned} S_1 &= a \\ S_2 &= \frac{2}{2} [2a + d] \\ S_3 &= \frac{3}{2} [2a + 2d] \\ S_4 &= \frac{4}{2} [2a + 3d] \\ &\vdots \end{aligned} \right\} S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_n = \frac{n}{2} (a + T_n)$$

First term \rightarrow Last term

GEOMETRIC PROGRESSION

(i) features



$$\therefore r = \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = \dots = \frac{T_n}{T_{n-1}} \rightarrow r \neq 1; n > 1 @ n \geq 2$$

(ii) The n^{th} term, T_n

$$\left. \begin{aligned} T_1 &= a \\ T_2 &= ar \\ T_3 &= ar^2 \\ T_4 &= ar^3 \\ &\vdots \end{aligned} \right\} T_n = ar^{n-1}$$

Positive integer

$$\text{Min Geometri} \\ (T_n)^2 = (T_{n-1})(T_{n+1})$$

Recurring decimal

(iii) Sum of the first n term, S_n

$$S_n = \frac{a(r^n - 1)}{r - 1}, |r| > 1$$

@

$$S_n = \frac{a(1 - r^n)}{1 - r}, |r| < 1$$

Sum to infinity

$$S_\infty = \frac{a}{1 - r}, \text{ for } |r| < 1$$

$-1 < r < 1$

WORKSHEET

TOPIC 5 : PROGRESSIONS [2 – 4 questions → 7 – 11 marks]

5.1 Arithmetic progressions

5.1.1 Identify a sequence as an arithmetic progression and provide justification.

⇒ **conditions** → **has common difference, d & $d \neq 0$**

- 1 (a) Determine whether the following sequence is an arithmetic progression.

$$\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \dots$$

- (b) Given a reason for the answer in (a).

[2 marks] **[Forecast]**

Answer :

- (a) (b)

- 2 Given x , y and z are three successive terms in an arithmetic progression. Express y in terms of x and z .

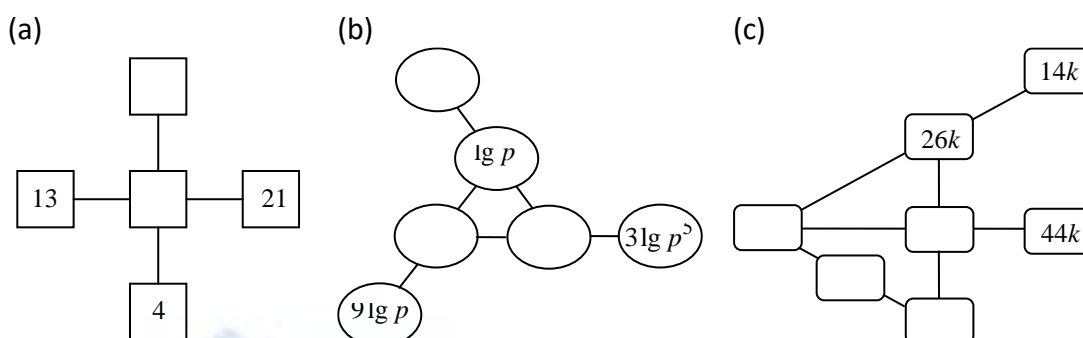
[2 marks] **[Forecast]**

Answer :

- 3 Complete the following network diagram, where the relationship of the network is a consecutive term in an arithmetic progression.

[2 marks] **[Forecast]**

Answer :



MIND think :

- Arithmetic Progression

→ a sequence of number where each term is obtained by a constant to the term before

→ has common , $d = T_n - T_{n-1}$, where $d \neq$; $n >$ @ $n \geq$

- Arithmetic Mean → $\frac{T_{n-1} + T_{n+1}}{2} =$, where n is a positive integer and $n \neq$.

5.1.2 Derive the formula of the n^{th} term, T_n , of arithmetic progressions, and hence use the formula in various situations.

5.1.3 Derive the formula of sum of the first n terms, S_n , of arithmetic progressions, and hence use the formula in various situations.

- 4** Complete the following table, to derive the formula of the n^{th} term, T_n , of arithmetic progressions :

An arithmetic progression with first term, a and common difference, d	
T_1	a
T_2	$a + d$
T_3	
T_4	
T_5	
\vdots	\vdots
T_n	

- 5** Complete the following table, to derive the formula of sum of the first n terms, S_n , of arithmetic progressions :

		Sum	
T_1	T_n	$= a + a + (n-1)d$	$= 2a + (n-1)d$
T_2	T_{n-1}	$=$	$=$
T_3	T_{n-2}	$=$	$=$
T_4	T_{n-3}	$=$	$=$
T_5	T_{n-4}	$=$	$=$
\vdots	\vdots	\vdots	\vdots
T_{n-1}	T_2	$=$	$=$
T_n	T_1	$=$	$=$
Sum	S_n		$=$

<p style="text-align: center;">Sum of first n terms, S_n</p> <p style="text-align: center;">↓</p> <p style="text-align: center;">where</p> <p>n is a <input type="text"/> integer,</p> <p><math>n > \text{<input type="text"/>>}</math> @ <math>n \geq \text{<input type="text"/>>}</math></p>	$S_n = \text{>}$ <div style="text-align: center;">↓</div> $S_n = \text{>}$
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MIND think :

<div style="margin-bottom: 10px;"><input type="text"/></div> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> $T_1 + T_2 + T_3$ <input type="text"/> </div> <div style="text-align: center;"> $T_4 + T_5 + T_6 + T_7 + T_8 + T_9 + T_{10} + T_{11} + T_{12} + T_{13} + T_{14} + T_{15}$ <input type="text"/> </div> </div>	$T_1 = S_1 = a$ $T_2 = S_2 - S_1$ $T_3 = S_3 - S_2$ \vdots $T_n = \text{>}$
--	---

\Rightarrow given three @ more consecutive terms \rightarrow concept d , T_n , S_n / arithmetic mean 1

- 6** The first three terms of an arithmetic progression are $k - 3$, $k + 3$, $2k + 2$. Find

- (a) the value of k ,
 (Ans : 7)
- (b) the sum of the first 9 terms of the progression.
 (Ans : 252)

[3 marks]
[2003, No.7]

Answer :

- (a) (b)

- 7** Three consecutive terms of an arithmetic progression are $5 - x$, 8 , $2x$. Find the common difference of the progression.

(Ans : 14)

[3 marks] **[2007, No.10]**

Answer :

- 8** The first three terms of an arithmetic progression are $3h$, k , $h + 2$.

(a) Express k in terms of h .

(Ans : $k = 2h + 1$)

(b) Find the 10th term of the progression in terms of h .

(Ans : $9 - 6h$)

[4 marks] [2010, No.11]

Answer :

(a)

(b)

- 9** It is given that x , 5, 8, ..., 41, ... is an arithmetic progression.

(a) State the value of x .

(b) Write the three consecutive terms after 41.

[3 marks] [2011, No.9]

Answer :

(a)

(b)

- 10** It is given that 11, $y + 4$ and $3y - x$ are three consecutive terms of an arithmetic progression.

(a) Express y in terms of x .

(Ans : $y = x - 3$)

(b) Find the common difference if $x = 8$.

(Ans : -2)

[4 marks] [2012, No.10]

Answer :

(a)

(b)

- 11 (a) State the conditions for a sequence is an arithmetic progression. **
 (b) The first three terms of a arithmetic progression are h , 8 and k . Find the value of $h + k$.

(Ans : 16) [2 marks] [2013, No.9]

Answer :

(a)

(b)

⇒ **arithmetic mean 2**

- 12 If 25 terms are added into between 4 and 82 to form an arithmetic progression, find the sum of the 25 terms that are added.

(Ans : 1075)

[2 marks] [Forecast]

Answer :

- 13 Given an arithmetic progression, $p, 6\frac{2}{5}, \dots, 13\frac{3}{5}, q$. The sum of all the terms is 270.

Find the number of terms in the progression.

(Ans : 27)

[2 marks] [Forecast]

Answer :

⇒ **concept, formula 1**

- 14 The volume of water in a tank is 450 litres on the first day. Subsequently, 10 litres of water is added to the tank everyday. Calculate the volume, in litres, of water in the tank at the end of the 7th day.

(Ans : 510) [2 marks] [2004, No.11]

Answer :

- 15 The first three terms of an arithmetic progression are 5, 9, 13. Find

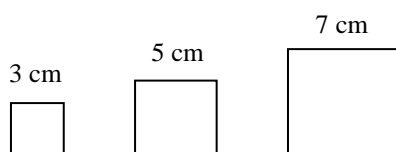
- (a) the common difference of the progression,
(Ans : 4)
- (b) the sum of the first 20 terms after the 3rd term.
(Ans : 1100)

[4 marks] [2005, No.11]

Answer :

- (a) (b)

- 16 The diagram shows three square cards



The perimeters of the cards form an arithmetic progression. The terms of the progression are in ascending order.

- (a) Write down the first three terms of the progression,
- (b) Find the common difference of the progression.

[3 marks] [2009, No.10]

Answer :

- (a) (b)

\Rightarrow concept, formula 2 [given 3 values of consecutive terms \rightarrow solve equation]

- 17 Given an arithmetic progression $-7, -3, 1, \dots$, state three consecutive terms in this progression with sum up to 75.

(Ans : 21, 25, 29)

[3 marks] [2004, No.10]

Answer :

- 18** The first three terms of an arithmetic progression are 46, 43, 40. The n th term of this progression is negative. Find the least value of n .
(Ans : 17)
[3 marks] [2008, No.10]

Answer :

\Rightarrow concept, formula 3 [solve an equation]

- 19** In an arithmetic progression, the common difference is -5 . Given the sum of the first 10 terms of the progression is 45, find
- (a) the first term of the progression, (Ans : 27)
 - (b) the tenth term of the progression. (Ans : -18)

[4 marks] [2013, No.10]

Answer :

(a) (b)

- 20** The third term of an arithmetic progression is 4 and the fourth term is 7.

- (a) State the common difference of the progression.
- (b) Find the sum of the first 25 terms of the progression.

(Ans : 850)

[4 marks] [2019, No.6]

Answer :

(a) (b)

\Rightarrow concept, formula 4 [solve 2 equations]

- 21** The 9th term of an arithmetic progression is $4 + 5p$, and the sum of the first four term of the progression is $7p - 10$, where p is a constant. Given that the common difference of the progression is 5, find the value of p .
(Ans : 8)

[3 marks] [2006, No.9]

Answer :

- 22** The second term of an arithmetic progression is -3 and the sixth term is 13. Find the first term and the common difference of the progression.
(Ans : $a = -7$, $d = 4$)

[3 marks] [2011,

No.10]

Answer :

- 23** In an arithmetic progression, the sum of the first four terms is 14 and the sixth term is -7 . Find the first term and the common difference of the progression.
(Ans : $a = 8$, $d = -3$)

[3 marks] [2015, No.9]

Answer :

- 24** The seventh term of an arithmetic progression is 12. The tenth term of the progression is greater than the second term by 8. Find the first term and the common difference of the progression. (Ans : $a = 6$, $d = 1$)

[4 marks] [Forecast]

Answer :

- 25 The sum of the first and seventh terms of an arithmetic progression is 6, and the ninth term of the series is double the sixth term. Find the first term and the common difference of the progression.

(Ans : $a = -6$, $d = 3$) [4 marks] [**Forecast**]

Answer :

- 26 An arithmetic progression has 14 terms. The sum of the odd terms is 140, and the sum of the even terms is 161. Find the first term and the common difference of the progression.

(Ans : $a = 2$, $d = 3$)
[4marks] [**Forecast**]

Answer :

\Rightarrow given formula T_n

- 27 The n -th term of an arithmetic progression is given by $T_n = 11 - 3n$. Find

(a) the common difference,

(Ans : -3)

(b) the sum of the second five terms.

(Ans : -65)

[3 marks] [**Forecast**]

Answer :

(a)

(b)

28 The n^{th} term, T_n , of a progression is given by $4n - 7$.

(a) Show that the progression is an arithmetic progression.

(b) Express T_{n-1} in term of n .

(Ans : $4n - 11$)

[4 marks] [*Forecast*]

Answer :

(a)

(b)

\Rightarrow *given formula S_n*

29 The sum of the first n terms of an arithmetic progression is given by $S_n = \frac{n}{2}(3n + 1)$. Find

(a) the sum of the first 5 terms,

(Ans : 40)

(b) the 5th term.

(Ans : 14)

[4 marks] [*2010, No.9*]

Answer :

(a)

(b)

30 It is given that the sum of the first n terms of an arithmetic progression is $S_n = \frac{n}{2} [13 - 3n]$. Find the n term.

(Ans : $8 - 3n$)

[3 marks] [*2017, No.8*]

Answer :

- 31 It is given that the sum of the first m terms of an arithmetic progression is $S_m = \frac{k+1}{2}(a+7)$, such that k is a constant, a is the first term and 7 is the last term.

(a) Express k in terms of m . (Ans : $k = m - 1$)

(b) State the range of values of k . (Ans : $k > 0$ @ $k \geq 1$)

[2 marks] [2019, No.5]

Answer :

(a)

(b)

5.1.4 Solve problems involving arithmetic progressions.

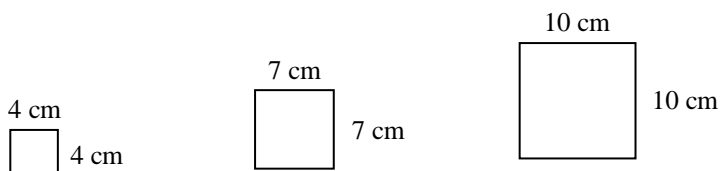
\Rightarrow **daily problems**

- 32 A stall selling 'teh tarik' gives choice to customers of using either condensed milk or evaporated milk in their drink. On a particular day the stall has 70 cans of condensed milk dan 48 cans of evaporated milk. The stall used 5 cans of condensed milk and 3 cans of evaporated milk in a day. After how many days, the remainder cans of both milk are the same ? (Ans : 11)

[3 marks] [2016, No.21]

Answer :

- 33 A student has a wire with the length of 13.16 m. The student divided the wire into several pieces. Each piece is to form a square. The diagram shows the first three squares formed by the student.



How many squares can be formed by the student ?

(Ans : 14)

[3 marks] [2018, No.15]

Answer :

- 34 Find the total number of integers between 100 and 200 that are multiples of 9. (Ans : 11)
[3 marks] [**Forecast**]

Answer :

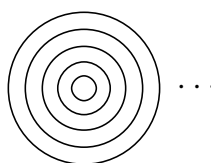
- 35 Given that $5^2 \cdot 5^4 \cdot 5^6 \cdot 5^8 \cdot \dots \cdot 5^{2n} = (0.04)^{-28}$. Find the value of n . (Ans : 7)
[4 marks] [**Forecast**]

Answer :

- 36 In a game, Julita is given 130 cuboid to form a pyramid. She needs to arrange a cuboid in the first row, three cuboids in the second row, five cuboids in the third row, and so on. Find the number of cuboids there were left. (Ans : 9)
[4 marks] [**Forecast**]

Answer :

- 37 A piece of wire is used to form 20 circles as shown in the diagram.



The radius of the circles form an arithmetic progression. Given that the smallest circle has a radius of 10 cm and the biggest circle has a circumference of 96π m. Find the radius, in m, for second circle.

(Ans : 12) [4 marks] [**Forecast**]

Answer :

- 38 A wire of length p cm is cut into 30 pieces. The length of these pieces form an arithmetic progression. Given that the length of the longest piece is 99 cm, and the sum of the three shortest pieces is 45 cm. Find the length, in cm, of the shortest piece.
(Ans : 12)

[4 marks] [Forecast]

Answer :

- 39 Two particles are moves simultaneously from the both end of a straight tube with a length of 14.3 m. One of the particle moves 51 cm in the 1st second, 49 cm in the 2nd second, 47 cm in the 3rd second, and so on. The other particle moves 30cm in the 1st second, 31 cm in the 2nd second, 32 cm in the 3rd second, and so on. Find the time, in second, that it would take for the two particles to meet ?
(Ans : 20)

[4 marks] [Forecast]

Answer :

5.2 Geometric progressions

5.2.1 Identify a sequence as a geometric progression and provide justification.

\Rightarrow conditions \rightarrow has common ratio, r & $r \neq 1$

- 40 (a) Determine whether the following sequence is an arithmetic progression or a geometric progression.

16x, 8x, 4x,

- (b) Give a reason for the answer in (a).

[2 marks] [2007, No.9]

Answer :

(a)

(b)

- 41 Given x , y and z are three successive terms in a geometric progression. Express y in terms of x and z .

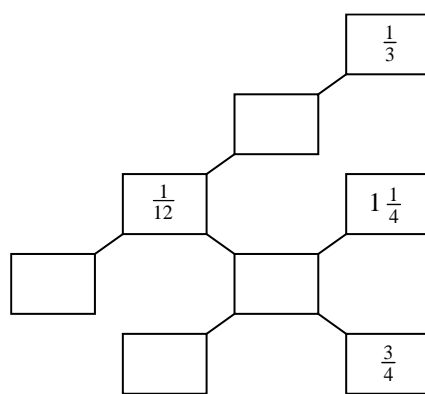
[2 marks] [*Forecast*]

Answer :

- 42 Complete the following network diagram, where the relationship of the network is a consecutive term in a geometric progression.

[2 marks] [*Forecast*]

Answer :



MIND think :

- Geometric Progression

→ a sequence of number where each term is obtained by a constant with the previous

→ has common , $r = \frac{T_n}{T_{n-1}}$, where $r \neq$; $n >$ @ $n \geq$.

- Geometric Mean → $\pm\sqrt{(T_{n-1})(T_{n+1})} =$, where n is a positive integer and $n \neq$.

5.2.2 Derive the formula of the n^{th} term, T_n , of geometric progressions, and hence use the formula in various situations.

5.2.3 Derive the formula of sum of the first n terms, S_n , of geometric progressions, and hence use the formula in various situations.

43 Complete the following table, to derive the formula of the n^{th} term, T_n , of geometric progressions :

A geometric progression with first term, a and common ratio, r	
T_1	a
T_2	ar
T_3	
T_4	
T_5	
	\vdots
T_n	

44 Complete the following table, to derive the formula of sum of the first n terms, S_n , of geometric progressions :

	n^{th} term	$r \times n^{\text{th}}$ term
	$T_1 = a$	$rT_1 = ar$
	$T_2 = ar$	$rT_2 =$
	$T_3 =$	$rT_3 =$
	$T_4 =$	\vdots
	\vdots	$rT_{n-2} =$
	$T_{n-1} =$	$rT_{n-1} =$
	$T_n =$	$rT_n =$
Sum	S_n (1)	(2)
<p>Sum of first n terms, S_n</p> <p style="text-align: center;">\downarrow</p> <p style="text-align: center;">where</p> <p>n is a <input type="text"/> integer,</p> <p><math>n > \text{<input type="text"/></math> @ <math>n \geq \text{<input type="text"/></math></p>	<p style="text-align: center;">(1) – (2)</p> <p style="text-align: center;">commonly used when $r < 1 \rightarrow -1 < r < 1$</p>	<p style="text-align: center;">(2) – (1)</p> <p style="text-align: center;">commonly used when $r > 1 \rightarrow r < -1, r > 1$</p>

MIND think :

<div style="display: flex; align-items: center; justify-content: center;"> <div style="border: 1px solid black; width: 40px; height: 20px; margin-bottom: 10px;"></div> <div style="display: flex; align-items: center; justify-content: center;"> $T_1 + T_2 + T_3 + T_4 + T_5 + T_6 + T_7 + T_8 + T_9 + T_{10} + T_{11} + T_{12} + T_{13} + T_{14} + T_{15}$ </div> <div style="display: flex; align-items: center; justify-content: center;"> <div style="border: 1px solid black; width: 40px; height: 20px; margin-bottom: 10px;"></div> <div style="border: 1px solid black; width: 80px; height: 20px; margin-bottom: 10px;"></div> </div> </div>	$T_1 = S_1 = a$ $T_2 = S_2 - S_1$ $T_3 = S_3 - S_2$ \vdots $T_n = \boxed{}$
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\Rightarrow given three @ more consecutive terms \rightarrow concept r, T_n, S_n / geometric mean

- 45** Given a geometric progression $y, 2, \frac{4}{y}, p, \dots$, express p in term of y . (Ans : $p = \frac{8}{y^2}$)

[2 marks] [2004, No.9]

Answer :

- 46** The first three terms of a sequence are 2, x , 8. Find the positive value of x so that the sequence is

- (a) an arithmetic progression,
(Ans : 5)
- (b) a geometric progression.
(Ans : 4)

[2 marks] [2005, No.10]

Answer :

- (a) (b)

- 47** It is given that the first four terms of a geometric are 3, -6, 12 and x . Find the value of x .
(Ans : -24) [2 marks] [2008, No.9]

Answer :

- 48** The first three terms of a geometric progression are x , 6, 12. Find

(a) the value of x ,

(Ans : 3)

(b) the sum from the fourth term to the ninth term.

(Ans : 15)

[4 marks] [2009, No.11]

Answer :

(a)

(b)

- 49** The first three positive terms of a geometric progression are 2, p and 18. Find the value of p and the common ratio of the progression.

(Ans : $p = 6$, $r = 3$)

[3 marks] [2012, No.9]

Answer :

- 50** It is given that $(x + 1)$, $(2x - 7)$ and $\left(\frac{x+1}{4}\right)$ are three consecutive terms of a geometric progression with a common ratio of $\frac{1}{2}$. Find

(a) the value of x ,

(Ans : 5)

(b) the first term if $(x + 1)$ is the 12th term of the progression.

(Ans : 12288)

[4 marks] [2016, No.22]

Answer :

(a)

(b)

- 51** The ratio of three numbers f , g , h is $1 : 3 : 7$. If each of the numbers is added by 2, its form a geometric progression. Find the value of $f + g + h$.

(Ans : 22)

[3 marks] [Forecast]

Answer :

⇒ **concept, formula 1 [given 3 values of consecutive terms → solve equation]**

52 The sum of the first n term of the progression 8, 24, 72, is 8744. Find

(a) the common ratio of the progression,

(Ans : 3)

(b) the value of n .

(Ans : 7)

[4 marks] [2005, No.12]

Answer :

(a)

(b)

⇒ **concept, formula 2 [solve an equation]**

53 In a geometric progression, the first term is a and the common ratio is r . Given that the third term of the progression exceeds the second term by $12a$, find the values of r . (Ans : -3, 4)

[3 marks] [2012, No.11]

Answer :

54 A geometric progression has a positive terms. The sixth term is four times the fourth term, and the third term is 1, find

(a) the common ratio,

(Ans : 2)

(b) the first term.

(Ans : $\frac{1}{4}$)

[3 marks] [Forecast]

Answer :

(a)

(b)

- 55 The common ratio of a geometric progression is $\frac{1}{2}$. The sum of the first four terms after the third term is 15. Find the first term of the progression. (Ans : 64)
[3 marks] [**Forecast**]

Answer :

- 56 A geometric progression and arithmetic progression have the same first term of 3. The common ratio and the common difference of both progressions are also the same. The fifth term of the geometric progression is 48, and the sum of the first n terms of the arithmetic progression is equal to the fourth term of the geometric progression. Find

- (a) the common ratio, (Ans : 2)
(b) the value of n . (Ans : 4)

[4 marks] [**Forecast**]

Answer :

- (a) (b)

- 57 The ratio of the sum of the first two terms of a geometric progression to the sum of its first and third terms is 2 : 5. Find the possible value of the common ratio. (Ans : $-\frac{1}{2}$, 3)

[3 marks] [**Forecast**]

Answer :

- 58 The sum of the first six terms of a geometric progression is 28 times the sum of the first three terms. Find the common ratio of the progression. (Ans : 3)

[3 marks] [**Forecast**]

Answer :

\Rightarrow concept, formula 3 [solve 2 equations ~ 1]

- 59** In a geometric progression, the third term is -3 and the sixth term is 24 . Find the sum of the first eleven terms.

(Ans : $-512\frac{1}{4}$)

[4 marks] [**Forecast**]

Answer :

- 60** The sum of the first and third terms of a geometric progression is 20 , whereas the sum of the fourth and sixth terms is 540 . Find the first term and the common ratio of the progression.

(Ans : $a = 2, r = 3$)

[4 marks] [**Forecast**]

Answer :

- 61** The third term of a geometric progression exceeds the second term by 4 , and the fourth term of the progression exceeds the third term by 3 . Find the first term and the common ratio of the progression.

(Ans : $a = -\frac{64}{3}, r = \frac{3}{4}$)

[4 marks] [**Forecast**]

Answer :

- 62** The sum of the first three term of a geometric progression is 21 , and the sum of the next three terms is 168 . Find the first term and the common ratio of the progression.

(Ans : $a = 3, r = 2$)

[3 marks] [**Forecast**]

Answer :

\Rightarrow concept, formula 4 [solve 2 equations ~ 2]

- 63** Three consecutive terms of a geometric progression are 32, p , q . It is given that the sum of these three terms is 26. Find the possible values of p and of q .
 (Ans : $p = -8$, $q = 2$; $p = -24$, $q = 18$)
 [3 marks] [2019, No.7]

Answer :

- 64** Given that p , 20, q , where $p < q$, are three consecutive numbers of an arithmetic progression, whereas p , 12, q are three consecutive numbers of a geometric progression. Find the values of p and q .
 (Ans : $p = 4$, $q = 36$)
 [4 marks] [Forecast]

Answer :

5.2.4 Determine the sum to infinity of geometric progressions, and hence use the formula in various situations, S_{∞} and hence use the formula in various situations.

- 65** (a) Consider a geometric progression with $|r| < 1$, state the formula of S_n , that commonly used.
 (b) If the value of n increases and get closer to infinity ($n \rightarrow \infty$) :
 (i) state the value of r^n ,
 (ii) hence, determine the sum to infinity of geometric progressions.

Answer :

(a)

(b) (i)

(ii)

\Rightarrow sum to infinity 1

- 66** The first three terms of a geometric progression are 27, 18, 12. Find the sum to infinity of the geometric progression.

(Ans : 81)

[3 marks] [2007, No.II]

Answer :

- 67** Given the geometric progression $-5, \frac{10}{3}, -\frac{20}{9}, \dots$, find the sum to infinity of the progression.

(Ans : -3) [3 marks] [2009, No.9]

Answer :

- 68** Find the sum to infinity of the geometric series $1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \dots$

(Ans : $\frac{4}{5}$)

[3 marks] [Forecast]

Answer :

- 69** Given $k = a + 4 + \frac{4}{5} + \frac{4}{25} + \dots$ is a infinite geometric series.

Find the values of a and k .

(Ans : $a = 20, k = 25$)

[3 marks] [Forecast]

Answer :

⇒ **sum to infinity 2**

70 In a geometric progression, the first term is 4 and the common ratio is r .

- (a) State the condition for r such that the sum to infinity of the progression is exist. **
 (b) Given that the sum to infinity of this progression is 16, find the value of r . (Ans : 0.75)
 [2 marks] [2008, No.11]

Answer :

- (a) (b)

71 It is given that $1, x^2, x^4, x^6, \dots$ is a geometric progression and its sum to infinity is 3. Find

- (a) the common ratio in terms of x ,
 (b) the positive value of x . (Ans : $\sqrt{\frac{2}{3}}$)

[3 marks] [2010, No.10]

Answer :

- (a) (b)

72 It is given that $x^2, x^4, x^6, x^8, \dots$ is a geometric progression such that $0 < x < 1$. The sum to infinity of this progression is $\frac{1}{3}$. Find

- (a) the common ratio of this progression in terms of x ,
 (b) the value of x . (Ans : $\frac{1}{2}$)

[3 marks] [2011, No.11]

Answer :

- (a) (b)

\Rightarrow sum to infinity 3 \rightarrow problems solving

73 In a geometric progression, the first term is 64 and the fourth term is 27. Calculate

(a) the common ratio,

(Ans : $\frac{3}{4}$)

(b) the sum to infinity of the geometric progression.

(Ans : 256)

[4 marks] [2003, No.8]

Answer :

(a)

(b)

74 The third term of a geometric progression is 16. The sum of the third term and the fourth term is 8. Find

(a) the first term and the common ratio of the progression,

(Ans : $a = 64, r = -\frac{1}{2}$)

(b) the sum to infinity of the progression.

(Ans : $42\frac{2}{3}$)

[4 marks] [2006, No.10]

Answer :

(a)

(b)

75 It is given that $p, 2, q$ are the first three terms of a geometric progression. Express in terms of q

(a) the first term and the common ratio of the progression,

(Ans : $a = \frac{4}{q}, r = \frac{q}{2}$)

(b) the sum to infinity of the progression.

[Ans : $\frac{8}{q(2-q)}$]

[4 marks] [2018, No.14]

Answer :

(a)

(b)

- 76 A geometric progression has a positive terms. The first term of the progression exceeds the second term by 16, and the sum to infinity of the progression is 36. Find the first term and the common ratio of the progression.

(Ans : $a = 24$, $r = \frac{1}{3}$)

[4 marks] [*Forecast*]

Answer :

\Rightarrow *sum to infinity 4 \rightarrow recurring decimal*

- 77 Express the recurring decimal 0.969696 as a fraction in its simplest form.

(Ans : $\frac{32}{33}$)

[4 marks] [*2004, No.12*]

Answer :

- 78 Given $\frac{h}{k} = 0.1\dot{6}$ is a recurring decimal where h and k are positive integers. Find the value of $h + k$.

(Ans : 7)

[4 marks] [*Forecast*]

Answer :

- 79 Given $p = 1.05\dot{4}\dot{5}\dot{6}$ is a recurring decimal. Express p as a fraction in its simplest form. (Ans : $\frac{35117}{33300}$)
[4 marks] [**Forecast**]

Answer :

\Rightarrow given formula T_n

- 80 It is given that the n -term of a geometric progression is $T_n = \frac{3r^{n-1}}{2}$, $r \neq k$. State

- (a) the value of k ,
(b) the first term of the progression.

[2 marks] [**2017, No.7**]

Answer :

- (a) (b)

- 81 The n^{th} term of a geometric progression is given by $T_n = b(2^{2n-1})$. If the third term of the progression is 96, find

- (a) the value of b , (Ans : 3)
(b) the sixth term of the progression. (Ans : 6144)

[4 marks] [**Forecast**]

Answer :

- (a) (b)

⇒ *given formula S_n*

- 82 It is given the sum of the first n terms of the geometric progression is $S_n = \frac{5}{2}(3^n - 1)$. Find

- (a) the first term of the progression,
 (b) the common ratio of the progression.

(Ans : 5)

(Ans : 3)

[3 marks] [2014, No.8]

Answer :

(a)

(b)

5.2.5 Solve problems involving geometric progressions.

⇒ *daily problems*

- 83 Adam has just completed his diploma in engineering field. He was offered a job from two different companies. Syarikat Satria offered him an initial salary of RM 36000 per annum with 5% yearly increment from the basic salary. Syarikat Perdana offered an initial salary of RM 30000 per annum with 9% yearly increment from the basic salary. Adam decided to choose the company which offered higher income and save 20% of his salary for further study after working for 10 years.

Which company should Adam choose and how much his total saving after working for 10 years. [Round off your answer to the nearest RM]

(Ans : Perdana, 91158)

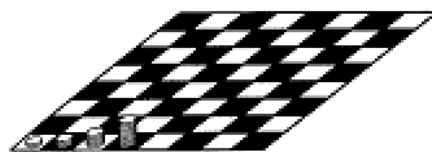
[4 marks] [2014, No.10]

Answer :

- 84 Mohan took 4 minutes to complete the first kilometer of a 15 km run. He could not sustain his stamina thus for each subsequent kilometer, he took $\frac{1}{8}$ more time compared to the time he took for the previous kilometer. The participants who finished the run more than two hours are not qualified for the state level run. Did Mohan qualified ? Show calculation to support your answer. (Ans : 155.27 > 2 hours)
[3 marks] [2016, No.23]

Answer :

- 85 The diagram shows a chess board.

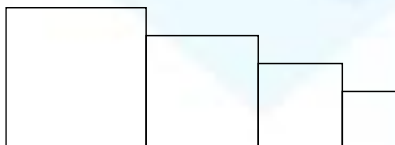


Bervelly puts RM1, RM2, RM4, RM8, . . . consecutively in the first square, second square, third square, . . . , and so on. If Bervelly has RM2500, find the number of squares on the chess board that would fill the money.
(Ans : 11)

[3 marks] [Forecast]

Answer :

- 86 The diagram shows part of the squares which is drawn consecutively. The length of the sides of the squares form a geometric progression.



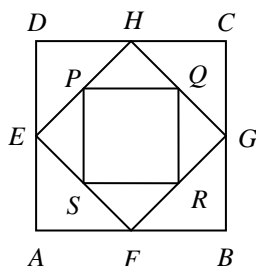
Given that the first square has a side of x cm, and the ratio of the length of the sides of the fourth square to the length of side of the first square is $8 : 27$. If the sum of the areas of the first three square is $\frac{3325}{81} \text{ cm}^2$, find the value of x .

(Ans : 5)

[4 marks] [*Forecast*]

Answer :

- 87 The diagram a square $ABCD$ with side length of 14 cm.



The second square is formed by connecting the midpoints of the sides of the given square. The third square is formed by connecting the midpoints of the sides of the second square and so on. Find the sum to infinity of the areas, in cm^2 , of the squares.
(Ans : 392)

[4 marks] [*Forecast*]

Answer :

- 88 The increasement of the number of tourists to a resort form a geometric progression. In the second month, the number of tourist increased by 120, the third month increased by 240. The incresement of the subsequent month is 480, and so on. The number of tourists in the fifth month is 2040. If the number of tourists is exceeds 10000, a special promotion will be given. At which month, the special promotion will be started ? (Ans : 9)

[4 marks] [**Forecast**]

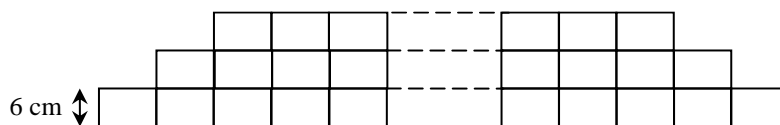
Answer :

PAPER 2

⇒ Part A → 6 – 8 marks

Arithmetic Progression

- 89 The diagram shows part of an arrangement of bricks of equal size.



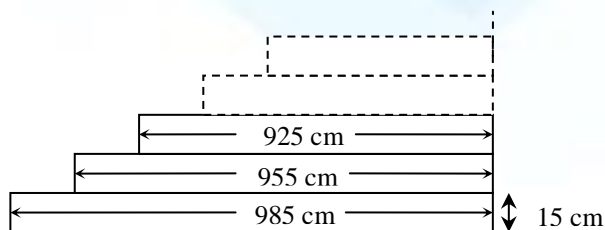
The number of bricks in the lowest row is 100. For each of the other rows, the number of bricks is 2 less than in the row below. The height of each brick is 6 cm. Ali builds a wall by arranging bricks in this way. The number of bricks in the highest row is 4. Calculate

- (a) the height, in cm, of the wall, (Ans : 294) [3 marks]
 (b) the total price of the bricks used if the price of one brick is 40 sen.
 (Ans : 1019.20) [3 marks]

[2005, No.3]

Answer :

- 90 The diagram shows the side elevation of part of stairs built of cement blocks.



The thickness of each block is 15 cm. The length of the first block is 985 cm. The length of each subsequent block is 30 cm less than the preceding block as shown in the diagram.

- (a) If the height of the stairs to be built is 3 m, calculate

- (i) the length of the top most block,
(Ans : 415 cm)
- (ii) the total length of the block.
(Ans : 14000 cm)

[5 marks]

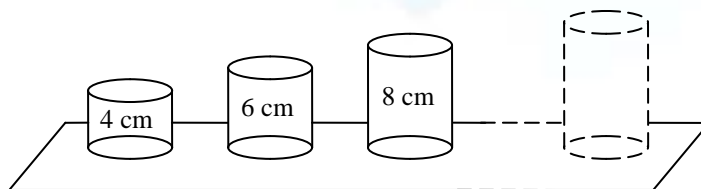
- (b) Calculate the maximum height of the stairs.

(Ans : 495 cm) [3 marks]

[2007, No.6]

Answer :

- 91 The diagram shows the arrangement of cylinders having the same radius, 3 cm. The height of the first cylinder is 4 cm and the height of each subsequent cylinder increase by 2. [volume of cylinder = $\pi r^2 h$]



- (a) Calculate the volume, in cm^3 , of the 17th cylinder, in term of π . (Ans : 324π) [3 marks]
 (b) Given the total volume of the first n cylinders is $1620\pi \text{ cm}^3$, find the value of n . (Ans : 12) [3 marks]

[2010, No.3]

Answer :

- 92 Two companies, Delta and Omega, start to sell cars at the same time.
- (a) Delta sells k cars in the first month and its sales increase constantly by m cars every subsequent month. It sells 240 car in the 8th month and the total sales for the first 10 month are 1900 cars. Find the value of k and m . (Ans : $k = 100$, $m = 20$) [5 marks]
 (b) Omega sells 80 cars in the first month and its sales increase constantly by 22 cars every subsequent month. If both companies sell the same number of car in the n^{th} , find the value of n . (Ans : 11) [2 marks]

[2006, No.3]

Answer :

- 93** At a certain day, a breeder has 3000 ducks in his farm to supply to a wholesaler. He starts selling 250 ducks on the next day and subsequently for the following days. The breeder feeds the ducks before selling. If the cost to breed a duck is RM0.50 per day, calculate the total cost until his remaining ducks are 500.
(Ans : 9625) [6 marks]
[2015, No.4]

Answer :

- 94** Height of the wall is 2 m. The side length of the first coloured rectangle is 5 cm and the side length of each subsequent coloured rectangle increases by 3 cm.

R	B	Y	R	B	Y	R	B	Y	
---	---	---	---	---	---	---	---	---	--

It is given that the total number of the coloured rectangles is 54.

- (a) Find
- (i) the side length, in cm, of the last coloured rectangle, (Ans : 164)
 - (ii) the total length, in cm, of the painted wall. (Ans : 4563)
[4 marks]
- (b) Which coloured rectangle has an area of 28000 cm². Hence, state the colour of that particular rectangle.
(Ans : 46, red)
[3 marks] [2017, No.4]

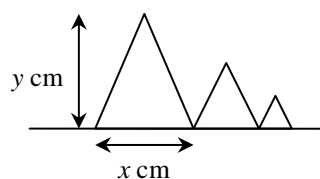
Answer :

- 95 The sum of the first n terms of an arithmetic progression, S_n is given by $S_n = \frac{3n(n-33)}{2}$. Find
- (a) the sum of the first 10 terms, (Ans : -345) [1 mark]
 - (b) the first term and the common difference, (Ans : $a = -48$, $d = 3$) [3 marks]
 - (c) the value of q , given that the q^{th} term is the first positive term of the progression. (Ans : 18) [2 marks]
- [2018, No.1]**

Answer :

Geometric Progression

- 96 The diagram shows the arrangement of the first three of an infinite series of similar triangles. The first triangle has a base of x cm and a height of y cm. The measurements of the base and height of each subsequent triangle are half of the measurements of its previous one



- (a) Show that the areas of the triangles form a geometric progression and state the common ratio. (Ans : $\frac{1}{4}$) [3 marks]
 - (b) Given that $x = 80$ cm and $y = 40$ cm,
 - (i) determine which triangle has an area of $6\frac{1}{4}$ cm², (Ans : 5)
 - (ii) find the sum to infinity of the areas, in cm², of the triangles. (Ans : $2133\frac{1}{3}$)
- [5 marks][2004, No.6]

Answer :

- 97 Muthu started working for a company on 1 January 2002 with an initial annual salary of RM 18000. Every January, company increased his salary by 5 % of the previous year's salary. Calculate
- (a) his annual salary, to the nearest RM, for the year 2007, (Ans : 22973) [3 marks]
 - (b) the minimum value of n such that his annual salary in the n^{th} year will exceed RM 36000, (Ans : 16) [2 marks]
 - (c) the total salary, to the nearest RM, paid to him by the company, for the years 2002 to 2007. (Ans : 122434) [2 marks]
[2008, No.3]

Answer :

- 98 Amir drops a ball from a height of H cm above the floor. After the first bounce, the ball reaches a height of H_1 cm = $0.8H$. After the second bounce, the ball reaches a height of H_2 , where H_2 cm = $0.8H_1$. The ball continuous bouncing in this way until it stops. Given that $H = 200$, find
- (a) the number of bounces when the maximum height of the ball from the floor is less than 50 cm for the first time, (Ans : 7) [4 marks]
 - (b) the total distance, in cm, travelled by the ball until it stops. (Ans : 1800) [2 marks]
[2009, No.6]

Answer :

- 99** It is given that $\dots, 567, y, 5103, \dots$ is part of geometric progression and the sum of the first five terms of the progression is 847. Find

- (a) the common ratio, (Ans : 3) [2 marks]
 (b) the first term, (Ans : 7) [2 marks]
 (c) the smallest value of n such that the n -th term exceeds 10000.
 (Ans : 8) [2 marks]

[2011, No.3]

Answer :

- 100** A wire is cut into n parts. The length of each part increase and form a geometri progression. It is given that the length of the fifth part of the wire is 4 times the length of the third part of the wire.

- (a) Calculate the common ratio. (Ans : 2) [2 marks]
 (b) If the total length of the wire is 1533 cm and the length of the first part of the wire is 3 cm, calculate
 (i) the value of n , (Ans : 9)
 (ii) the length, in cm, of the last part of the wire. (Ans : 768)

[4 marks]

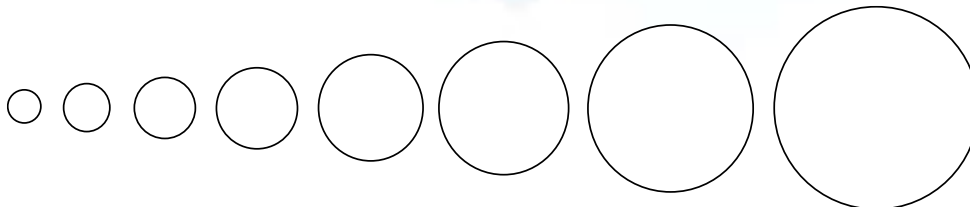
[2013, No.2]

Answer :

FORECAST

Arithmetic Progression

- 101 A wire of length 108π cm was cut to form eight circles as shown in the diagram.

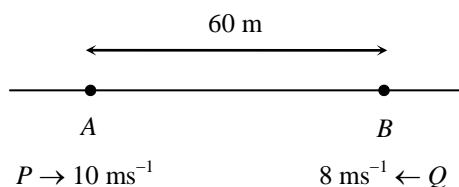


The difference of the diameter between the subsequent circle is 1 cm. Find

- (a) the diameter of the largest circle, Ans : 17) [4 marks]
 (b) the maximum number of circles that it would form is the length of the wire is 300π cm. (Ans : 16) [3 marks]

Answer :

- 102 The diagram shows the position of A and B on a straight line such that the distance of AB is 60 m.



Particle P moves from A to B with a initial speed of 10ms^{-1} , and the subsequent speed increase constantly by 2ms^{-1} each second. The particle Q moves with a constant speed of 8ms^{-1} from B to A . Given particles P and Q moves at the same time. Find

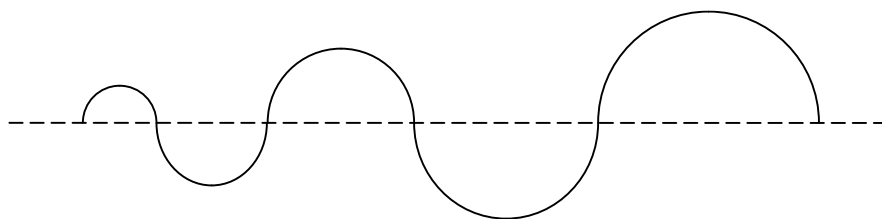
- (a) the value of t , if particle P and Q meets after t seconds, (Ans : 3) [5 marks]
 (b) the distance travelled by particle P from the beginning until it meets particle Q .
 (Ans : 36) [2 marks]

Answer :

- 103** A wire of length 262.5 cm is cut into 30 pieces. The length of these pieces form an arithmetic progression. If the difference between the length of the longest piece and the shortest piece is 14.5 cm, find
- (a) the length of the shortest piece, (Ans : 1.5) [4 marks]
- (b) the difference between the lengths of the fifth piece and the tenth piece. (Ans : 2.5) [3 marks]

Answer :

- 104** A wire of length y cm is bent to form the arcs of semi-circles as shown in the diagram



Given that the radius of the smallest semi circle is 2 cm, and the radius of the subsequent semi circle increase constantly by 3 cm over the preceding semi circle, until the biggest semi circle has a radius of 59 cm

- (a) Find the length of the wire that used to form the first eight semi circle. (Ans : 100π) [4 marks]
- (b) Determine whether exist the possible value of y is 500π cm. (Ans : no) [3 marks]

Answer :

- 105 Find the sum of integers between 10 and 150 that *cannot* be exactly divided by 3 or 4. (Ans : 5599)
[8 marks] [Forecast]

Answer :

- 106 The diagram shows the arrangement of a few chairs. The height of each chair is 68 cm. When the chairs are arranged, there is a 5 cm gap in between two chairs. The arranged chairs will kept in the store.



- (a) Find the maximum number of chairs that can be arranged if the height of the store is 3 m. (Ans : 47) [3 marks]
- (b) 15 stack of chairs were kept in the store with the condition that the first stack will have the maximum number of chair and the arrangement of chairs for subsequent stacks decrease by 2. Do you agree that there are 500 chairs in the store ? Show your calculations. (Ans : 495) [3 marks]

[clon text book form 4]

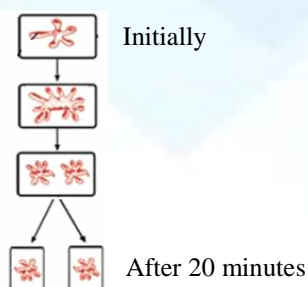
Answer :

Geometric Progression

- 107 Tin is extracted from tin ores at a mine in Pahang. In its first year of operation, the mine was able to produce 8000 kg of tin a year. With increasing difficulty in mining, tin production in each subsequent year decreased by 10% from the previous year. Assume this mining situation lasts for an indefinite period of time.
- (a) Calculate the maximum quantity of tin that can be extracted. (Ans : 80000) [3 marks]
- (b) Due to economic factors, tin mining will be discontinued if its annual production is less than 1000 kg. Calculate the number of years that the mine will operate. (Ans : 20) [4 marks]

Answer :

- 108 Bacteria *Y* reproduce by binary fission process as shown in the diagram. Under the optimum condition, the process will take 20 minutes.



A study on reproduction is carried out in 10 samples of bacteria *Y*. The first sample contains 2 bacteria *Y*. The number of bacteria in the next sample is 3 times the number of its previous sample.

- (a) What is the total number of bacteria *Y* in the entire sample at the beginning of the study ?
(Ans : 59048) [3 marks]
- (b) After a certain period, it is found that the number of bacteria in the 6th sample is 124416. Determine the period, in minutes.
(Ans : 160) [4 marks]

Answer :

- 109 The table dan diagram show the knock-off system for eight teams which participating in a competition

Round	Number of match
1	4
2	2
3	1

The diagram shows a knockout tournament bracket for eight teams. Teams 1 through 4 are listed on the left, and Teams 5 through 8 are listed on the right. Lines connect the teams to represent matches. In the first round, Team 1 plays Team 2, Team 3 plays Team 4, Team 5 plays Team 6, and Team 7 plays Team 8. The winners of these four matches advance to the second round. In the second round, the winner of the first match plays the winner of the second match, and the winner of the third match plays the winner of the fourth match. The winners of these two matches advance to the final match, which is labeled 'Champion'.

To determine the winning team, three rounds are required with the total matches of $4 + 2 + 1 = 7$. In a competition, there are 256 teams participating. Find

- (a) the number of rounds, (Ans : 8) [3 marks]
- (b) the total matches. (Ans : 255) [3 marks]

Answer :

Arithmetic Progression & Geometric Progression

- 110** The first three terms of a geometric progression are also the first, ninth and eleventh terms respectively of an arithmetic progression.

(a) Find the common ratio of the geometric progressions. (Ans : $\frac{1}{4}$) [4 marks]

(b) If the sum to infinity of the geometric progression is 8, find

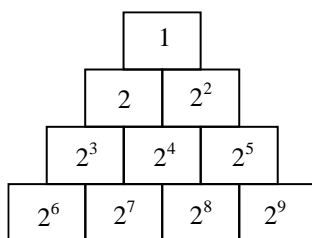
(i) the first term, (Ans : 6)

(ii) the common difference of the arithmetic progression. (Ans : $-\frac{9}{16}$)

[4 marks]

Answer :

- 111** The diagrams show the numbers in base two which are arranged in a few rows.

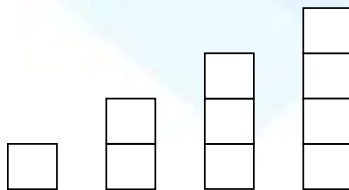


(a) Find the total number in the first n row, in term of n . [Ans : $S_n = \frac{n}{2}(n+1)$] [3 marks]

(b) Hence, find the sum of the numbers of the first six row. (Ans : 2097151) [4 marks]

Answer :

- 112 Diagram shows a series of shapes produced by a combination of rectangles with sides of 2 cm.



If y represents the perimeter of the shape produced by x equal rectangular pieces.

- (a) Form an equation relating x dan y .

(Ans: $y = 4x + 4$)

- (b) Hence, find the perimeter of the shape that has 40 equal squares.

(Ans: 164)

[4 marks] [**Forecast**]

Jawapan :

(a)

(b)

LINEAR LAW

- ONE PAGE NOTE (OPN)

- WORKSHEET

Encik Mohd Zulkarnain bin Zulkifli

ONE PAGE NOTES (Coding Method)
" LINEAR LAW "

STEPS OF SOLUTION → PAPER 2

- construct table, based on the given X -axis and Y -axis.
- plot graph Y against X , based on given scale
- draw the line of best fit. ... (a)
 - ⇒ find m ~ choose two points on the line of best fit
 - ⇒ find c ~ y -intercept of the line of best fit
- convert non linear equation to linear equation. ... (b)
 - ⇒ determine m
 - ⇒ determine c
- compare m and c from (a) and (b). Hence, solve.
- from the graph :
 - ⇒ given value of x ~ find the value of y
 - ⇒ given value of y ~ find the value of x

refer

gradient, m

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

SPM 2019

"correcting" the value of y that misrecorded

the line of best fit

[extreme point]

CONVERT "NON-LINEAR" EQUATION TO "LINEAR" → $Y = mX + c$

"Method 1" - by using " \times ", " \div ", square, factorization, rearrange ...

$$y = px + \frac{r}{x} \rightarrow Y = xy, X = x^2$$

$$xy = px^2 + \frac{r}{x}$$

$$\Rightarrow m = p, c = \frac{r}{x}$$

$$\frac{p}{y} = \frac{q}{x^2} - 1 \rightarrow Y = \frac{1}{y}, X = \frac{1}{x^2}$$

$$\frac{1}{y} = \left(\frac{q}{p} \right) \left(\frac{1}{x^2} \right) - \frac{1}{p}$$

$$\frac{x+3}{a} - \frac{y^2}{b} = 1 \rightarrow Y = y^2, X = x+3$$

$$\frac{y^2}{b} = - \left(\frac{x+3}{a} \right) + 1$$

$$y^2 = - \left(\frac{b}{a} \right) (x+3) + b$$

$$\Rightarrow m = - \left(\frac{b}{a} \right), c = b$$

$$nx = py + xy \rightarrow Y = \frac{1}{y}, X = \frac{1}{x}$$

$$nx = y(p+x)$$

$$\frac{1}{y} = \frac{p+x}{nx}$$

$$\frac{1}{y} = \left(\frac{p}{n} \right) \left(\frac{1}{x} \right) + \frac{1}{n}$$

$$\Rightarrow m = \frac{p}{n}, c = \frac{1}{n}$$

$$Y = \frac{1}{y}, X = \frac{1}{x^2}$$

$$\rightarrow Y = \frac{d}{x-b} = \frac{x-b}{a} = \frac{1}{a} \left(\frac{x}{x} \right) - \frac{b}{a}$$

$$Y = \frac{d}{x-b} = \frac{x-b}{a} = \frac{1}{a} \left(\frac{x}{x} \right) - \frac{b}{a} \Rightarrow m = \frac{1}{a}, c = -\frac{b}{a}$$

$$T = 2\pi \sqrt{\frac{L}{g}} \rightarrow Y = T^2, X = \frac{L}{g}$$

$$T^2 = 4\pi^2 \left(\frac{L}{g} \right)$$

$$T^2 = \frac{4\pi^2}{g} \left(\frac{L}{g} \right)$$

$$\Rightarrow m = \frac{4\pi^2}{g}, c = 0 \text{ (straight line passes O)}$$

"METHOD 2" - by using 'log'

$$y = ab^x \rightarrow Y = \log_{10} y, X = x$$

$$\log_{10} y = \log_{10} a + \log_{10} b^x$$

$$\log_{10} y = \log_{10} a + (x \log_{10} b)$$

$$\Rightarrow m = \log_{10} b, c = \log_{10} a$$

$$y = pk^{x+1} \rightarrow Y = \log_{10} y, X = x+1$$

$$\log_{10} y = \log_{10} p + \log_{10} k^{x+1}$$

$$\log_{10} y = \log_{10} p + (x+1) \log_{10} k$$

$$\Rightarrow m = \log_{10} k, c = \log_{10} p$$

$$y = 100^a + bx^2 \rightarrow Y = \log_{10} y, X = x^2$$

$$\log_{10} y = \log_{10} (10^a + bx^2)$$

$$\log_{10} y = (a + bx^2) \log_{10} 10$$

$$\log_{10} y = 2a + 2bx^2$$

$$\Rightarrow m = 2b, c = 2a \dots [\log_{10} 100 = 2]$$

$$y = \frac{k}{h^{2x}} \rightarrow Y = \log_{10} y, X = x$$

$$\log_{10} y = \log_{10} k - \log_{10} h^{2x}$$

$$\log_{10} y = \log_{10} k - (2 \log_{10} h)(x)$$

$$\Rightarrow m = -2 \log_{10} h, c = \log_{10} k$$

Express $T + 10 = \frac{k}{h^x}$ in linear form.

$$\log_{10} (T + 10) = \log_{10} k - \log_{10} h^x$$

$$\log_{10} (T + 10) = \log_{10} k - (x \log_{10} h)$$

$$\Rightarrow m = -\log_{10} h, c = \log_{10} k$$

$$\Rightarrow Y = \log_{10} (T + 10), X = x$$

WORKSHEET
TOPIC 6 : LINEAR LAW
[0 – 1 questions → 0 – 4 marks]

6.1 Linear and non-linear relations

6.1.1 Differentiate between linear and non-linear relations based on tables of data and graphs.

6.1.2 Draw lines of best fit for graph of linear relations with and without the use of digital technology.

[lines of best fit need not necessarily pass through any of the points]

6.1.3 Form equations of lines of best fit.

6.1.4 Interpret information based on lines of best fit.

6.2 Linear law and non-linear relations

6.2.1 Apply linear law to non-linear relations

6.3 Application of linear law

6.3.1 Solve problems involving linear law.

[problem-based learning may be involved]

MIND think :

Match the following :

The graph which forms a straight line.

A linear relation.

The graph which does not forms a straight line.

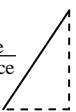
A non-linear relation.

Equation of Straight Line, $y = mx + c$; where m = gradient, c = y-intercept

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$$

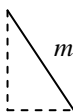
$$m = - \left(\frac{y - \text{intercept}}{x - \text{intercept}} \right)$$

$m = \frac{\text{vertical distance}}{\text{horizontal distance}}$



vertical distance
horizontal distance

$m = - \left(\frac{\text{vertical distance}}{\text{horizontal distance}} \right)$



vertical distance
horizontal distance

Laws of Logarithms

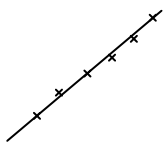
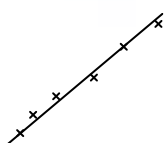
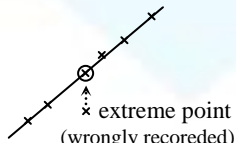
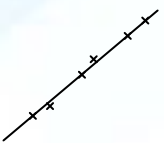
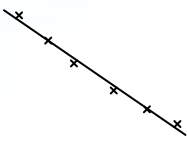
$$\log_a a = 1$$

$$\log_a 1 = 0$$

$$\log_a x^n = n \log_a x$$

$$\log_a (xy) = \log_a x + \log_a y$$

$$\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y$$

Line of Best Fit				
<p>passes 3 points</p>  <p>SPM 2010</p>	<p>passes 2 points</p>  <p>SPM 2016</p>	<p>passes 4 points</p>  <p>SPM 2017</p>	<p>passes 4 points</p>  <p>SPM 2018</p>	<p>passes 2 points</p>  <p>SPM 2019</p>

6.2 Linear law and non-linear relations

6.2.1 Apply linear law to non-linear relations

- 1 Reduce the following no-linear relations to the linear form, $Y = mX + c$

⇒ **type 1 ~ USING MULTIPLICATION @ DIVISION**

$y = ax^2 + bx \rightarrow Y = \frac{y}{x}, X = x$	$\frac{y}{x} = px + \frac{q}{x} \rightarrow Y = y, X = x^2$
$y = \frac{p}{\sqrt{x}} + q\sqrt{x} \rightarrow Y = y\sqrt{x}, X = x$	$y - \sqrt{h} = \frac{hk}{x} \rightarrow Y = xy, X = x$
$\frac{x+3}{a} + \frac{y^2}{b} = 1 \rightarrow Y = y^2, X = x + 3$	$\frac{p}{y} = \frac{q}{x^2} - 1 \rightarrow Y = \frac{1}{y}, X = \frac{1}{x^2}$

$y = \frac{a}{x-b} \rightarrow Y = \frac{1}{y}, X = x$	$nx = py + xy \rightarrow Y = \frac{1}{y}, X = \frac{1}{x}$
--	---

$T = 2\pi \sqrt{\frac{L}{g}} \rightarrow Y = T^2, X = L$	$4kx = (y-h)^2 \rightarrow Y = y, X = \sqrt{x}$
--	---

\Rightarrow type 2 ~ USING LAW OF LOGARITHMS

$y = ab^x \rightarrow Y = \log_{10} y, X = x$	$y = 10^{a+bx^2} \rightarrow Y = \log_{10} y, X = x^2$
$y = hk^{2x} \rightarrow Y = \log_{10} y, X = x$	$y = \frac{k}{h^x} \rightarrow Y = \log_{10} y, X = x$

$y = pq^{x-2} \rightarrow Y = \log_{10} y, X = x - 2$	$y = \frac{p}{k^{x+1}} \rightarrow Y = \log_{10} y, X = x + 1$
$y = (1+k)x^{\frac{h}{2}} \rightarrow Y = \log_{10} y, X = \log_{10} x$	$y = \frac{a}{b\sqrt{x}} \rightarrow Y = \log_{10} y, X = \log_{10} x$

\Rightarrow **apply 1**

- 2 Diagram (a) shows the curve $y = -3x^2 + 5$. Diagram (b) shows the straight line graph obtained when $y = -3x^2 + 5$ is expressed in the linear form $Y = 5X + c$.

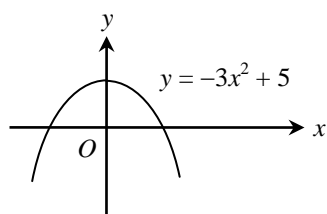


Diagram (a)

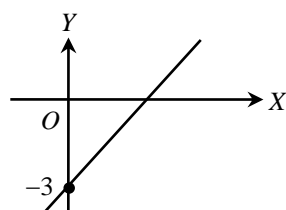


Diagram (b)

Express X and Y in terms of x and / or y .

(Ans : $Y = \frac{y}{x^2}, X = \frac{1}{x^2}$)

[3 marks] [2006, No.11]

Answer :

- 3 The variables x and y are related by the equation $\frac{x}{y} = \frac{p}{x} + 5x$, where p is a constant. When the equation $\frac{x}{y} = \frac{p}{x} + 5x$ is express in linear form, the straight line obtained is $Y = pX + \frac{q}{2}$.

(a) Express X and Y in term of x and / or y .

(Ans : $Y = \frac{1}{y}$, $X = \frac{1}{x^2}$)

(b) Find the value of q .

(Ans : 10)

[4 marks] [Forecast]

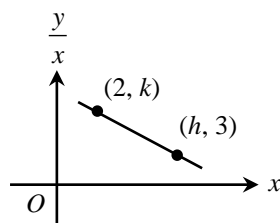
Answer :

(a)

(b)

\Rightarrow apply 2

- 4 The diagram shows a straight line graph of $\frac{y}{x}$ against x .



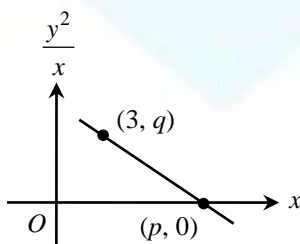
Given that $y = 6x - x^2$, calculate the values of k and h .

(Ans : $h = 3$, $k = 4$)

[4 marks] [2004, No.13]

Answer :

- 5 The variables x and y are related by the equation $y^2 = 2x(10 - x)$. A straight line graph is obtained by plotting $\frac{y^2}{x}$ against x , as shown in diagram.



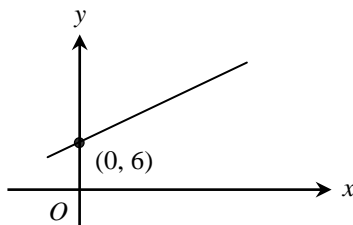
Find the value of p and of q .

(Ans : $p = 10$, $q = 14$)
[3 marks] [2007, No.12]

Answer :

\Rightarrow **apply 3 ~ 1**

- 6 The variables x and y are related by the equation $hy = kx^2 + hk$. A straight line graph is obtained by plotting y against x^2 as shown in the diagram.

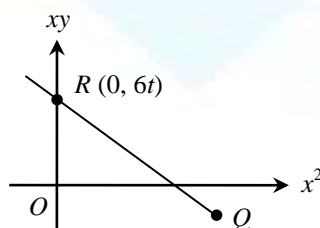


Given the gradient of the straight line is 3, find the value of h and of k .

(Ans : $h = 2$, $k = 6$)
[3 marks] [2010, No.12]

Answer :

- 7 The variables x and y are related by the equation $3y = (p-1)x + \frac{12}{x}$, where p is a constant. The diagram shows the straight line QR obtained by plotting xy against x^2 .



- (a) Express the equation $3y = (p-1)x + \frac{12}{x}$ in its linear form, which is used to obtain the straight line graph shown in diagram. [Ans : $xy = \left(\frac{p-1}{3}\right)x^2 + 4$]
- (b) Given that the gradient of QR is -2 , find the value of p and of t . (Ans : $p = -5$, $t = \frac{2}{3}$)
- [4 marks] [2011, No.12]

Answer :

(a)

(b)

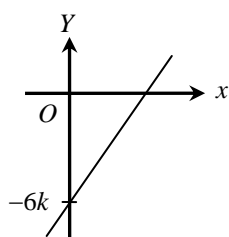
- 8 Given the variable x and y are related by the equation $x - py = qxy$, where p and q are constants. If the vertical-axis is represented by $\frac{1}{y}$. Explain, how the p and q can be obtained.

[3 marks] [Forecast]

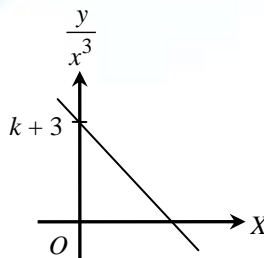
Answer :

⇒ **apply 4**

- 9 The variables x and y are related by the equation $\frac{y}{x} = px^2 - qx$ where p and q are constants. Diagram (i) and Diagram (ii) show the straight line graphs obtained by plotting the relations from the equation.



(i)



(ii)

Express p in terms of q .

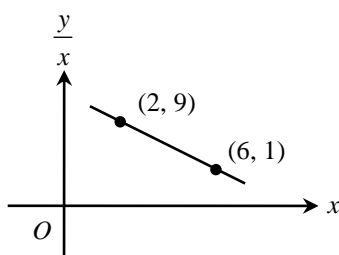
(Ans : $p = \frac{q+18}{6}$)

[3 marks] [2019, No.11]

Answer :

⇒ **apply 5**

- 10 x and y are related by the equation $y = px^2 + qx$, where p and q are constants. A straight line is obtained by plotting $\frac{y}{x}$ against x , as shown in diagram below.



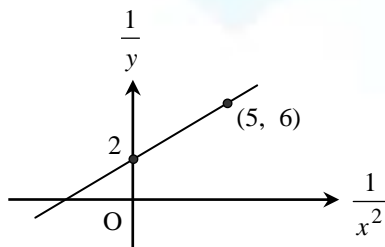
Find the values of p and q .

(Ans : $p = -2, q = 13$)

[4 marks] [2003, No.10]

Answer :

- 11 The variables x and y are related by the equation $\frac{p}{y} = 1 - \frac{q}{x^2}$. The diagram shows the straight line graph obtained by plotting $\frac{1}{y}$ against $\frac{1}{x^2}$.



Find the value of

(a) p ,

(Ans : $\frac{1}{2}$)

(b) q .

(Ans : $-\frac{2}{5}$)

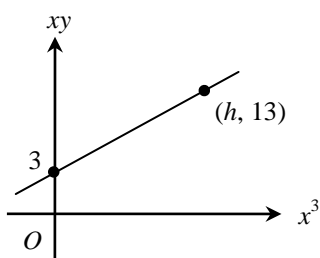
[4 marks] [2012, No.12]

Answer :

(a)

(b)

- 12 The variable x and y are related by the equation $y = 2x^2 - \frac{q}{x}$, where q is a constant. A straight line is obtained by plotting xy against x^3 , as shown in Diagram.



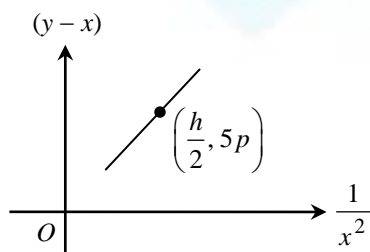
Find the value of h and of q .

(Ans : $h = 5, q = -3$)

[3 marks] [2016, No.16]

Answer :

- 13 The variables x and y are related by the equation $y = x + \frac{r}{x^2}$, where r is a constant. The diagram shows a straight line graph obtained by plotting $(y - x)$ against $\frac{1}{x^2}$.



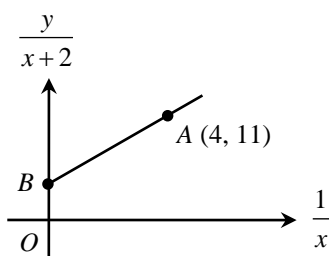
Express h in terms of p and r .

(Ans : $h = \frac{10p}{r}$)

[3 marks] [2017, No.19]

Answer :

- 14 The variables x and y are related by the equation $y = (x + 2) \left(\frac{m}{x} + 3 \right)$, where m is a constant. When the graph of $\frac{y}{x+2}$ is plotted against $\frac{1}{x}$, a straight line passing through the points A and B is obtained.



Find

- (a) the coordinates of point B ,
(b) the value of m .

(Ans : 2)

[4 marks]
[Forecast]

Answer :

- (a) (b)

- 15 x and y are related by the equation $\frac{y}{x^2} = ax + \frac{b}{x}$, where a and b are constants. A straight line passes through point $(4, 6)$ with a gradient of $\frac{1}{2}$ is obtained by plotting $\frac{y}{x}$ against x^2 . Calculate the values of a and b .

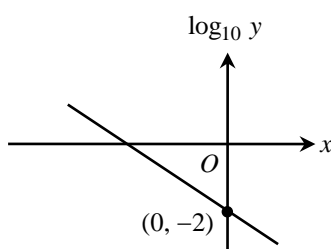
(Ans : $a = \frac{1}{2}$, $b = 4$)

[4 marks] [Forecast]

Answer :

\Rightarrow apply 3 ~ 2

- 16 The variables x and y are related by the equation $y = \frac{k}{5^x}$, where k is a constant. Diagram below shows the straight line graph obtained by plotting $\log_{10} y$ against x .



- (a) Express the equation $y = \frac{k}{5^x}$ in its linear form used to obtain the straight line graph shown in diagram.
- (b) Find the value of k .

(Ans : $\frac{1}{100}$)

[4 marks] [2008, No.12]

Answer :

(a)

(b)

- 17 The variables x and y are related by $y = \frac{1}{2}p^x$, where p is a constant. A straight line with a gradient of 3 is obtained by plotting $\log_2 y$ against x . Find

(a) the value of p ,

(Ans : 8)

(b) the y -intercept of the straight line.

(Ans : -1)

[4 marks] [Forecast]

Answer :

(a)

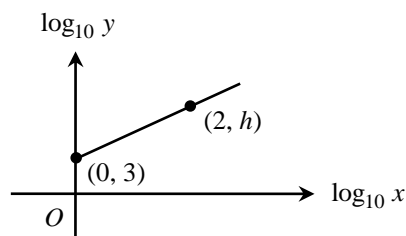
(b)

\Rightarrow apply 5 ~ 2

- 18 The variables x and y are related by the equation $y = kx^4$, where k is a constant.

(a) Convert the equation $y = kx^4$ to linear form.

(b) The diagram shows the straight line obtained by plotting $\log_{10} y$ against $\log_{10} x$.



Find the value of

(i) $\log_{10} k$,

(Ans : 3)

(ii) h .

(Ans : 11)

[4 marks] [2005, No.13]

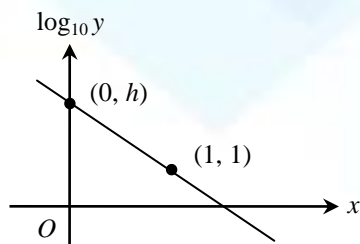
Answer :

(a)

(b) (i)

(ii)

- 19 The variables x and y are related by the equation $y = 1000p^x$, where p is a constant. The diagram shows the straight line graph obtained by plotting $\log_{10}y$ against x .



- (a) Express the equation $y = 1000p^x$ in linear form used to obtain the straight line graph shown above.
 [Ans : $y = (\log_{10} p)(x) + 3$]
- (b) Find the value of h and of p .
 (Ans : $h = 3, p = \frac{1}{100}$)

[4 marks] [2013, No.12]

Answer :

- (a) (b)

- 20 x and y are related by the equation $y = ax^n$, where a and n are constants. When the graph of $\log_2 y$ is plotted against $\log_2 x$, a straight line passing through the points $(1, 5)$ and $(3, 11)$ is obtained. Find the value of a and of n .

(Ans : $a = 4, n = 3$)

[4 marks] [Forecast]

Answer :

⇒ **apply 6**

- 21 Diagram (a) shows the graph of a non linear equation. Diagram (b) shows the straight line graph obtained when the non linear equation is expressed in linear form.

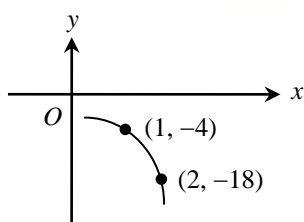


Diagram (a)

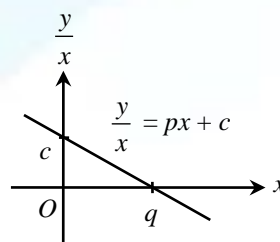


Diagram (b)

Calculate the value of c , p and q .

(Ans : $p = -5$, $q = \frac{1}{5}$, $c = 1$)

[4 marks] [Forecast]

Answer :

- 22 The diagram (a) shows part of the curve $y = ax^2 + bx$, where a and b are positive constants

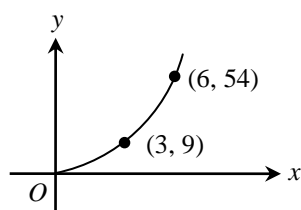


Diagram (a)

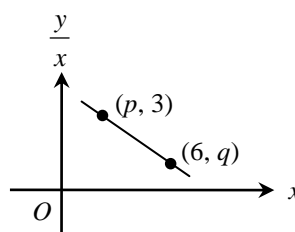


Diagram (b)

The graph of the curve $y = ax^2 + bx$ is converted to its linear form, a straight line is obtained as shown in diagram (b). Find

(a) the values of p and q ,

(Ans : $p = 3$, $q = 9$)

(b) the values of a and b .

(Ans : $a = 2$, $b = -3$)

[4 marks] [Forecast]

Answer :

(a)

(b)

- 23 Diagram (a) shows the curve $y = ab^x$, where a and b are constants. Diagram (b) shows the straight line graph obtained when $y = ab^x$ is expressed in the linear form.

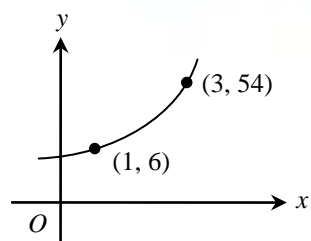


Diagram (a)

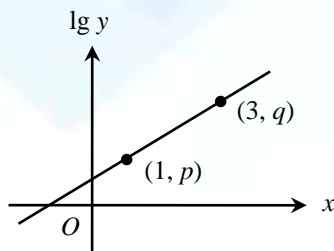


Diagram (b)

Find

- (a) the values of p and q ,
 (b) the values of a and b .

(Ans : $p = 0.7782$, $q = 1.7324$)

(Ans : $a = 2$, $b = 3$)

[4 marks] [Forecast]

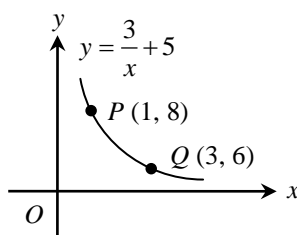
Answer :

(a)

(b)

⇒ **apply 7**

- 24 The diagram shows part of the graph $y = \frac{3}{x} + 5$ which passes points $P(1, 8)$ and $Q(3, 6)$.

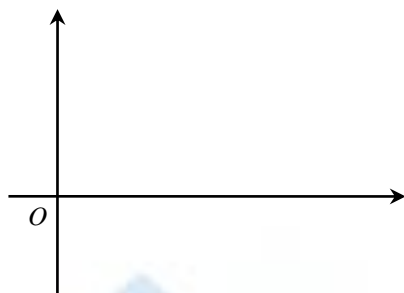


Sketch the graph xy against x that shows the coordinates of point P and point Q .

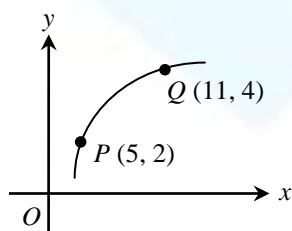
[Ans : $P(1, 8)$, $Q(3, 18)$]

[4 marks] [Forecast]

Answer :



- 25 The diagram shows a curve. The variables x and y are related by the linear equation $y^2 = Ax + B$, where A and B are constants.



- (a) Calculate the value of A and B .
 (b) Sketch the graph linear of y^2 against x .

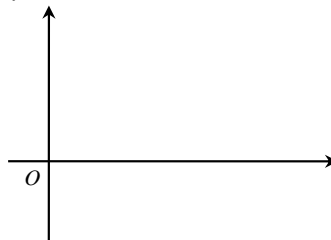
(Ans : $A = 2$, $B = -6$)

[4 marks] [Forecast]

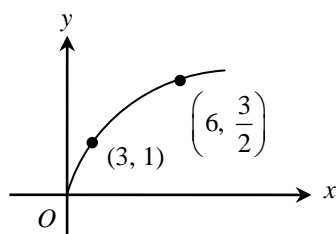
Answer :

(a)

(b)



- 26 The diagram shows a curve. The variables x and y are related by the equation $y = \frac{bx}{x+ab}$, where a and b are constants.



- (a) Sketch the graph linear of $\frac{1}{y}$ against $\frac{1}{x}$.

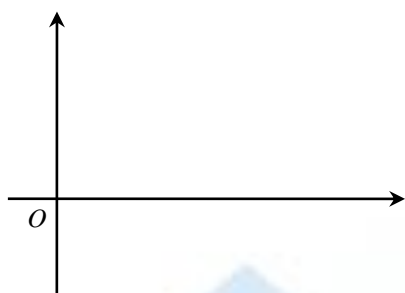
- (b) Calculate the values of a and b .

(Ans : $a = 2$, $b = 3$)
 [4 marks] [Forecast]

Answer :

(a)

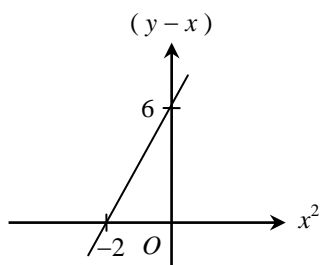
(b)



6.1 Linear and non-linear relations
6.1.3 Form equations of lines of best fit.

⇒ **type 1**

- 27** The diagram shows the straight line graph obtained by plotting $(y - x)$ against x^2 .

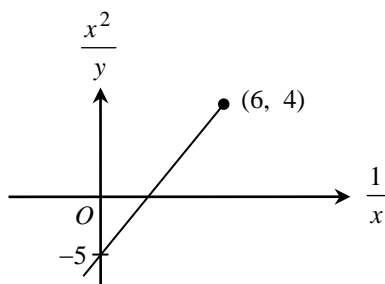


Express y in terms of x .

(Ans : $y = 3x^2 + x + 6$)
[3 marks] [2015, No.10]

Answer :

- 28** The diagram shows the graph of a straight line $\frac{x^2}{y}$ against $\frac{1}{x}$.

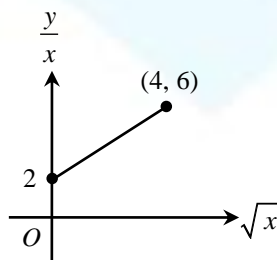


Based on the diagram, express y in terms of x .

(Ans : $y = \frac{2x^3}{3-10x}$)
[3 marks] [2018, No.13]

Answer :

- 29 The diagram shows a straight line graph of $\frac{y}{x}$ against \sqrt{x} .



Express y in term of x .

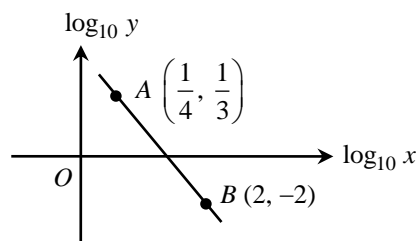
(Ans : $y = x^{\frac{3}{2}} + 2x$)

[3 marks] [Forecast]

Answer :

\Rightarrow type 2

- 30 A straight line is obtained by plotting $\log_{10} y$ against $\log_{10} x$, as shown in diagram below.



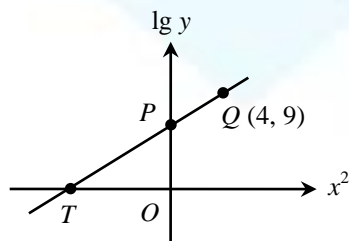
Find the relation between y and x .

(Ans : $y^3 = \frac{100}{x^4}$)

[4 marks] [Forecast]

Answer :

- 31 The diagram shows a straight line graph $\lg y$ against x^2 .



Given the gradient of the straight line PQ is 2, and P lies on $\lg y$ -axis.

(a) Find the coordinates of point P .

(b) Express y in term of x .

(Ans : $y = 10^{2x^2+1}$)

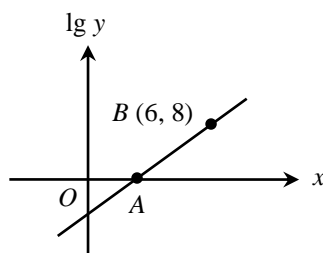
[4 marks] [Forecast]

Answer :

(a)

(b)

- 32 The diagram shows the graf $\lg y$ against x . Given the length of AB is $4\sqrt{5}$ unit and point A lies on x -axis



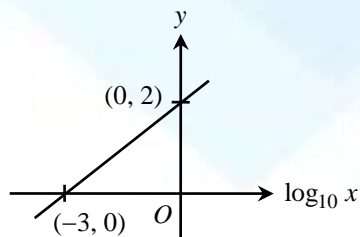
Express y in term of x .

(Ans : $y = 10^{2x-4}$)

[4 marks] [Forecast]

Answer :

- 33 The diagram shows a straight line graph y against $\log_{10} x$.



Given the straight line passes through points $(-3, 0)$ and $(0, 2)$.
Find the value of y when $x = 1000$.

(Ans : 4)
[4 marks] [Forecast]

Answer :

PAPER 2

⇒ **Part B → 10 marks**

6.1 Linear and non-linear relations

6.1.2 Draw lines of best fit for graph of linear relations with and without the use of digital technology.

6.1.4 Interpret information based on lines of best fit.

6.3 Application of linear law

6.3.1 Solve problems involving linear law.

⇒ **type 1a ~ apply linear law to non-linear relations**

- 34 Use graph paper to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. Variables x and y are related by the equation $y = px + \frac{r}{px}$, where p and r are constants

x	1.0	2.0	3.0	4.0	5.0	5.5
y	5.5	4.7	5.0	6.5	7.7	8.4

- (a) Plot xy against x^2 , by using a scale of 2 cm to 5 unit on both axes.

Hence, draw the line of best fit.

[5 marks]

- (b) Use the graph from (a) to find the value of

(i) p ,

(Ans : 1.373)

(ii) r .

(Ans : 5.488)

[5 marks][2005, No. 7]

Answer : REFER GRAPH

35 Use graph paper to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. Variables x and y are related by the equation $y = 2kx^2 + \frac{p}{k}x$, where p and k are constants.

x	2	3	4	5	6	7
y	8	13.2	20	27.5	36.6	45.5

- (a) Plot $\frac{y}{x}$ against x , using a scale of 2 cm to 1 unit on both axes.

Hence, draw the line of best fit.

[4 marks]

- (b) Use the graph from (a) to find the value of

(i) p ,

(Ans : 0.75)

(ii) k ,

(Ans : 0.25)

(iii) y when $x = 1.2$.

(Ans : 4.32)

[6 marks]

[2007, No.7]

Answer : REFER GRAPH

36 Use graph paper to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. Variables x and y are related by the equation $\frac{k}{y} = \frac{p}{x} + 1$, where k and p are constants.

x	1.5	2.0	3.0	4.0	5.0	6.0
y	2.502	0.770	0.465	0.385	0.351	0.328

- (a) Based on the table, construct a table for the values of $\frac{1}{x}$ and $\frac{1}{y}$. [2 marks]

- (b) Plot $\frac{1}{y}$ against $\frac{1}{x}$, using a scale of 2 cm to 0.1 unit on the $\frac{1}{x}$ -axis and 2 cm to 0.5 unit on the $\frac{1}{y}$ -axis. Hence, draw the line of best fit. [3 marks]

- (c) Use the graph from (b) to find the value of

(i) k ,

(Ans : 0.2564)

(ii) p ,

(Ans : -1.333)

[5 marks]

[2009, No.8]

Answer : REFER GRAPH

37 Use graph paper to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. Variables x and y are related by the equation $\frac{n}{y} = px + 1$, where n and p are constants.

x	0.1	0.2	0.3	0.4	0.5	0.6
y	0.303	0.364	0.465	0.588	0.909	1.818

- (a) Based on the table, construct a table for the values of $\frac{1}{y}$. [1 mark]
- (b) Plot $\frac{1}{y}$ against x , using a scale of 2 cm to 0.1 unit on the x -axis and 2 cm to 0.5 unit on the $\frac{1}{y}$ -axis. Hence, draw the line of best fit. [3 marks]
- (c) Use the graph from (b) to find the value of
- (i) y when $x = 0.38$, (Ans : 0.5714)
 - (ii) n , (Ans : 0.2597)
 - (iii) p , (Ans : -1.428)
- [6 marks]
[2011, No.7]

Answer : REFER GRAPH

38 Use graph paper to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. Variables x and y are related by the equation $y = \frac{h}{kx} + \frac{1}{kx^2}$, where h and k are constants.

x	1	2	3	4	5	6
y	2.601	0.551	0.194	0.089	0.040	0.017

- (a) Based on the above table, construct a table for the values of x^2y . [1 mark]
- (b) Plot x^2y against x , using a scale of 2 cm to 1 unit on the x -axis and 2 cm to 0.5 unit on the x^2y -axis. Hence, draw the line of best fit. [3 marks]
- (c) Use the graph from (b) to find the value of
- (i) y when $x = 2.5$, (Ans : 0.32)
 - (ii) k , (Ans : $\frac{1}{3}$)
 - (iii) h , (Ans : -0.1333)
- [6 marks]
[2012, No.7]

Answer : REFER GRAPH

39 Use the graph paper to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. Variables x and y are related by the equation $nx = py + xy$, where n and p are constants.

x	1.5	2.0	3.0	4.0	5.0	6.0
y	2.020	0.770	0.465	0.385	0.351	0.339

- (a) Based on the table above, construct a table for the values of $\frac{1}{y}$ and $\frac{1}{x}$. [2 marks]

- (b) Plot $\frac{1}{y}$ against $\frac{1}{x}$, using a scale of 2 cm to 0.1 unit on $\frac{1}{x}$ -axis and 2 cm to 0.5 unit on $\frac{1}{y}$ -axis. Hence, draw the line of best fit. [3 marks]

- (c) Using the graph in (b), find the value of

(i) n ,

(Ans : 0.2632)

(ii) p ,

(Ans : -1.316)

[5 marks]

[2015, No.11]

Answer : REFER GRAPH

40 Use the graph paper provided to answer this question.

The table shows the values of two variables, x and y , obtained by an experiment. The variables x and y are related by the equation $y - \sqrt{h} = \frac{hk}{x}$, where h and k are constants.

x	1.5	2.0	3.5	4.5	5.0	6.0
y	4.5	5.25	5.5	6.3	6.34	6.5

- (a) Plot xy against x , using a scale of 2 cm to 1 unit on the x -axis and 2 cm to 5 units on the xy -axis.

Hence, draw the line of best fit.

[4 marks]

- (b) Using the graph in (a), find

(i) the value of h and of k ,

(Ans : $h = 50 \frac{49}{64} / 50.766$, $k = -\frac{256}{3249} / -0.079$)

(ii) the correct value of y if one of the values of y has been wrongly recorded during the experiment

(Ans : 6)

[6 marks]

[2017, No.9]

Answer : REFER GRAPH

⇒ **type 1b** ~ form equations of lines of best fit

41 Use the graph paper provided to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. A straight line will be obtained when a graph of $\frac{y^2}{x}$ against $\frac{1}{x}$ is plotted.

x	1.25	1.43	2.00	2.50	4.00	5.00
y	4.47	4.38	4.18	3.87	2.83	2.24

(a) Based on the table, construct a table for the values of $\frac{1}{x}$ and $\frac{y^2}{x}$. [2 marks]

(b) Plot graph $\frac{y^2}{x}$ against $\frac{1}{x}$, using a scale of 2 cm to 0.1 unit on the $\frac{1}{x}$ - axis and 2 cm to 2 units on the $\frac{y^2}{x}$ - axis.

Hence, draw the line of best fit. [3 marks]

(c) Using the graph in (b),

(i) find the value of y when $x = 2.7$, (Ans : 3.747)

(ii) express y in terms of x . (Ans : $y = \sqrt{25 - 4x}$) [5 marks]

2018, No.11]

Answer : REFER GRAPH

42 Use the graph paper provided to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment.

x	1	1.2	1.4	1.6	1.8
y	79	56	36.5	18.4	0.9

(a) Plot xy against x^3 , by using a scale of 2 cm to 1 unit on the x^3 -axis and 2 cm to 10 units on the xy -axis.

Hence, draw the line of best fit. [4 marks]

(b) Use the graph from (a),

(i) determine the linear equation connecting x and y , (Ans : $xy = -15.89x^3 + 94.5$)

(ii) find the value of x , when $y = \frac{50}{x}$. (Ans : 1.409)

[6 marks]
[forecast]

Answer : REFER GRAPH

⇒ **type 2 ~ USING LAW OF LOGARITHMS**

43 Use graph paper to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. It is known that x and y are related by the equation $y = pk^{x^2}$, where p and k are constants.

x	1.5	2.0	2.5	3.0	3.5	4.0
y	1.59	1.86	2.40	3.17	4.36	6.76

- (a) Plot $\log y$ against x^2 , by using a scale of 2 cm to 2 units on the x^2 -axis and 2 cm to 0.1 unit on the $\log_{10} y$ -axis.

Hence, draw the line of best fit.

[5 marks]

- (b) Use the graph in (a) to find the value of

(i) p ,

(Ans : 1.259)

(ii) k .

(Ans : 1.109)

[5 marks]

[2003, No.7]

Answer : REFER GRAPH

44 Use graph paper to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. Variables x and y are related by the equation $y = pk^x$, where p and k are constants.

x	2	4	6	8	10	12
y	3.16	5.50	9.12	16.22	28.84	46.77

- (a) Plot $\log_{10} y$ against x , by using a scale of 2 cm to 2 units on the x -axis and 2 cm to 0.2 unit on the $\log_{10} y$ -axis.

Hence, draw the line of best fit.

[4 marks]

- (b) Use the graph from (a) to find the value of

(i) p ,

(Ans : 1.820)

(ii) k .

(Ans : 1.309)

[6 marks]

[2004, No.7]

Answer : REFER GRAPH

45 Use graph paper to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. Variables x and y are related by the equation $y = pk^{x+1}$, where p and k are constants.

x	1	2	3	4	5	6
y	4.0	5.7	8.7	13.2	20.0	28.8

- (a) Plot $\log y$ against $(x + 1)$, using a scale of 2 cm to 1 unit on the $(x + 1)$ -axis and 2 cm to 0.2 unit on the $\log_{10} y$ -axis.

Hence, draw the line of best fit.

[5 marks]

- (b) Use the graph from (a) to find the value of

(i) p ,

(Ans : 1.778)

(ii) k .

(Ans : 1.483)

[5 marks]

2006, No.7]

Answer : REFER GRAPH

46 Use graph paper to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. Variables x and y are related by the equation $y = hk^{2x}$, where h and k are constants.

x	1.5	3.0	4.5	6.0	7.5	9.0
y	2.51	3.24	4.37	5.75	7.76	10.00

- (a) Based on the table, construct a table for the values of $\log_{10} y$. [1 mark]
- (b) Plot $\log_{10} y$ against x , using a scale of 2 cm to 1 unit on the x -axis and 2 cm to 0.1 unit on the $\log_{10} y$ -axis.

Hence, draw the line of best fit.

[4 marks]

- (c) Use the graph from (b) to find the value of

(i) x when $y = 4.8$,

(Ans : 5)

(ii) h ,

(Ans : 1.905)

(iii) k ,

(Ans : 1.096)

[5 marks]

[2008, No.8]

Answer : REFER GRAPH

47 Use graph paper to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. Variables x and y are related by the equation $y = \frac{h^x}{k}$, where h and k are constants.

x	3	4	5	6	7	8
y	2.57	3.31	4.07	4.90	6.31	7.94

- (a) Plot $\log_{10} y$ against x , using a scale of 2 cm to 1 unit on the x -axis and 2 cm to 0.1 unit on the $\log_{10} y$ -axis.

Hence, draw the line of best fit.

[4 marks]

- (b) Use the graph from (a) to find the value of

(i) h ,

(Ans : 1.252)

(ii) k ,

(Ans : 0.7586)

(iii) y when $x = 2.7$.

(Ans : 2.40)

[6 marks]

2010, No.7]

Answer : REFER GRAPH

48 Use the graph paper to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. Variables x and y are related by the equation $y = \frac{h}{k^x}$, where h and k are constants.

x	4	6	8	10	12	14
y	2.82	2.05	1.58	1.23	0.89	0.66

- (a) Based on the table above, construct a table for the values of $\log_{10} y$.

[1 mark]

- (b) Plot $\log_{10} y$ against x , using a scale of 2 cm to 2 units on the x -axis and 2 cm to 0.1 unit on the $\log_{10} y$ -axis.

Hence, draw the line of best fit.

[3 marks]

- (c) Using the graph in (b), find the value of

(i) y when $x = 2$,

(Ans : 3.758)

(ii) h ,

(Ans : 5.012)

(iii) k ,

(Ans : 1.155)

[6 marks]

2014, No.9]

Answer : REFER GRAPH

49 Use the graph paper provided to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. The variables x and y are related by the equation $y = \frac{a}{b\sqrt{x}}$, where a and b are constants.

x	0.34	0.43	0.55	0.85	1.08	1.42
y	47.68	25.12	12.58	4.17	2.51	1.38

(a) Based on the table, construct a table for the values of $\log_{10} x$ and $\log_{10} y$. [2 marks]

(b) Plot $\log_{10} y$ against $\log_{10} x$, using a scale of 2 cm to 0.1 unit on the X -axis and 2 cm to 0.2 unit on the Y -axis.

Hence, Draw the line of best fit. [3 marks]

(c) Using the graph in (b), find the value of

(i) a ,

(Ans : -3.020)

(ii) b .

(Ans : 0.400)

[5 marks]

[2019, No.11]

Answer : REFER GRAPH

FORECAST

⇒ **type 1a** ~ apply linear law to non-linear relations

50 Use the graph paper to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. Variables x and y are related by the equation $\sqrt{y} = a\sqrt{x} + \frac{b}{\sqrt{x}}$, where a and b are constants.

x	1	2	3	4	5
y	0.64	4.79	9.67	14.82	19.89

(a) Plot \sqrt{xy} against x , by using a scale of 2 cm to 1 unit on the both axes.

Hence, draw the line of best fit. [4 marks]

(b) Use the graph in (a), find

(i) the values of a and b ,

(Ans : $a = 2.3$, $b = -1.5$)

(ii) the value of y , when $x = 3.6$.

(Ans : 12.66)

[6 marks]

Answer : REFER GRAPH

51 Use the graph paper provided to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. Variables x and y are related by the equation $4k^2x = (y - c)^2$, where k and c are constants.

x	0	100	400	900	1600	2500
y	20	30	40	50	60	70

- (a) Plot y against \sqrt{x} , by using a scale of 2 cm to 10 unit on the both axes.

Hence, draw the line of best fit.

[4 marks]

- (b) Use the graph in (a), find

- (i) the value of k and c , (Ans : $k = \frac{1}{2}$, $c = 20$)
 (ii) the value of x , when $y = 55$, (Ans : 1225)
 (iii) the value of y , when $x = 500$. (Ans : 42)

[6 marks]

Answer : REFER GRAPH

52 Use the graph paper to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. Variables x and y are related by the linear equation $\frac{x+3}{a} + \frac{y^2}{b} = 1$, where a and b are constants.

x	1	2	3	4	5
y	1.31	1.39	1.47	1.55	1.62

- (a) Plot a linear graph to represent the data.

[5 marks]

[HINT : $\frac{x}{a} + \frac{y}{b} = 1$ ~ equation of straight line in intercept form]

- (b) Use the graph from (a), find

- (i) the value of a and b , (Ans : $a = -3.5$, $b = 0.8$)
 (ii) the corresponding value of x , when $y = 1.5$. (Ans : 3.4)

[5 marks]

Answer : REFER GRAPH

53 Use the graph paper to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. Variables x and y are related by the equation $y^2 = m(x^2 + 2x) + n$, where m and n are constants.

x	1	2	3	4	5
y	2.18	2.65	3.24	3.84	4.53

- (a) Plot y^2 against $(x + 1)^2$, using a scale of 2 cm to 4 units on the $(x + 1)$ -axis and 2 cm to 2 units on the y^2 -axis.

Hence, draw the line of best fit.

[5 marks]

- (b) Use the graph from (a), to find the value of

(i) m ,

(Ans : 0.4792 ~ 0.4971)

(ii) n .

(Ans : 3.1971 ~ 3.2792)

[5 marks]

Answer : REFER GRAPH

54 Use the graph paper to answer this question.

The table shows the values of displacement, s meter and time, t seconds of a moving particle, obtained from an experiment. Given s and t are related by the equation $s = ut + \frac{1}{2}at^2$, where u and a are the initial velocity and acceleration of the particle respectively. A pair of data is misrecorded because of a mistake.

Time, t seconds	20	50	80	110	140	180
Displacement, s meter	6	22.5	48	x	119	198

- (a) Plot $\frac{s}{t}$ against t , using a scale of 2 cm to 40 units on the t -axis and 2 cm to 0.1 unit on the $\frac{s}{t}$ -axis.

Hence, draw the line of best fit.

[4 marks]

- (b) Marks \otimes on the graph to represent the exactly point of the misrecorded data.

[1 mark]

- (c) Use the graph from (a) to find the value of

(i) initial velocity,

(Ans : 0.2)

(ii) acceleration,

(Ans : 0.01)

(iii) x .

(Ans : 82.5)

[5 marks]

Answer : REFER GRAPH

55 Use the graph paper to answer this question.

The swing time of a pendulum with a length of L , is known to satisfy the non linear relation $T = 2\pi \sqrt{\frac{L}{g}}$, where g is a constant. The table shows the corresponding values of L and T obtained from an experiment.

L	20	40	60	80	100
T	0.95	1.28	1.58	1.81	2.04

- (a) Plot T^2 against L , by using a scale of 2 cm to 20 units on the L -axis and 2 cm to 1 unit on the T^2 -axis.

Hence, draw the line of best fit.

[4 marks]

- (b) Use the graph in (a), find

- (i) the value of g , (Ans : 947.7 ~ 995.7)
 (ii) the value of T , when $L = 50\text{cm}$, (Ans : 1.449)
 (iii) the value of L , when $T = 1.79\text{s}$. (Ans : 77 ~ 78)

[6 marks]

Answer : REFER GRAPH

56 Use the graph paper to answer this question.

Some load with a mass of m kg is hung at the end of a spring and swung vertically. The swing rate, f swings per second, for each load is determined. The table shows the results of the experiment.

Mass of load, m kg	0.01	0.02	0.04	0.06	0.08
Swing rate, f swing per second	20	14	10	8	7

It is known that the rate of swing, f swings per second, and mass of load, m kg, is related by the equation $f^2 km = 1$, where k is a constant.

- (a) Plot f^2 against $\frac{1}{m}$, by using a scale of 2 cm to 20 units on the $\frac{1}{m}$ -axis and 2 cm to 50 units on the f^2 -axis. Hence, draw the line of best fit. [4 marks]

- (b) Use the graph in (a), find

- (i) the swing rate for a load with a mass of 0.05kg, (Ans : 8.944)
 (ii) the mass of the load that makes 15 swings per second, (Ans : 18)
 (Give your answer correct to nearest gram)
 (iii) the value of k . (Ans : 0.2558)

[5 marks]

- (c) When the spring is replaced by another spring, it is found that the relation between f and m

becomes $f^2 = \frac{1}{2m}$. Draw the graph that is formed on the same axis. [1 mark]

Answer : REFER GRAPH

⇒ **type 2a** ~ apply linear law to non-linear relations

57 Use the graph paper to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. Variables x and y are related by the equation $y = ab^{-x}$, where a and b are constants.

x	1	2	3	4	5	6
y	41.7	34.7	28.9	27.5	20.1	16.7

- (a) Plot $\log_{10} y$ against x by using a scale of 2 cm to 1 unit on the x -axis and 2 cm to 0.2 unit on the $\log_{10} y$ -axis. Hence, draw the line of best fit. [4 marks]
- (b) Use your graph from (a) to find
- and mark \otimes on the graph to represent the exactly point of the misrecorded data, then estimate a more accurate value for the data, (Ans : 23.99)
 - the value of a and of b , (Ans : $a = 50.12$, $b = 1.202$)
 - the value of y when $x = 3.5$. (Ans : 26.3)
- [6 marks]

Answer : REFER GRAPH

58 Use the graph paper to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. Variables x and y are related by the equation $y = pq^{x-1}$, where p and q are constants.

x	3	4	5	6	7	8
y	12.1	6.46	3.47	1.89	0.95	0.53

- (a) Plot $\log_{10} y$ against $(x - 1)$, by using a scale of 2 cm to 1 unit on the $(x - 1)$ -axis and 2 cm to 0.2 unit on the $\log_{10} y$ -axis. Hence, draw the line of best fit. [4 marks]
- (b) From the graph paper in (a), find the value of
- p and q , (Ans : $p = 42.66$, $q = 0.5346$)
 - x when $y = 5.0$. (Ans : 4.4)
- [6 marks]

Answer : REFER GRAPH

59 Use the graph paper to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. Variables x and y are related by the equation $y = (1 + k) x^{\frac{c}{2}}$, where k and c are constants.

x	3.2	6.3	10	30	50	80	100
y	17.8	25	31.6	54.7	70.1	89.4	100

- (a) Express the non linear equation to linear form. [2 marks]
- (b) Plot a linear graph to represent the data. [4 marks]
- (c) Use the graph from (a), to find the value of
- (i) k , (Ans : $k = 9$)
- (ii) c . (Ans : 1) [4 marks]

Answer : REFER GRAPH

60 Use the graph paper to answer this question.

At time $t = 0$, a bacteria colony has 1000 bacteria. The bacteria population of the colony, y at any time t hours is given by the formula $y = y_0 e^{nt}$. The bacteria population of the colony at certain times t are recorded in the table.

t	0.5	1	1.5	2	2.5
y	2718	7389	20086	54598	148410

- (a) Show that $y_0 = 1000$. [1 mark]
- (b) Plot $\log_{10} y$ against t by using a scale of 2 cm to 0.5 unit on the both axes.
Hence, draw the line of best fit. [4 marks]
- (c) Use the graph from (b), find [Use $\log_{10} e = 0.4343$]
- (i) the value of n , (Ans : 1.996)
- (ii) the minimum time taken for the bacteria population to exceed 100000. (Ans : 2.3) [5 marks]

Answer : REFER GRAPH

61 Use the graph paper to answer this question.

The table shows the data obtained by Lea and Rowena in a Chemical experiment. The data show the relationship between the reaction rate of a chemical of $X \text{ mol s}^{-1}$ and the temperature of $T^\circ\text{C}$.

Reaction rate of chemical, $X \text{ mol s}^{-1}$	0.5	2.2	4.0	5.9	7.8
Temperature, $T^\circ\text{C}$	-7.5	-4.4	3.21	21.4	68.2

It is known that the temperature T and the reaction X are related by the equation $T+10 = Ab^X$, where A and b are constants.

- (a) Write $T + 10 = Ab^X$ in the form of linear equation.

[Ans : $\log_{10}(T+10) = \log_{10} A + (\log_{10} b)X$] [1 mark]

- (b) Plot $\log_{10}(T+10)$ against X by using a scale of 2 cm to 1 unit on the X -axis and 2 cm to 0.2 unit on the $\log_{10}(T+10)$ -axis.

Hence, draw the line of best fit. [4 marks]

- (c) Use the graph from (b), find

(i) the value of A and of b , (Ans : $A = 2.0895$, $b = 1.585$)

(ii) the value of X when $T = 0^\circ\text{C}$. (Ans : 3.45)

[5 marks]

Answer : REFER GRAPH

CONTINUOUS EXERCISES

62 Use the graph paper to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. The variables x and y are related by the equation $y = a^2x^2 + 2abx + b^2$, where a and b are constants.

x	2	4	6	8	10
y	40.23	25.81	14.58	6.54	1.68

- (a) Express $y = a^2x^2 + 2abx + b^2$ in linear form, $Y = mX + c$. (Ans : $\sqrt{y} = ax + b$) [2 marks]

[HINT : $(a + b)^2 = a^2 + 2ab + b^2$]

- (b) Using a scale of 2 cm to 2 units on the x -axis and 2 cm to 1 unit on the \sqrt{y} -axis, plot \sqrt{y} against x and draw the line of best fit. [4 marks]

- (c) Use the graph from (b),

(i) estimate the value of a and b . (Ans : $a = -0.6286$, $b = 7.6$)

(ii) find the value of x when $y = 17.64$. (Ans : 5.4)

[4 marks]

[Kedah2020, No.11]

Answer : REFER GRAPH

63 Use graph paper to answer this question.

The table shows the values of two variables, x and y , obtained from an experiment. The variables x and y are related by the equation $y = \frac{m}{x+n}$, where m and n are constants.

x	0.1	1.5	2.5	3.5	4.5	6.5
y	3.9	2.4	1.8	1.5	1.3	1.0

- (a) Based on the table, construct a table for the values of xy . [1 mark]
- (b) plot xy against y , using a scale of 2 cm to 0.5 unit on the horizontal axis and 2 cm to 1 unit on the vertical axis.

Hence, draw the line of best fit. [3 marks]

- (c) Use the graph from (b) to find the value of
- (i) m , (Ans : 2.085)
 - (ii) n , (Ans : 8.5)
 - (iii) the gradient of the straight line obtained if $\frac{1}{y}$ is plotted against x . (Ans : 0.1176)

[6 marks]
[YIK2020, No.11]

Answer : REFER GRAPH

COORDINATE GEOMETRY

- ONE PAGE NOTE (OPN) - WORKSHEET

Encik Suhairul bin Hadlee

ONE PAGE NOTES

“ COORDINATE GEOMETRY ”

Equation of PR = Equation of perpendicular bisector QS

- (1) m_{QS} (A) Rhombus
 (2) $m_{QS} \times m_{PR} = -1$ (B) Kite
 (3) midpoint QS
 (4) $y = (m_{PR})x + c$



$P(x_1, y_1)$ M $Q(x, y)$ N $R(x_2, y_2)$

$$\text{Distance } PR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Midpoint } PR = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Line Segment

$$Q(x, y) = \left(\frac{Mx_2 + Nx_1}{M+N}, \frac{My_2 + Ny_1}{M+N} \right)$$

$$\frac{Mx_2 + Nx_1}{M+N} = x \quad \& \quad \frac{My_2 + Ny_1}{M+N} = y$$

$$\text{Gradient of } PR, m_{PR} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

Note :

$$P, Q, R \text{ collinear} \rightarrow m_{PQ} = m_{PR} = m_{QR} = m_{QR}$$

$$\rightarrow \text{Area of } \triangle PQR = 0$$

Equation of straight line PR
[in gradient form]

$$y = mx + c$$

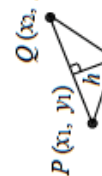
\uparrow y -intercept
 \downarrow gradient of PR

Equation of straight line [in general form]

$$ax + by + c = 0$$

Arrange in gradient form

If point (h, k) passes through @ lies on $ax + by = c$,
 the point satisfy the equation, which is $ah + bk = c$



$$\text{Area of } \triangle PQR = \frac{1}{2} \left| \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} \right|$$

Note : (1) arrange the points 'clockwise', the value in the absolute is negative, and vice-versa

(2) find the nearest distance / perpendicular R to PQ , $h \rightarrow \frac{1}{2} \times \text{distance of } PQ \times h = \text{Area of } \triangle PQR$

$$\text{Area of quadrilateral} = \frac{1}{2} \left| \begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{vmatrix} \right| = \frac{1}{2} \left| (x_1)(y_2) + (x_2)(y_3) + (x_3)(y_4) + (x_4)(y_1) - (y_1)(x_2) - (y_2)(x_3) - (y_3)(x_4) - (y_4)(x_1) \right|$$

$$\text{Area of polygon with } n \text{ sides} = \frac{1}{2} \begin{vmatrix} x_1 & x_2 & \dots & x_n \\ y_1 & y_2 & \dots & y_n \end{vmatrix} = \frac{1}{2} \left| (x_1)(y_2) + (x_2)(y_3) + \dots + (x_n)(y_1) - (y_1)(x_2) - (y_2)(x_3) - \dots - (y_n)(x_1) \right|$$

Concept of parallel / Equation of parallel

$$y = m_1x + c$$

$$y = m_2x + c$$

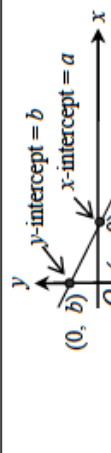
$$PQ \parallel RS \Rightarrow m_1 = m_2$$

Concept of perpendicular / Equation of perpendicular

$$y = m_1x + c$$

$$y = m_2x + c$$

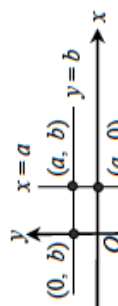
$$PQ \perp QR \Rightarrow m_1 \times m_2 = -1$$

Equation of straight line
[in intercept form]

$$m = -\left(\frac{b}{a}\right) = -\left(\frac{y\text{-intercept}}{x\text{-intercept}}\right)$$

Equation of straight line

\rightarrow parallel with x -axis $\sim y$ -coordinate similar $\sim y = b$
 \rightarrow parallel with y -axis $\sim x$ -coordinate similar $\sim x = a$



To determine either locus (1) @ (3) intersect x/y -axis \rightarrow
 substitute $y/x = 0$ and find the value of $b^2 - 4ac$

Equation of a locus \sim involving distance

$P(x, y) \rightarrow$ locus of point P from 1 fixed point with a constant distance

$PO = j$ \sim circle with centre O [midpoint of diameter], with radius j unit

$P(x, y) \rightarrow$ locus of point P with distance from 2 fixed points are equal

$PQ = PR \sim$ perpendicular bisector of QR

$P(x, y) \rightarrow$ locus of point P with distance ratio from two fixed point is $M:N$

$PQ:PR = M:N \rightarrow N(PQ) = M(PR) \sim$ is a circle

WORKSHEET
TOPIC 7 : COORDINATE GEOMETRY
[1 – 2 questions → 2 – 6 marks]

Revision [Distance, Midpoint, Gradient, Equation of straight line, Intersection point of two straight lines]

$$\Rightarrow \text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- 1** Given the points $A(3, 3)$, $B(5, -2)$, and $C(-2, 5)$. Show that the triangle ABC is isosceles.

[2 marks] [Forecast]

Answer :

- 2** Given $P(1, 7)$, Q and $R(6, 8)$ are three points on a Cartesian plane. If the area of triangle PQR is 6 unit^2 , find the perpendicular distance from Q to PR .

(Ans : 2.353)

[3 marks] [Forecast]

Answer :

- 3** The distance between points $C(-k, 5k)$ and $D(k, 4k)$ is $\sqrt{80}$, find the values of k . (Ans : -4, 4)

[3 marks] [Forecast]

Answer :

- 4** Given $P(2, 4)$, $Q(7, 3)$, and $R(t, 6)$ are three points on a Cartesian plane. If the length of PQ is half the length of PR , find the possible values of t .

(Ans : -8, 12)

[3 marks] [Forecast]

Answer :

$$\Rightarrow \text{midpoint, } (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

- 5 Show that the line joining $A(-2, 2)$ and $B(6, 10)$ and the line joining $C(0, 8)$ and $D(4, 4)$ bisect one another. [Hint : bisect one another \Rightarrow have same midpoint]

[3 marks] [Forecast]

Answer :

- 6 Given $A(2, 3)$, $B(5, 4)$, $C(6, 7)$, and $D(h, k)$ are vertices of a parallelogram. Find the values of h and k .

(Ans : $h = 3$, $k = 6$)

[3 marks] [Forecast]

Answer :

- 7 The coordinates of points A , B and C are $(-3, 4)$, $(-1, -2)$ and $(k, 4)$ respectively. Find the possible values of k if the distance between the midpoint of AB and C is $3\sqrt{2}$.

(Ans : $-5, 1$)

[3 marks] [Forecast]

Answer :

$$\Rightarrow \text{gradient, } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

- 8 Point $(h, 3)$ lies on the straight line that joins the points $(3, 9)$ and $(-2, 6)$. Find the value of h .

(Ans : -7)

[3 marks] [Forecast]

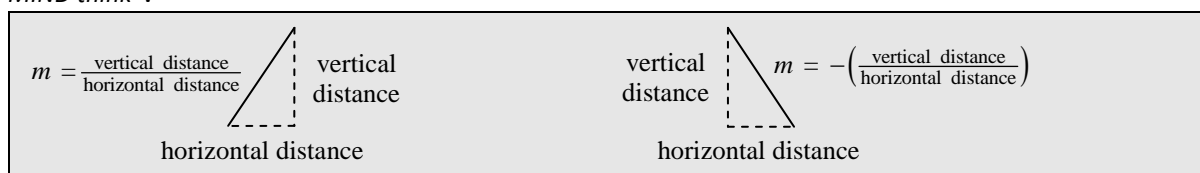
Answer :

- 9 Given points $P(-5, m)$, $Q(3, n)$, and $R(11, 1)$ are collinear. Show that $m - 2n + 1 = 0$.

[3 marks] [Forecast]

Answer :

MIND think :



\Rightarrow the equation of a straight line ~ gradient form, $y = mx + c$ — $\begin{cases} m = \text{gradient} \\ c = y\text{-intercept} \end{cases}$

- 10 The straight line $2y = 3x + h + 4$ intersect the y -axis at $5k$, where h and k are constants. Express h in terms of k .

(Ans : $h = 10k - 4$)

[2 marks] [2016, No.8]

Answer :

- 11 Given that the line $2x - 3y - 12 = 0$ meets the y -axis at P . Express the equation of the line in gradient form. Hence, state the coordinates of P .

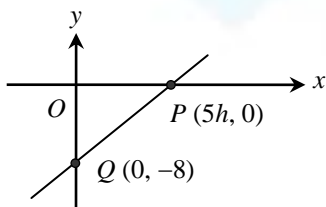
(Ans : $y = \frac{2}{3}x - 4$)

[2 marks] [Forecast]

Answer :

\Rightarrow the equation of a straight line ~ intercept form, $\frac{x}{a} + \frac{y}{b} = 1$ — $\left. \begin{array}{l} a = x\text{-intercept} \\ b = y\text{-intercept} \end{array} \right\} m = -\left(\frac{y\text{-intercept}}{x\text{-intercept}}\right)$

- 12 The diagram shows a straight line PQ with the equation $\frac{x}{10} + \frac{y}{2k} = 1$.



(a) h ,

(Ans : 2)

(b) k .

(Ans : -4)

[3 marks] [2012, No.14]

Answer :

(a)

(b)

- 13 A straight line which passes through point $(0, 8)$ has a gradient of 6, find the x -intercept of the straight line. Hence, state the equation of the straight line in intercept form.
(Ans : $-\frac{4}{3}$) [3 marks] [Forecast]

Answer :

- 14 Express the equation $6x - 7y - 5 = 0$ in the intercept form. Hence, state the gradient of the straight line.
(Ans : $\frac{x}{\frac{5}{6}} - \frac{y}{\frac{5}{7}} = 1$, $\frac{6}{7}$)

[2 marks] [Forecast]

Answer :

\Rightarrow **intersection point of two straight lines**

- 15 The straight line that has a gradient of 2 and passes through the point (4, -1) intersect the straight line $x + y + 4 = 0$ at point Q. Find the coordinates of point Q. [Ans : $(\frac{5}{3}, -\frac{17}{3})$]

[4 marks] [Forecast]

Answer :

- 16 The straight line AB passes through the points (-6, -2) and (2, 6). The straight line CD has a gradient of 3 and passes through the point (2, 8). Find the intersection point of the straight line AB and CD. [Ans : (1, 5)]





[4 marks] [Forecast]



Answer :

7.1 Divisor of a line segment

7.1.1 Relate the position of a point that divides a line segment with the related ratio.

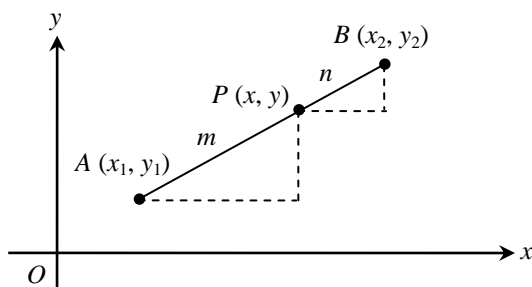
- 17 Sketch the following situasi on the given line segments. Hence, state the ratio of the following case.

<p>(a) B divides AC internally in the ratio 2 : 3.</p>  <p>AB : AC = <input type="text"/></p>	<p>(b) Point B lies on AC such that AB : AC = 3 : 4.</p>  <p>BC : AC = <input type="text"/></p>
<p>(c) The straight line AB is extended to point C such that its distance from point B is twice the distance of AB.</p>  <p>AB : BC = <input type="text"/></p>	<p>(d) Point R divides the line segment PQ such that $2PQ = 3RQ$.</p>  <p>RQ : PR = <input type="text"/></p>

<p>(e) A line segment such that $\frac{KM}{ML} = \frac{5}{3}$.</p>  <p>$ML : KL =$ <input type="text"/></p>	<p>(f) A line segment such that $PM = \frac{3}{8}PQ$.</p>  <p>$PM : MQ =$ <input type="text"/></p>
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7.1.2 Derive the formula for divisor of a line segment on a Cartesian plane, and hence use the formula in various situations.

18 In the diagram, point $P(x, y)$ is a point which divides line segment AB in the ratio $m : n$.



Show that $P(x, y) = \left(\frac{nx_1 + mx_2}{m+n}, \frac{ny_1 + my_2}{m+n} \right)$.

[3 marks] [Forecast]

Answer :

MIND think :

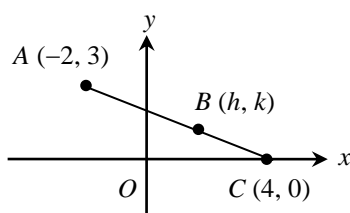
- If $m = n$, P will become the of line segment AB .

- 19 The points $A(2h, h)$, $B(p, t)$ and $C(2p, 3t)$ are on a straight line. B divides AC internally in the ratio 2 : 3. Express p in term of t .

(Ans : $p = -2t$)
[3 marks] [2003, No.9]

Answer :

- 20 The diagram shows a straight line AC .



The point B lies on AC such that $AB : BC = 3 : 1$. Find the coordinates of B . [Ans : $(\frac{5}{2}, \frac{3}{4})$]

[3 marks] [2009, No.15]

Answer :

- 21 A straight line passes through $A(-2, -5)$ and $B(6, 7)$

(a) Given $C(h, 10)$ lies on the straight line AB , find the value of h . [HINT : collinear] (Ans : 8)

(b) Point D divides the line segment AB in the ratio 1 : 3. Find the coordinates of D .

[Ans : $(0, -2)$]

[4 marks] [2010, No.13]

Answer :

(a)

(b)

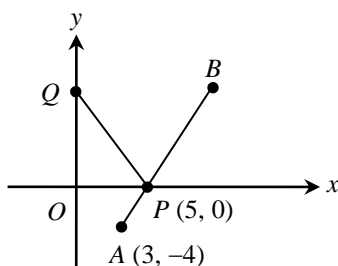
- 22** A straight line passes through $P(3, 1)$ and $Q(12, 7)$. The point R divides the line segment PQ such that $2PQ = 3RQ$. Find the coordinates of R .

[Ans : (6, 3)]

[3 marks] [2017, No.18]

Answer :

- 23** The diagram shows the straight line PQ with equation $\frac{x}{5} + \frac{y}{7} = 1$ intersects the straight line AB at point P .



- (a) State the y -intercept of PQ .
 (b) Find the coordinates of B if $BP = 2PA$.

[Ans : (9, 8)]

[3 marks] [2014, No.12]

Answer :

- (a)
 (b)

- 24** It is given that $A(1, 3)$ and $B(4, 7)$ lie on a Cartesian plane.

- (a) State the distance AB .
 (b) The straight line AB is extended to point C such that its distance from point B is twice the distance of AB . Find the coordinates of C .

(Ans : 5)

[Ans : (10, 15)]

[3 marks] [2019, No.13]

Answer :

- (a)
 (b)

- 25** Point $R(-3, 5)$ internally divides the line segment joining the points $P(-6, 7)$ and $Q(a, b)$ such that $PR = \frac{1}{4}PQ$. Find the values of a and b .
(Ans : $a = 6, b = -1$)

[3 marks] [Forecast]

Answer :

- 26** The points $A(-9, -2)$, $B(h, 0)$, and $C(3, 6)$ are on a straight line. Find

(a) $\frac{AB}{BC}$,

(Ans : $\frac{1}{3}$)

(b) the value of h .

(Ans : $h = -6$)

[3 marks] [Forecast]

Answer :

(a)

(b)

- 27** $C(p, -1)$ is the point dividing the line joining $A(-6, -5)$ and $B(1, 9)$ internally in the ratio $h : k$. Find

(a) the values of h and k ,
(Ans : $h = 2, k = 5$)

(b) the value of p .
(Ans : -4)

[3 marks] [Forecast]

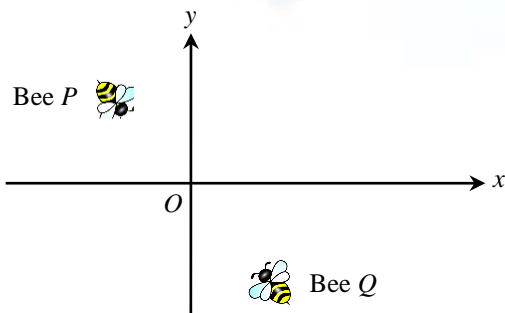
Answer :

(a)

(b)

7.1.3 Solve problems involving divisor of a line segment.

- 28** The diagram shows the position of two bees, P and Q .

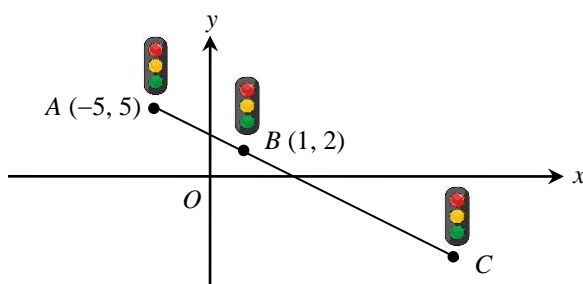


The coordinates of bee P and bee Q are $(-4, 4)$ and $(8, -12)$ respectively. Both bees fly towards each other on a straight line with different velocity. The velocity of bee P is three times more than velocity of bee Q . Find the distance of bee P from its initial point when it meets with bee Q . (Ans : 15)

[3 marks] [2015, No.12]

Answer :

- 29** The diagram show three traffic lights A , B and C along a straight line on a Cartesian plane.



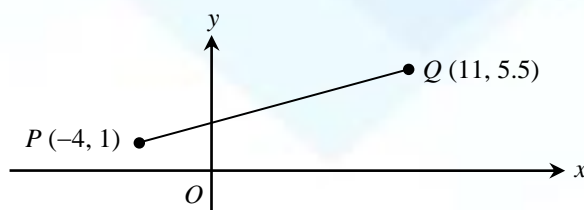
It is given that the distance from B to C is $\frac{5}{8}$ times the distance from A to C . Find the coordinates of traffic light C .

[Ans : $(11, -3)$]

[3 marks] [Forecast]

Answer :

- 30** The diagram shows the plan of a straight highway between two towns, P and Q on the Cartesian plane.



An engineer wants to build two rest houses between the two towns such that the two rest houses divide the road into three equal parts of equal distance.

Find the coordinates of the two rest houses.

[Ans : (1, 2.5), (6, 4)]
[3 marks] [Clon textbook form 4]

Answer :

7.2 Parallel lines and perpendicular lines

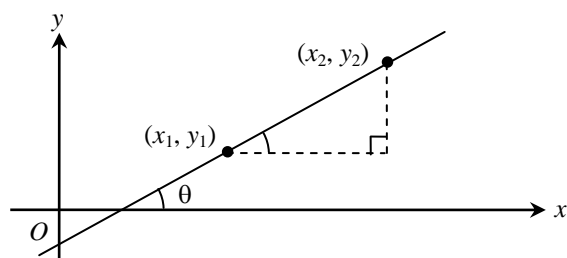
7.2.1 Make and verify conjectures about gradient of :

(i) parallel lines,

(ii) perpendicular lines

and hence, make generalisations.

- 31** The diagram shows a line segment on a Cartesian plane.



Show that the gradient of a straight line, $m = \tan \theta$.

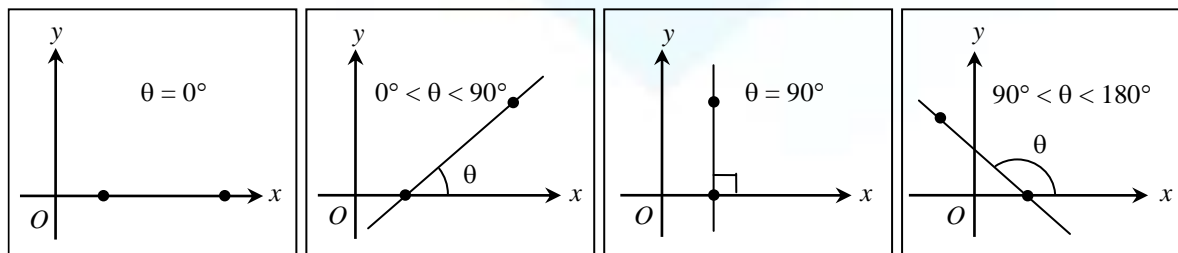
[2 marks] [Forecast]

Answer :

32 Match the following straight lines with the correct value of $\tan \theta$.

[2 marks] [Forecast]

Answer :

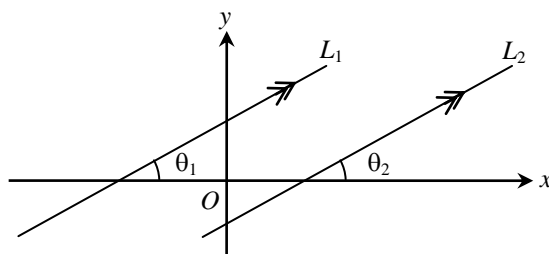


$\tan \theta < 0$	$\tan \theta = 0$	$\tan \theta > 0$	$\tan \theta = \infty$ (undefined)
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MIND think :

- $m = \boxed{}$, with θ being the angle formed between the straight line and the positive x -axis, and $0^\circ \leq \theta \leq 180^\circ$.

33 The diagram shows two straight lines, L_1 and L_2 on a Cartesian plane.

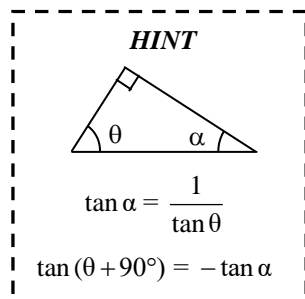
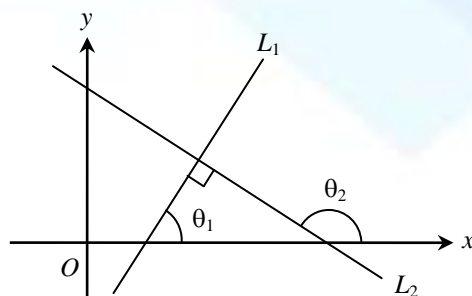


Given L_1 is parallel to L_2 , show that $m_1 = m_2$, where m_1 is the gradient of L_1 and m_2 is the gradient of L_2 .

[2 marks] [Forecast]

Answer :

34 The diagram shows two straight lines, L_1 and L_2 on a Cartesian plane.



Given L_1 is perpendicular to L_2 , show that $m_1 \times m_2 = -1$, where m_1 is the gradient of L_1 and m_2 is the gradient of L_2 .

[3 marks] [Forecast]

Answer :

MIND think :

- Two straight lines, L_1 and L_2 are parallel to each other \Leftrightarrow
- Two straight lines, L_1 and L_2 are perpendicular to each other \Leftrightarrow

\Rightarrow **parallel lines**

35 The equation of two straight lines are $\frac{x}{3} + \frac{y}{4} = 2$ and $3y = 1 - 4x$. Determine whether the lines are parallel to each other.

(Ans : parallel)

[3 marks] [Forecast]

Answer :

- 36 The straight line $\frac{x}{6} + \frac{y}{h} = 1$ has a y -intercept of 2 and is parallel to the straight line $y + kx = 0$.

Determine the value of h and of k .

(Ans : $h = 2$, $k = \frac{1}{3}$)

[3 marks] [2007, No.13]

Answer :

- 37 The straight line $y = -3x + 8$ is parallel to the straight line $y = (k + 2)x + 7$, where k is a constant. Determine the value of k .

(Ans : -5)

[2 marks] [2014, No.13]

Answer :

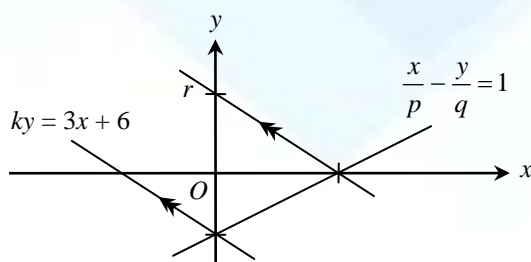
- 38 Given the points $P(2, -3)$, $Q(5, -1)$, $R(-8, m)$, and $S(-5, 4)$. Find the value of m if the straight lines PQ and RS are parallel.

(Ans : 2)

[3 marks] [Forecast]

Answer :

- 39 The diagram shows three straight lines, such that k , p , q and r are constants.



Express

- (a) k in terms of q ,

(Ans : $k = -\frac{6}{q}$)

- (b) r in terms of k and p .

(Ans : $r = -\frac{3p}{k}$)

[3 marks] [2019, No.14]

Answer :

(a)

(b)

- 40 The straight line which joining the points $(2k, -k)$ and $(2, -4)$ is parallel to the straight line $y = 1 - 3x$. Find the value of k .

(Ans : $\frac{2}{5}$)

[3 marks] [Forecast]

Answer :

- 41 The table shows three equations of straight line, KL , MN and PQ .

Straight Line	Equation
KL	$3x - 4y = 7$
MN	$y = \frac{3}{4}x - 18$
PQ	$3x - 2y - 17 = 0$

Which straight line did not intersect to each other ?
Give reason for your answer.

[2 marks] [Forecast]

Answer :

\Rightarrow **perpendicular lines**

- 42** The equation of two straight lines are $\frac{y}{5} + \frac{x}{3} = 1$ and $5y = 3x + 24$. Determine whether the lines are perpendicular to each other.

(Ans : perpendicular)
[3 marks] [2003, No.11]

Answer :

- 43** The following information refers to the equations of two straight lines, JK and RT , which are perpendicular to each other.

$$\begin{aligned} JK &: y = px + k \\ RT &: y = (k - 2)x + p \\ \text{where } p \text{ and } k &\text{ are constants.} \end{aligned}$$

Express p in terms of k .

(Ans : $p = \frac{1}{2-k}$)
[2 marks] [2005, No.14]

Answer :

- 44** The following information refers to the equation of two straight lines, AB and CD .

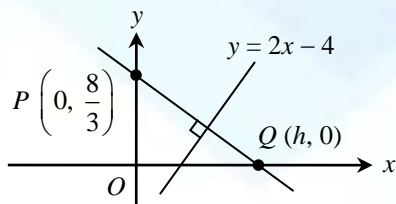
$$\begin{aligned} AB &: y - 2kx - 3 = 0 \\ CD &: \frac{x}{3h} + \frac{y}{4} = 1 \\ \text{where } h \text{ and } k &\text{ are constants.} \end{aligned}$$

Given the straight lines AB and CD are perpendicular to each other, express h in terms of k .

(Ans : $h = \frac{8}{3}k$)
[3 marks] [2018, No.10]

Answer :

- 45 The straight line $y = 2x - 4$ is perpendicular to the line segment which joins the points P and Q .



Find the value of h

(Ans : $5\frac{1}{3}$)

[3 marks] [Forecast]

Answer :

- 46 If the straight line $y = 8x - 6$ is perpendicular to the straight line which joins points $(2, 3)$ and $(4, p)$, find the value of p .

(Ans : $\frac{11}{4}$)

[3 marks] [Forecast]

Answer :

- 47 The coordinate of three points P , Q , and R are $(1, 1)$, $(2, m)$, and $(5, -1)$ respectively. If $\angle PQR = 90^\circ$, find the possible values of m .

(Ans : ± 2)

[3 marks] [Forecast]

Answer :

- 48 Two points have coordinates $E(2, 4)$ and $F(8, 6)$. If $C(2, t)$ lies on the perpendicular bisector of EF , find the value of t .

(Ans : 14)

[4 marks] [Forecast]

Answer :

- 49 The coordinates of the points A , B , and C are $(-4, 2)$, $B(-1, 4)$, and $C(3, -2)$ respectively. Show that ABC is right-angled triangle.

[3 marks] [Forecast]

Answer :

- 50 The line $y = px + q$ is perpendicular to the line $y - 2x = 5$ and passes through the point $(1, -5)$. Find the values of p and q .

(Ans : $p = -\frac{1}{2}$, $q = -4\frac{1}{2}$)

[3 marks] [Forecast]

Answer :

7.2.2 Solve problems involving equations of parallel and perpendicular lines.
[the use of dynamic software is encouraged]

\Rightarrow the equation of a straight line that passes through a fixed point and parallel to a given line

- 51 Find the equation of the line that passing through the point $(-4, 8)$ and is parallel to the line which joins points $(3, 5)$ and $(4, -1)$.

(Ans : $y = -6x - 16$)

[3 marks] [Forecast]

Answer :

- 52 Find the equation of the straight line which passes through the point $P(-3, 6)$ and is parallel to the straight line $4x - 2y + 1 = 0$.

(Ans : $y = 2x + 12$)

[3 marks] [Forecast]

Answer :

- 53 Find the equation of the line that is parallel to $x + 3y = 2$ and passing through the x-intercept of $\frac{x}{4} - \frac{y}{5} = 1$.

(Ans : $y = -\frac{1}{3}x + \frac{4}{3}$)

[3 marks] [Forecast]

Answer :

- 54 Find the equation of the straight line that is parallel to $5x + 2y = 8$ and bisect the line segment joining the points $(3, 1)$ and $(-1, -5)$.

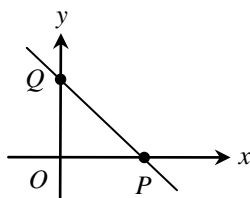
(Ans : $y = -\frac{5}{2}x + \frac{1}{2}$)

[3 marks] [Forecast]

Answer :

\Rightarrow the equation of a straight line that passes through a fixed point and perpendicular to a given line.

- 55 The diagram shows a straight line PQ with the equation $\frac{x}{2} + \frac{y}{3} = 1$. The point P lies on the x -axis and the point Q lies on the y -axis.



Find the equation of the straight line that is perpendicular to PQ and passing through the point Q .

(Ans : $y = \frac{2}{3}x + 3$)

[3 marks] [2004, No.14]

Answer :

- 56 Find the equation of the straight line that passes through point $(-1, 9)$ and perpendicular to the line

$$\frac{x}{3} - \frac{y}{6} = 1.$$

$$(Ans : y = -\frac{1}{2}x + \frac{17}{2})$$

[3 marks] [Forecast]

Answer :

- 57 Given the points $A(3, 3)$, $B(5, -2)$, and $C(-2, 5)$. Find the equation of the straight line that passes through point A and perpendicular to the straight line BC .
(Ans : $y = x$)

[3 marks] [Forecast]

Answer :

- 58 The coordinates of point A and point B are $(-2, 6)$ and $(4, 2)$ respectively. Point P divides AB internally in the ratio $3 : 1$. Find the equation of the straight line which passes through point P and perpendicular to the straight line AB .

$$(Ans : y = \frac{3}{2}x - \frac{3}{4})$$

[4 marks] [Forecast]

Answer :

\Rightarrow **the equation of the perpendicular bisector of a straight line**

59 A straight line $\frac{x}{2} + \frac{y}{6} = 1$ cuts the x-axis at P and y-axis at Q . Find

(a) the gradient of the straight line,

(Ans : -3)

(b) the equation of the perpendicular bisector of the straight line.

(Ans : $y = \frac{1}{3}x + \frac{8}{3}$)

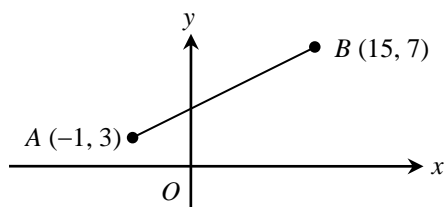
[3 marks] [2011, No.13]

Answer :

(a)

(b)

60 The diagram shows a straight line AB .



Find

(a) the midpoint of AB ,

[Ans : $(7, 5)$]

(b) the equation of the perpendicular bisector of AB .

(Ans : $y = -4x + 33$)

[4 marks] [2012, No.13]

Answer :

(a)

(b)

61 Given $ABCD$ is a rhombus with vertices $A(-1, 1)$ and $C(5, 7)$. Find the equation of the straight line BD .

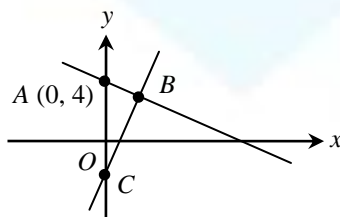
(Ans : $y = -x + 6$)

[3 marks] [Forecast]

Answer :

\Rightarrow the intersection point of two straight lines which perpendicular to each others

62 The diagram shows the straight line AB which is perpendicular to the straight line CB at the point B .



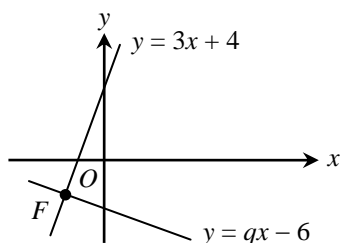
The equation of the straight line CB is $y = 2x - 1$. Find the coordinates of B .

[Ans : (2, 3)]

[3 marks] [2006, No.12]

Answer :

63 The diagram shows two straight lines on a Cartesian plane.



(a) State the value of q .

(Ans : $-\frac{1}{3}$)

(b) Find the coordinates of F .

[Ans : (-3, -5)]

[3 marks] [2016, No.9]

Answer :

(a)

(b)

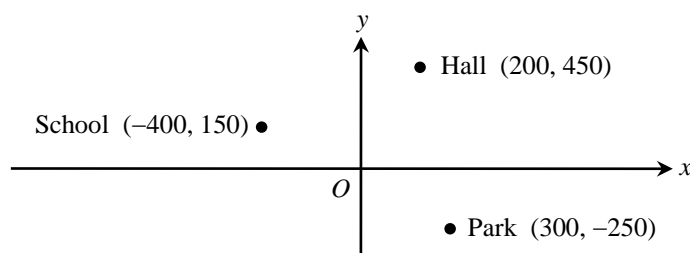
- 64 Find the equation of the straight line passing through point (4, 5) and perpendicular to the line $\frac{x}{4} + \frac{y}{2} = 1$. These two lines intersect at P . Find the coordinates of P .
[Ans : $y = 2x - 3$, (2, 1)]

[4 marks] [Forecast]

Answer :

⇒ **problems involving equations of parallel and perpendicular lines.**

- 65 The diagram shows the locations of the school, hall and park drawn on a Cartesian plane.



A clock tower will be built such as its distance from school and hall are the same, and the closest to the park. Find the coordinates of the clock tower.
[Ans : (200, -300)]

[7 marks] [Forecast]

Answer :

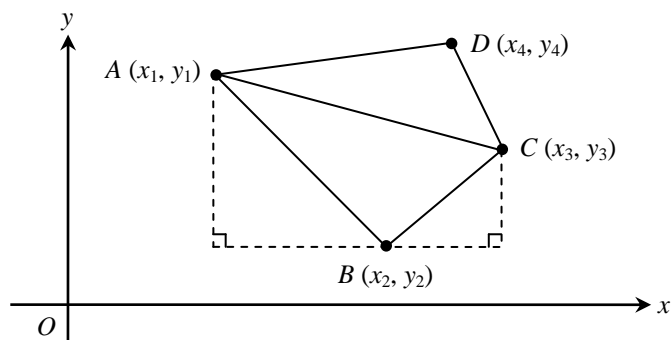
7.3 Areas of polygons

7.3.1 Derive the formula of area of triangles when the coordinates of each vertex are known.

[the use of digital technology is encouraged]

7.3.4 Make generalisation about the formula of area of polygons when the coordinates of each vertex are known, and hence use the formula to determine the area of polygons.

66 The diagram shows two triangles, ABC and ACD , drawn on a Cartesian plane.



(a) Show that :

$$(i) \quad \text{area of } \triangle ABC = \frac{1}{2} |x_1 y_2 + x_2 y_3 + x_3 y_1 - y_1 x_2 - y_2 x_3 - y_3 x_1| \Rightarrow \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$$

$$(ii) \quad \text{hence, area of quadrilateral } ABCD = \frac{1}{2} |x_1 y_2 + x_2 y_3 + x_3 y_4 + x_4 y_1 - y_1 x_2 - y_2 x_3 - y_3 x_4 - y_4 x_1|$$

(b). Based on (a), make a conclusion by induction for the area of a polygon with n sides.

[6 marks] [Forecast]

Answer :

MIND think :

Shoelace Algorithm :

- area of $\Delta = \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix} = \frac{1}{2} | \underbrace{(x_1 y_2 + x_2 y_3 + x_3 y_1) - (y_1 x_2 + y_2 x_3 + y_3 x_1)}_{\substack{(+)\text{ value} \sim \text{coordinates of vertices are arranged anticlockwise} \\ (-)\text{ value} \sim \text{coordinates of vertices are arranged clockwise}}} |$

~ If three points A , B and C are collinear \rightarrow area of $\triangle ABC = \square$

- area of quadrilateral $= \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix} = \frac{1}{2} | (x_1 y_2 + x_2 y_3 + x_3 y_4 + x_4 y_1) - (y_1 x_2 + y_2 x_3 + y_3 x_4 + y_4 x_1) |$
- area of polygon with n -sides $= \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & \dots & x_n & x_1 \\ y_1 & y_2 & y_3 & \dots & y_n & y_1 \end{vmatrix}$
 $= \frac{1}{2} | (x_1 y_2 + x_2 y_3 + x_3 y_4 + \dots + x_n y_1) - (y_1 x_2 + y_2 x_3 + y_3 x_4 + \dots + y_n x_1) |$

7.3.2 Determine the area of triangles by using the formula.

7.3.3 Determine the area of quadrilaterals by using the formula.

\Rightarrow **area 1**

- 67** By considering the area of triangle PQR , show that points $P(2, 5)$, $Q(-1, -1)$ and $R(-4, -7)$ are collinear.

[2 marks] [Forecast]

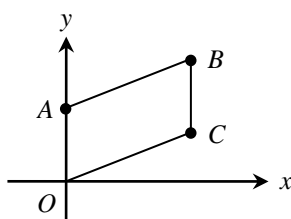
Answer :

- 68** By considering the area of quadrilateral $PQRS$, determine whether $P(-5, 5)$, $Q(-3, -2)$, $R(-1, 1)$, and $S(3, 7)$ are collinear.

[2 marks] [Forecast]

Answer :

- 69** The diagram shows a parallelogram $OABC$ with $A(0, 5)$ and $B(4, 7)$.



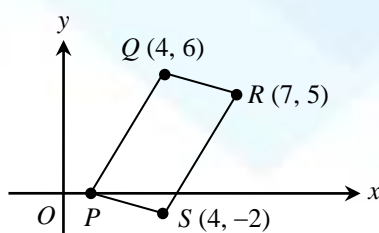
Find the area of parallelogram $OABC$.

(Ans : 20)

[3 marks] [Forecast]

Answer :

- 70 The diagram shows a parallelogram $PQRS$.



Find the area of parallelogram $PQRS$.

(Ans : 24)

[3 marks] [Forecast]

Answer :

- 71 Find the area of a pentagon $PQRST$ with vertices $P(-2, 1)$, $Q(1, 5)$, $R(4, 4)$, $S(6, -1)$ and $T(2, -3)$.

(Ans : 37.5)

[2 marks] [Forecast]

Answer :

\Rightarrow area 2

- 72 The vertices of a triangle are $A(5, 2)$, $B(4, 6)$ and $C(p, -2)$. Given that the area of the triangle is 30 units², find the values of p .

(Ans : -9, 21)

[3 marks] [2007, No.14]

Answer :

- 73 The points $(0, 3)$, $(2, t)$ and $(-2, -1)$ are the vertices of a triangle. Given that the area of the triangle is 4 units², find the values of t .

(Ans : 3, 11)

[3 marks] [2008, No.14]

Answer :

- 74 The points $P(k, 3)$, $Q(9, k)$, $R(2k, 7)$, and $S(-3, 3k + 2)$ are vertices of a quadrilateral. Find the value of k if the area of $PQRS$ is 30 unit^2 , where $k > 0$, and k is an integer.

(Ans : 2)

[3 marks] [Forecast]

Answer :

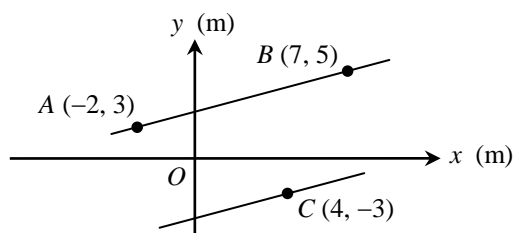
- 75 The area of a parallelogram with vertices $A(-1, -2)$, $B(1, -1)$, $C(3, 4)$, and $D(p, q)$ is 23 unit^2 . Show that $2q - 3p = 18$.

[3 marks] [Forecast]

Answer :

7.3.5 Solve problems involving areas of polygons.

- 76 The diagram shows the position of three campsites A , B and C at a park of a riverbank drawn on a Cartesian plane, such that A and B lie on the same straight riverbank.



Shah wants to cross the river from campsite C to the opposite riverbank where the campsites A and B located. Find the shortest distance, in m, that he can take to cross the river. Give your answer correct to four decimal places.

(Ans : 7.1587)

[4 marks] [2018, No.23]

Answer :

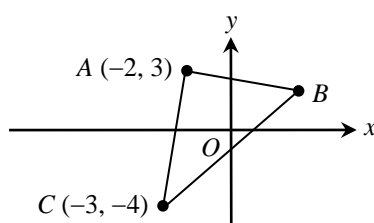
- 77 Given that the straight line $2y = x + 6$ intersects x -axis and y -axis at point A and point B respectively. If C is $(-4, 5)$, find the area of triangle ABC .
(Ans : 12)
[3 marks] [Forecast]

Answer :

- 78 Given that the straight line $2x + y - 8 = 0$ intersects the straight line $y = k$, x -axis and y -axis at points A , B and C respectively. If the area of $\triangle OAC$ is 12 unit^2 , find the value of k .
(Ans : 2)
[3 marks] [clon textbook form 4]

Answer :

- 79 The diagram shows a triangle ABC with an area of 18 unit^2 . The equation of straight line BC is $y - x + 1 = 0$.



Find the coordinates of point B .

[Ans : (3, 2)]
[3 marks] [Forecast]

Answer :

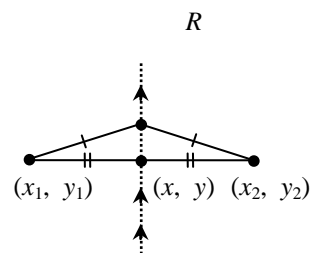
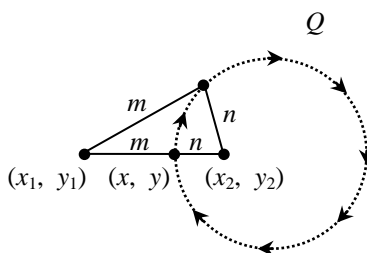
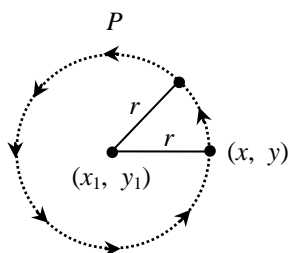
7.4 Equations of loci

7.2.1 Represent graphically, the locus that satisfies these conditions :

- (i) the distance of a moving point from a fixed point is constant,
 - (ii) the ratio of a moving point from two fixed points is constant,
- and hence determine the equation of the locus.

⇒ graphical of a locus

80 In the answer space, choose *P*, *Q* or *R*, which represents each of the following locus.



[3 marks] [Forecast]

Answer :

- (a) Locus of a moving point which the distance from a fixed point is always constant. ☐
- (b) Locus of a moving point which the distance from two fixed points is always constant in the ratio $m : n$. ☐
- (c) Locus of a moving point which the distance from two fixed points is always the same. ☐

MIND think :

Equation of circle with centre (a, b) and radius r units $\rightarrow (x-a)^2 + (y-b)^2 = r^2$

\Rightarrow **locus of a moving point which the distance from a fixed point is always constant**

- 81** Point P moves such that its distance is always 5 unit from $Q(-3, 4)$.

- (a) Describe fully the locus of P . **
 (b) Find the equation of the locus of P .

(Ans : $x^2 + y^2 + 6x - 8y = 0$)
 [3 marks] [2010, No.14]

Answer :

(a)

(b)

- 82** Find the equation of locus of the moving point P such that its distance from the midpoint of $A(2, 0)$ and $B(4, -2)$ is 3 unit.

(Ans : $x^2 + y^2 - 6x + 2y + 1 = 0$)
 [3 marks] [Forecast]

Answer :

- 83** A point P moves along the circumference of a circle with centre $A(2, 3)$. The circumference passes through point $Q(-2, 0)$. Find the equation of the locus of P .

(Ans : $x^2 + y^2 - 4x - 6y - 12 = 0$)
 [3 marks] [Forecast]

Answer :

- 84** The points $P(2, 6)$, $Q(-4, -2)$ and R lie on the circumference of the circle with diameter PQ . Find the equation of the locus of the moving point R .

(Ans : $x^2 + y^2 + 2x - 4y - 20 = 0$)
 [4 marks] [Forecast]

Answer :

- 85 Given the points $A(4, 6)$ and $B(2, 4)$. Find the equation of the locus of the moving point P such that the triangle APB always has a right angle at P .
 (Ans : $x^2 + y^2 - 6x - 10y + 32 = 0$)
 [4 marks] [Forecast]

Answer :

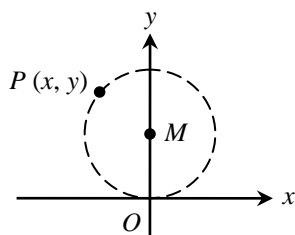
- 86 The coordinate of points A and B are $(-2, 1)$ and $(5, -6)$ respectively. Find the equation of the locus of the moving point R such that $\angle ARB$ is always 90° .
 (Ans : $x^2 + y^2 - 3x + 5y - 16 = 0$)
 [4 marks] [Forecast]

Answer :

- 87 Given the locus of P with centre $A(h, k)$ and diameter 13 units is $x^2 + y^2 - 5x - 12y = 0$. Find the coordinates of point A .
 [Ans : $(\frac{5}{2}, 6)$]
 [3 marks] [Forecast]

Answer :

- 88 The diagram show a locus $P(x, y)$ which moves such that its distance from point $M(0, h)$ is always equal.



Given the equation of the locus of P is $x^2 + y^2 - 10y = 0$. Find the value of h .

(Ans : 5)

[3 marks] [Forecast]

Answer :

\Rightarrow locus of a moving point which the distance from two fixed points is always constant in the ratio $m : n$.

- 89** The point A is $(-1, 3)$ and the point B is $(4, 6)$. The point P moves such that $PA : PB = 2 : 3$. Find the equation of the locus of P .
(Ans : $5x^2 + 5y^2 + 50x - 6y - 118 = 0$)

[3 marks] [2004, No.15]

Answer :

- 90** A point R moves such that its distance from the points $A(-2, 3)$ and $B(5, -1)$ are in the ratio $2 : 1$. Find the equation of the locus of R .
(Ans : $3x^2 + 3y^2 - 44x + 14y + 91 = 0$)

[3 marks] [Forecast]

Answer :

- 91** The coordinates of point A and B are $(-3, -5)$ and $B(1, -2)$ respectively. A point P moves such that $2AP = 3PB$. Find the equation of the locus of P .
(Ans : $5x^2 + 5y^2 - 42x - 4y - 91 = 0$)

[3 marks] [Forecast]

Answer :

- 92** Find the locus of the moving point M such that its distance from point $A(1, -1)$ is three times its distance from point $B(-2, 3)$.
(Ans : $8x^2 + 8y^2 + 38x - 56y + 115 = 0$)

[3 marks] [Forecast]

Answer :

- 93 A point R moves from two fixed points $P(1, 0)$ and $Q(-2, 3)$ such that $RP = \frac{1}{2} RQ$. Find the equation of the locus of R .
 $(x^2 + y^2 - 4x + 2y - 3 = 0)$
 [3 marks] [Forecast]

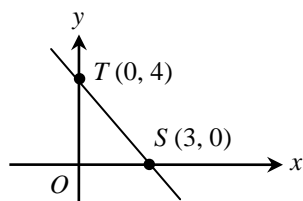
Answer :

- 94 Given the points $P(1, -3)$ and $Q(3, -1)$. Find the equation of the locus of Z if $PZ = 2RZ$ such that $PQ = 2QR$ and PQR is a straight line.
 $(Ans : x^2 + y^2 - 10x - 2y + 18 = 0)$
 [4 marks] [Forecast]

Answer :

\Rightarrow locus of a moving point which the distance from two fixed points is always the same.

- 95 The diagram shows a straight line passing through $S(3, 0)$ and $T(0, 4)$



- (a) Write down the equation of the straight line ST in the form $\frac{x}{a} + \frac{y}{b} = 1$.

- (b) A point $P(x, y)$ moves such that $PS = PT$.
 Find the equation of the locus of P .

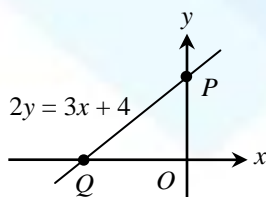
$(Ans : 6x - 8y + 7 = 0)$
 [4 marks] [2008, No.13]

Answer :

(a)

(b)

- 96 The diagram shows the straight line $2y = 3x + 4$ on a Cartesian plane.



A point T moves such that its distance from point P and point Q are equal.

(a) Describe fully the locus of T .

(b) Find the equation of the locus of T .

(Ans : $6x + 9y - 5 = 0$)

[4 marks] [Forecast]

Answer :

(a)

(b)

- 97 $A(6, 1)$ and $B(2, 4)$ are two fixed points. A point C moves such that $\angle CAB = \angle CBA$. Find the equation of the locus of C .

(Ans : $8x - 6y - 17 = 0$)

[3 marks] [Forecast]

Answer :

7.2.2 Solve problems involving equations of loci.
[usage of dynamic geometry software need to be involved]

- 98** A point R moves such that its distance from the point $M(2, 0)$ is equal to its distance from the line $x = -2$. Find the equation of the locus of R .

(Ans : $y^2 = 8x$)

[3 marks] [Forecast]

Answer :

- 99** A point P moves such that its distance from a point $A(2, 1)$ and its distance to the x -axis is always equal. Find the equation of the locus P .

($x^2 - 4x - 2y + 5 = 0$)

[3 marks] [Forecast]

Answer :

- 100** A point M moves such that its distance from the point $R(-\frac{3}{2}, 0)$ is twice its distance from the y -axis. Find the equation of the locus of M .

(Ans : $12x^2 - 4y^2 - 12x - 9 = 0$)

[3 marks] [Forecast]

Answer :

- 101** A point P moves such that the sum of distance OP and PN is always 2 units. O is the origin, N lies on x -axis and PN is parallel to y -axis. Show that the equation of the locus of P is $y = 1 - \frac{1}{4}x^2$.

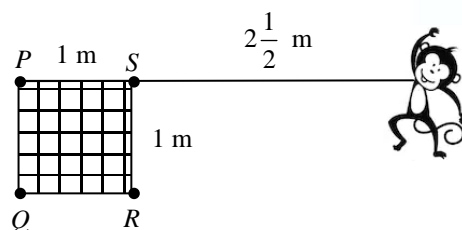
[4 marks] [Forecast]

Answer :

- 102** A monkey is tied to one vertex of the cage which measures $1\text{ m} \times 1\text{ m}$ with a rope. The length of the rope is $2\frac{1}{2}\text{ m}$. On the diagram in the answer space, sketch the locus if the monkey moves clockwise around the cage with the rope taut.

[3 marks] [clon textbook form 4]

Answer :



- 103** A point T moves such that its distance from point $(3, 1)$ is $2\sqrt{2}$ units. Show that the straight line $y - x = 2$ is the tangent of locus of T .

(Ans : $x^2 + y^2 - 6x - 2y + 2 = 0$)

[4 marks] [Forecast]

Answer :

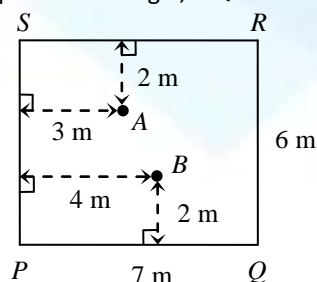
- 104** Given points $A(1, 3)$ and $B(4, 1)$. A point P moves such that its distance from B is twice its distance from A . Show that the locus of P does not intersect x -axis.

(Ans : $3x^2 + 3y^2 - 22y + 23 = 0$)

[4 marks] [Forecast]

Answer :

105 The diagram shows a field in the shape of a rectangle, $PQRS$ which measures $7\text{ m} \times 6\text{ m}$.



Rachel wants to move from one point on the PS so the distance from points A and B is always the same.

- (a) Find the distance from P where she should start. (Ans : $\frac{5}{4}$)
- (b) State the pattern of her journey.

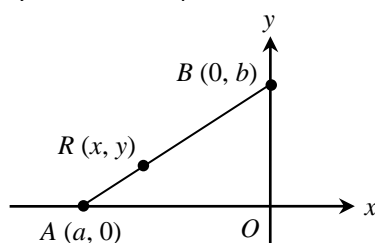
[4 marks] [Forecast]

Answer :

(a)

(b)

106 The diagram shows the x -axis and y -axis which represent the floor and wall respectively.



A ladder, AB of length 10 m leaning against the wall touches the floor and wall at points $A(a, 0)$ and $B(0, b)$ respectively.

- (a) Write the equation which relates a and b .
- (b) Given $R(x, y)$ is a point on the ladder AB such that the ratio $AR : RB = 1 : 3$. Show that the locus of point R when both ends of the ladder slide along the x -axis and y -axis is $\frac{16}{9}x^2 + 16y^2 = 100$.

[4 marks] [Forecast]

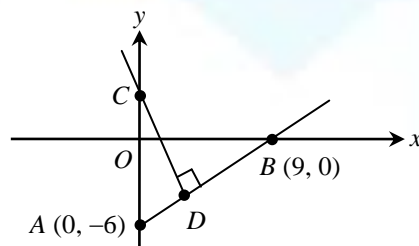
Answer :

(a)

(b)

PAPER 2 **\Rightarrow Part A ~ 6 – 8 marks**

107 The diagram shows a straight line CD which meets a straight line AB at point D . The point C lies on the y -axis.

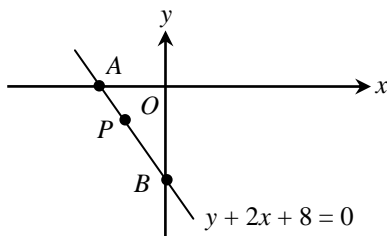


- (a) Write down the equation of AB in the form of intercepts. (Ans : $\frac{x}{9} - \frac{y}{6} = 1$) [1 mark]
- (b) Given that $2AD = DB$, find the coordinates of D . [Ans : $(3, -4)$] [2 marks]
- (c) Given that CD is perpendicular to AB , find the y -intercept of CD . (Ans : $\frac{1}{2}$) [3 marks]
- [2004, No.2]

Answer :

108 Solutions by scale drawing will not be accepted.

In the diagram, the straight line AB has an equation $y + 2x + 8 = 0$. AB intersects the x -axis at point A and intersects the y -axis at point B .



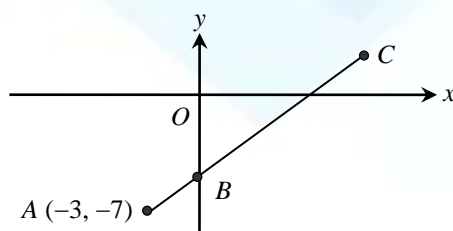
Point P lies on AB such that $AP : PB = 1 : 3$. Find

- (a) the coordinates of P , [Ans : $(-3, -2)$] [3 marks]
- (b) the equation of the straight line that passes through P and perpendicular to AB .
(Ans : $y = \frac{1}{2}x - \frac{1}{2}$) [3 marks][2007, No.2]

Answer :

109 Solutions by scale drawing will not be accepted.

The diagram shows the straight line AC which intersects the y -axis at the point B .



The equation of AC is $3y = 2x - 15$. Find

- (a) the equation of the straight line which passes through point A and perpendicular to AC .
 (Ans : $y = -\frac{3}{2}x - \frac{23}{2}$) [4 marks]

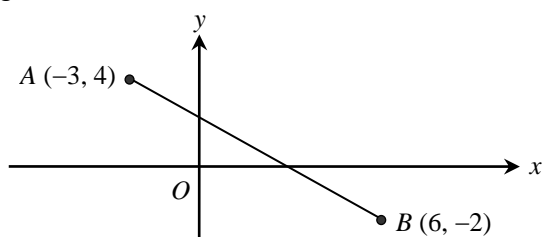
- (b) (i) the coordinates of B , [Ans : (0, -5)]
 (ii) the coordinates of C , given $AB : BC = 2 : 7$. [Ans : ($\frac{21}{2}$, 2)]

[3 marks][2010, No.5]

Answer :

110 Solutions by scale drawing will not be accepted.

The diagram shows a straight line AB .



- (a) Calculate the area of triangle AOB . (Ans : 9) [2 marks]

- (b) Point C divides the straight line AB internally in the ratio $AC : CB = 3 : 2$. Find coordinates of C .
 [Ans : ($\frac{12}{5}$, $\frac{2}{5}$)] [2 marks]

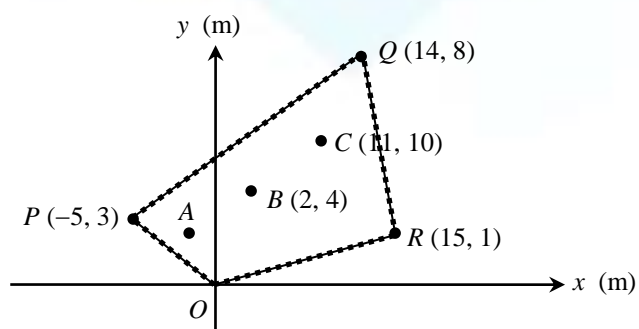
- (c) Point P moves such that its distance from A is always twice its distance from B . Find the equation of the locus of P .
 (Ans : $x^2 + y^2 - 18x + 8y + 45 = 0$) [3 marks]

[2011, No.5]

Answer :

111 Solutions by scale drawing is not accepted.

The diagram shows a campsite $OPQR$ that has been cleared by a group of scouts. Points A , B and C are the centre of the tents A , B and C . A , B and C are collinear.



Given the distance of point C is 3 times from point B and 4 times from point A .

(a) Find

- (i) the area, in m^2 , of the campsite $OPQR$,
(Ans : 94)
- (ii) the coordinates of A .
[Ans : $(-1, 2)$]

[4 marks]

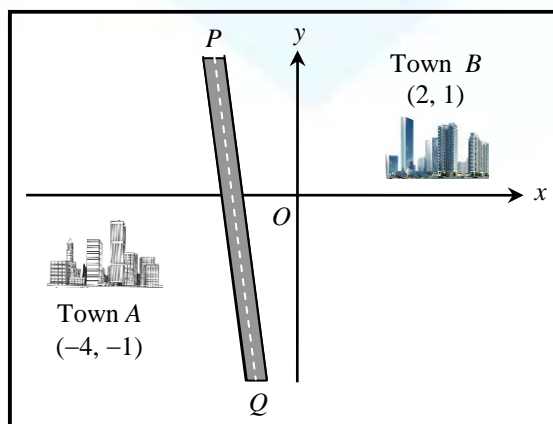
- (b) A scout spread sulphur powder around tent C such that the distance of the sulphur powder track from the centre of tent C is always 3 m. Find the equation of the track of the sulphur powder.
(Ans : $x^2 + y^2 - 22x - 20y + 212 = 0$) [3 marks]

[2015, No.3]

Answer :

112 Solutions by scale drawing is not accepted.

The diagram shows the locations of town A and town B drawn on a Cartesian plane.



PQ is a straight road such that the distance from town A and town B to any point on the road is always equal.

(a) Find the equation of PQ .

(Ans : $3x + y + 3 = 0$ / $y = -3x - 3$) [3 marks]

(b) Another straight road, ST with an equation $y = 2x + 7$ is to be built.

(i) A traffic light is to be installed at the crossroads of the two roads. Find the coordinates of the traffic light. [Ans : $(-2, 3)$]

(ii) Which of the two roads passes through town C $\left(-\frac{4}{3}, 1\right)$?

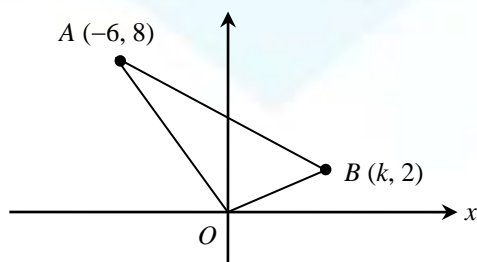
[4 marks]

[2017, No.5]

Answer :

113 Solution by scale drawing is not accepted

The diagram shows a triangle OAB .



- (a) Given the area of triangle OAB is 30 unit^2 , find the value of k .
 (Ans : 6) [2 marks]
- (b) Find the perpendicular distance from O to AB . ** (Ans : 4.472) [2 marks]
- (c) Point $Q(2, 4)$ lies on the straight line AB .
- (i) Find $AQ : QB$. (Ans : 2 : 1)
- (ii) Point P moves such that $PB = 2PQ$.
 Find the equation of the locus P . (Ans : $3x^2 + 3y^2 - 4x - 28y + 40 = 0$)

[5 marks]

[2018, No.3]

Answer :

⇒ **Part B ~ 10 marks**

114 Solutions to this question by scale drawing will not be accepted.

A point P moves along the arc of a circle with centre $A(2, 3)$. The arc passes through $Q(-2, 0)$ and $R(5, k)$.

(a) Find

(i) the equation of locus of the point P ,

(Ans : $x^2 + y^2 - 4x - 6y - 12 = 0$)

(ii) the values of k .

(Ans : $-1, 7$)

[6 marks]

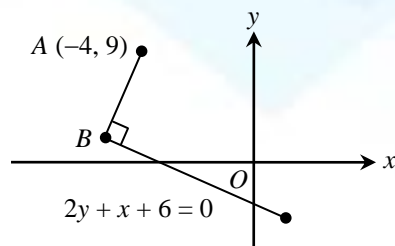
(b) The tangent of the circle at point Q intersects the y -axis at point T . Find the area of triangle OQT .

(Ans : $2\frac{2}{3}$) [4 marks]

[2003, No.11]

Answer :

115 Solution to this question by scale drawing will not be accepted



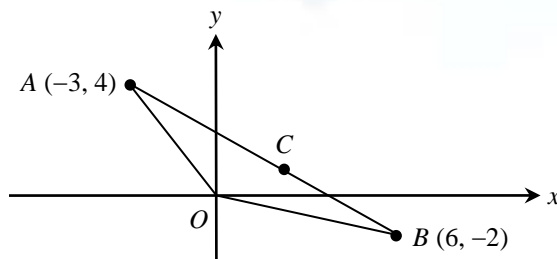
- (a) Find
- the equation of the straight line AB ,
 (Ans : $y = 2x + 17$) [2 marks]
 - the coordinates of point B .
 [Ans : $(-8, 1)$] [3 marks]
- (b) The straight line AB is extended to a point D such that $AB : BD = 2 : 3$. Find the coordinates of D .
 [Ans : $(-14, -11)$] [2 marks]
- (c) A point P moves such that its distance from point A is always 5 units. Find the equation of the locus of P .
 (Ans : $x^2 + y^2 + 8x - 18y + 72 = 0$) [3 marks]

[2005, No.9]

Answer :

116 Solutions to this question by scale drawing will not be accepted.

The diagram shows the triangle AOB where O is the origin. Point C lies on the straight line AB .

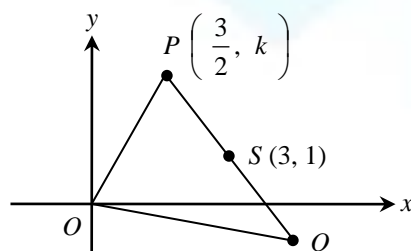


- (a) Calculate the area, in unit^2 , of triangle AOB . (Ans : 9) [2 marks]
- (b) Find the shortest distance from O to AB . ** (Ans : 1.664) [2 marks]
- (c) Given that $AC : CB = 3 : 2$, find the coordinates of C . [Ans : $(\frac{12}{5}, \frac{2}{5})$] [2 marks]
- (d) A point P moves such that its distance from point A is always twice its distance from point B .
- (i) Find the equation of the locus of P . (Ans: $x^2 + y^2 - 18x + 8y + 45 = 0$)
- (ii) Hence, determine whether or not this locus intercepts the y -axis. (Ans : no)
- [6 marks]
[2006, No.9]

Answer :

117 Solutions by scale drawing will not be accepted.

The diagram shows a triangle OPQ . Point S lies on the line PQ .



- (a) A point W moves such that its distance from point S is always $2\frac{1}{2}$ units. Find the equation of the locus of W .

(Ans : $4x^2 + 4y^2 - 24x - 8y + 15 = 0$) [3 marks]

- (b) It is given that point P and point Q lies on the locus of W . Calculate

(i) the value of k ,

(Ans : 3)

(ii) the coordinates of Q .

[Ans : $(4\frac{1}{2}, -1)$]

[5 marks]

- (c) Hence, find the area, in unit², of triangle OPQ .

(Ans : $\frac{15}{2}$)

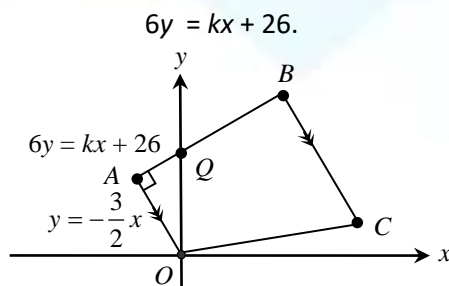
[2 marks]

[2008, No.10]

Answer :

118 Solutions by scale drawing will not be accepted.

The diagram shows a trapezium $OABC$. The line OA is perpendicular to the line AB , which intersect the y -axis at the point Q . It is given that the equation of OA is $y = -\frac{3}{2}x$ and the equation of AB is



(a) Find

- (i) the value of k ,
(Ans : 4)
- (ii) the coordinates of A .
[Ans : $(-2, 3)$]

[4 marks]

(b) Given $AQ : QB$ is $1 : 2$, find

- (i) the coordinates of B ,
- (ii) the equation of the straight line BC .

[Ans : $(4, 7)$]

(Ans : $y = -\frac{3}{2}x + 13$)

[4 marks]

(c) A point $P(x, y)$ moves such that $2PA = PB$. Find the equation of the locus of P .

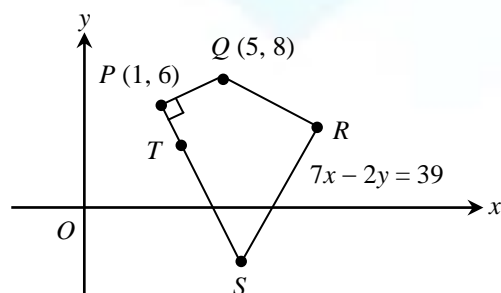
(Ans : $3x^2 + 3y^2 + 24x - 10y - 13 = 0$) [2 marks]

[2009, No.9]

Answer :

119 Solutions by scale drawing will not be accepted.

The diagram shows a quadrilateral $PQRS$. The straight line PQ is perpendicular to the straight line PS . Point T lies on the straight line PS .



Find

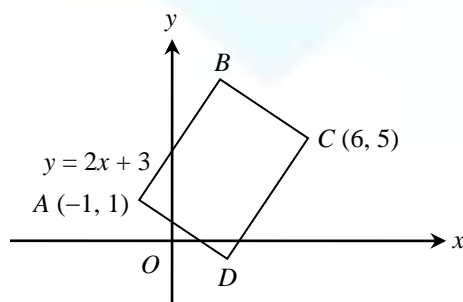
- (a) the equation of the straight line PS , (Ans : $y = -2x + 8$) [3 marks]
- (b) the coordinates of S , [Ans : (5, -2)] [2 marks]
- (c) the coordinates of T if $PT : TS = 1 : 3$, [Ans : (2, 4)] [2 marks]
- (d) the coordinate of R if the area of quadrilateral $PQRS$ is 30 unit².
[Ans : (7, 5)] [3 marks]

[2012, No.10]

Answer :

120 Solutions by scale drawing is not accepted.

The diagram shows a rectangle $ABCD$. The equation of the straight line AB is $y = 2x + 3$.

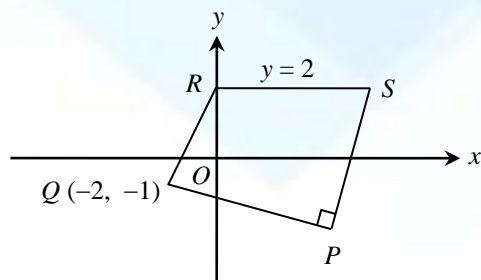


Find

- | | |
|--|---|
| (a) the equation of the straight line DC , | $(Ans : y = 2x - 7)$ [2 marks] |
| (b) the equation of the straight line AD , | $(Ans : y = -\frac{1}{2}x + \frac{1}{2})$ [3 marks] |
| (c) the coordinates of D , | $[Ans : (3, -1)]$ [2 marks] |
| (d) the area, in unit^2 , of rectangle $ABCD$. | $(Ans : 30)$ [3 marks]
[2013, No.9] |

Answer :

121 The diagram shows a quadrilateral $PQRS$. Point R lies on the y -axis



The equation of a straight line PS is $2y = 5x - 21$.

(a) Find

(i) the equation of straight line PQ ,

(Ans : $y = -\frac{2}{5}x - \frac{9}{5}$)

(ii) the coordinates of P .

[Ans : (3, -3)]

[6 marks]

(b) A point T moves such that its distance from point S is always 5 units. Find the equation of the locus of T .

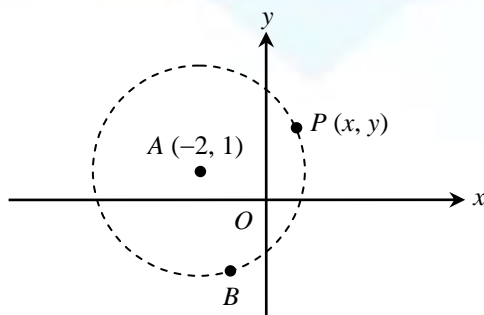
(Ans : $x^2 + y^2 - 10x - 4y + 4 = 0$) [4 marks]

[2014, No.10]

Answer :

122 *Solution by scale drawing is not accepted.*

The diagram shows the path of a moving point $P(x, y)$. P always moves at a constant distance from point A .



$B(-1, -2)$ and $R(-5, q)$ lie on path of point P . The straight line BC is a tangent to the path and intersects the x -axis at point C . Find

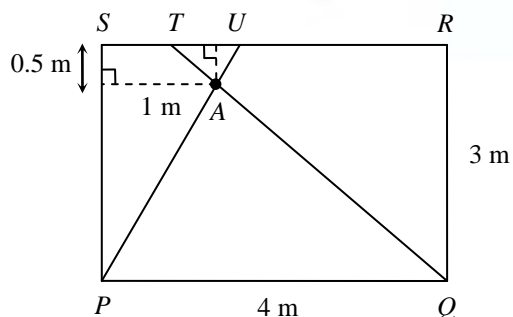
- (a) the equation of the path of point P , [3 marks] (Ans : $x^2 + y^2 + 4x - 2y - 5 = 0$)
- (b) the possible values of q , (Ans : 0, 2) [2 marks]
- (c) the area of $\triangle ABC$. (Ans : 10) [5 marks]
[2019, No.9]

Answer :

FORECAST

⇒ **Part A ~ 6 – 8 marks**

123 The diagram shows the surface of a rectangular wall, $PQRS$ which measures $4\text{ m} \times 3\text{ m}$.



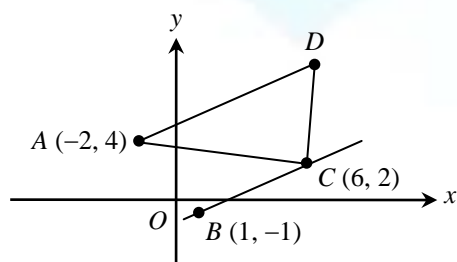
Syafiqah want to mark a point A so that the lamp can be installed at that position exactly. She has made the measurement using two ropes QT and PU which intersect at points A that required. Find

- the distance ST and TU so that the two ropes intersect at A ,
[4 marks] (Ans : $ST = \frac{2}{5}$, $TU = \frac{4}{5}$)
- the ratio of $TA : TQ$,
(Ans : $1 : 6$) [2 marks]
- the area of quadrilateral $AQRU$.
(Ans : 5.2) [2 marks]

Answer :

124 Solutions to this question by scale drawing will not be accepted.

The diagram shows the points $A(-2, 4)$, $B(1, -1)$, $C(6, 2)$ and D on a Cartesian plane.



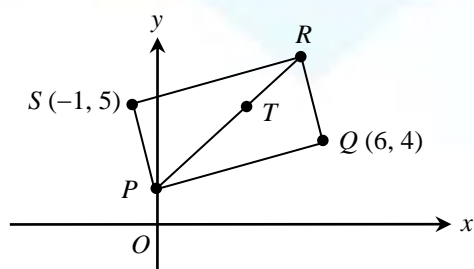
It is given that lines AD and BC are parallel and angle $ACD = 90^\circ$. Find

- (a) the equation of the line AD and CD , (Ans : $AD : y = \frac{3}{5}x + \frac{26}{5}$, $CD : y = 4x - 22$)
 [5 marks]
- (b) the coordinates of point D , [Ans : (8, 10)] [2 marks]

Answer :

125 Solutions to this question by scale drawing will not be accepted.

The diagram shows a rectangle $PQRS$.



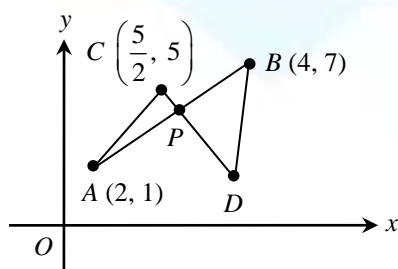
Given that the equation of the straight line PR is $y = 2 + x$. Point T lies on the straight line PR such that $PT : TR = 2 : 1$. Find

- (a) the equation of the straight line SR , (Ans : $y = \frac{1}{3}x + \frac{16}{3}$) [3 marks]
- (b) the coordinates of point T , [Ans : $(\frac{10}{3}, \frac{16}{3})$] [3 marks]
- (c) the area of triangle PST . (Ans : $\frac{20}{3}$) [2 marks]

Answer :

126 Solutions to this question by scale drawing will not be accepted.

In the diagram, APB and CPD are straight lines.



Given that P is the midpoint of AB , and $CD : PD = 4 : 3$. Find

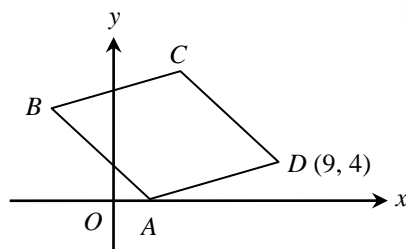
- the coordinates of point P , [Ans : (3, 4)] [1 mark]
- the coordinates of point D , [Ans : $(\frac{9}{2}, 1)$] [2 marks]
- the coordinates of the intersection point of the straight line AC and DB that are produced. [Ans : $(\frac{7}{2}, 13)$] [5 marks]

Answer :

⇒ **Part B ~ 10 marks**

127 Solutions to this question by scale drawing will not be accepted.

The diagram shows a rhombus on a Cartesian plane. Diagonal AC and BD intersect at point $(3, 6)$.



(a) Find

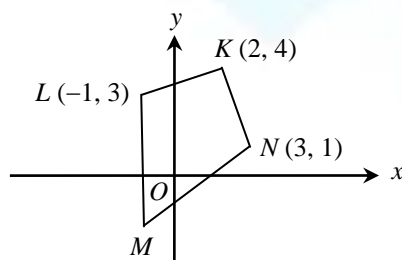
- (i) the coordinates of point B , [Ans : $(-3, 8)$] [1 mark]
- (ii) the equation of the line AC , (Ans : $y = 3x - 3$) [2 marks]
- (iii) the equation of the line CD . (Ans : $y = -2x + 22$) [3 marks]

(b) A point P moves such that its distance from point A is always 4 units. Find the equation of the locus of point P . (Ans : $x^2 + y^2 - 2x - 15 = 0$) [2 marks]

Answer :

128 Solutions to this question by scale drawing will not be accepted.

The diagram shows the vertices of a kite $KL MN$ on a Cartesian plane. LM is parallel to the y -axis.



Find

- (a) the coordinates of the point M ,
- (b) the area of the kite $KL MN$,
- (c) the equation of the locus of point P such that $KP = 2KL$.

[Ans : $(-1, -2)$] [4 marks]

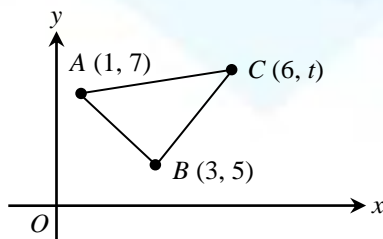
(Ans : 15) [3 marks]

(Ans : $x^2 + y^2 - 4x - 8y - 20 = 0$) [3 marks]

Answer :

129 Solutions to this question by scale drawing will not be accepted.

In the diagram, ABC is a triangle such that $\angle ABC = 90^\circ$.



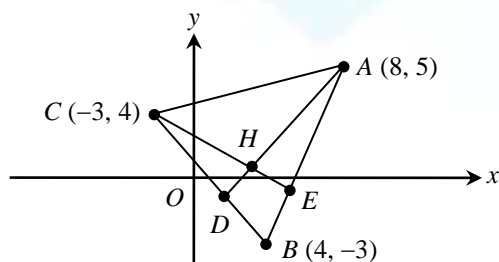
Find

- (a) the value of t , (Ans: 8) [2 marks]
- (b) the area of triangle ABC , (Ans : 6) [2 marks]
- (c) the perpendicular distance from B to AC , (Ans: 2.353) [3 marks]
- (d) the equation of the straight line that passes through point B and perpendicular to the straight line AC . (Ans: $y = -5x + 20$) [3 marks]

Answer :

130 Solutions to this question by scale drawing will not be accepted.

The diagram shows a triangle ABC that has the vertices $A(8, 5)$, $B(4, -3)$, and $C(-3, 4)$. AD is perpendicular to BC , and CE is perpendicular to AB . AD and CE intersect at point H .



Find

(a) the coordinates of point D ,

[Ans : $(2, -1)$] [3 marks]

(b) the coordinates of point H ,

[Ans : $(\frac{11}{3}, \frac{2}{3})$] [3 marks]

(c) the ratio $AD : HD$,

(Ans : $18 : 5$) [2 marks]

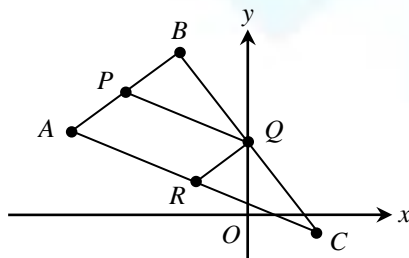
(d) the area of triangle AHC .

(Ans : $21\frac{2}{3}$) [2 marks]

Answer :

131 Solutions to this question by scale drawing will not be accepted.

In the diagram, $P(-4, 10)$, $Q(0, 6)$ and $R(-2, 2)$ are the midpoints of the straight lines AB , BC and AC respectively such that $ARQP$ forms a parallelogram.



- (a) Find
- the equation of the straight line AB , (Ans : $y = 2x + 18$) [2 marks]
 - the equation of the perpendicular bisector of the straight line AC .
(Ans : $y = x + 4$) [2 marks]
- (b) The straight line AB that is produced intersects the perpendicular bisector of the straight line AC at point S . Find the coordinates of point S .
[Ans : $(-14, -10)$] [3 marks]
- (c) Calculate the area of triangle PQR . (Ans : 12)
Hence, find the area of triangle ABC . (Ans : 48)
[3 marks]

Answer :

VECTORS

- ONE PAGE NOTE (OPN)

- WORKSHEET

Encik Hartono bin Jose

ONE PAGE NOTES “ VECTORS ”

VECTORS IN CARTESIAN PLANE

$$A(-3, 2)$$

$$\Rightarrow \vec{OA} = -3\mathbf{i} + 2\mathbf{j} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

$$\Rightarrow \vec{AO} = 3\mathbf{i} - 2\mathbf{j} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

TRIANGLE LAW



$$\vec{OA} + \vec{AB} = \vec{OB}$$

PARALLELOGRAM LAW



$$\vec{OA} + \vec{OB} = \vec{AB}$$

POLYGON LAW



$$\vec{OA} + \vec{AB} + \vec{BC} + \vec{CD} + \vec{DE} + \vec{EA} = \vec{0}$$

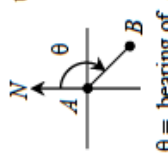
$$\vec{BF} + \vec{FD} = \vec{BD}$$

$$\vec{AB} + \vec{BC} = \vec{AC}$$

$$\vec{AB} = \vec{ED}$$

$$\vec{BC} = \vec{FE}$$

three-digit



θ = bearing of B from A

$$\vec{AB} : \vec{BC} = m : n$$

$$\vec{AB} = \left(\frac{m}{m+n} \right) \vec{AC}$$

$$\vec{CB} = \left(\frac{n}{m+n} \right) \vec{CA}$$

ADDITION AND SUBTRACTION OF VECTORS

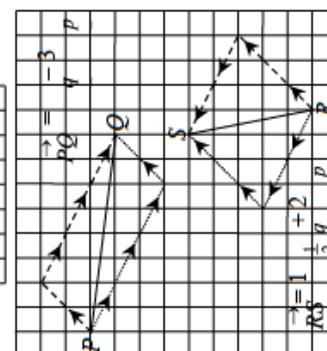
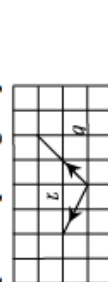
$$k \begin{pmatrix} a \\ b \\ h \end{pmatrix} = \begin{pmatrix} ka \\ kb \\ kh \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \\ h \end{pmatrix} + \begin{pmatrix} c \\ d \\ e \end{pmatrix} = \begin{pmatrix} a+c \\ b+d \\ h+e \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \\ h \end{pmatrix} - \begin{pmatrix} c \\ d \\ e \end{pmatrix} = \begin{pmatrix} a-c \\ b-d \\ h-e \end{pmatrix}$$

RESULTANT VECTORS

[on a square grid]



MAGNITUDE @ LENGTH OF VECTORS

$$\text{magnitude } r = \sqrt{x^2 + y^2}$$

UNIT VECTORS

$$\text{unit vector } \hat{r} = \frac{x\mathbf{i} + y\mathbf{j}}{\sqrt{x^2 + y^2}} = \frac{x}{r}\mathbf{i} + \frac{y}{r}\mathbf{j}$$

$$\text{Remark : } \left| \hat{r} \right| = 1 \rightarrow \text{magnitude of unit vector} = 1$$

NON- PARALLEL VECTORS

If \vec{a} and \vec{b} are non-parallel and non-zero, and $h\vec{a} = k\vec{b}$ where $h = k = 0$

PARALLEL VECTORS

If \vec{a} is parallel to \vec{b} where $\vec{a} = \lambda \vec{b}$ where λ is a constant

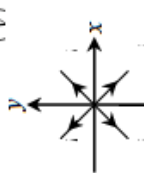
If $\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix}$ is parallel to x-axis $\Rightarrow y = 0$

If $\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix}$ is parallel to y-axis $\Rightarrow x = 0$

If A, B, C are collinear $\Rightarrow AB \parallel AC \parallel BC$

VECTORS - gradient

[translation $\begin{pmatrix} x \\ y \end{pmatrix}$]



$$\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$m_r = \frac{y}{x}$$

PERPENDICULAR VECTORS

If \vec{a} is perpendicular / orthogonal to \vec{b}

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = 0 \quad @ \quad a_1 b_2 - a_2 b_1 = 0$$

WORKSHEET
TOPIC 8 : VECTORS
[1 – 2 questions → 3 – 7 marks]

8.1 Vectors

8.1.1 Compare and contrast between vectors and scalars, and hence identify whether a quantity is a vector or a scalar by providing justifications.

1 Identify whether each of the following quantities is a scalar or a vector by marking (✓).

<i>Quantity</i>	<i>calar</i>	<i>ector</i>	<i>Quantity</i>	<i>calar</i>	<i>ector</i>
force			speed		
distance			resistance		
volume			displacement		
weight			mass		
work			area		
velocity			time		
length			momentum		
impulse			temperature		
power			energy		
pressure			acceleration		

2 Identify whether each of the following quantities is a scalar or a vector by marking (✓).

<i>Quantity</i>	<i>Scalar</i>	<i>Vector</i>
-----------------	---------------	---------------

A bus is moving at a speed of 80 kmh^{-1} due east.

Batrisyia walks 500 m from house to school.

A bag weight 250 N is moved as high as 2 m from the floor.

The body temperature of Maven is 36.5°C .

The density of a solid Y is 2.3 g cm^3 .

MIND think :

Match the following :

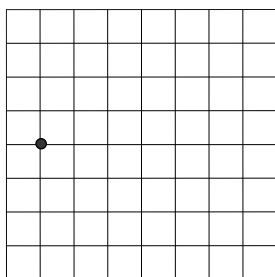
tensor at level zero	•	• <i>scalar quantity</i> •	• a quantity that has magnitude and direction
tensor at level one	•	• <i>vector quantity</i> •	• a quantity that has magnitude but no direction

8.1.2 Represent vectors by using directed line segments and vector notations, and hence determine the magnitude and direction of vectors.

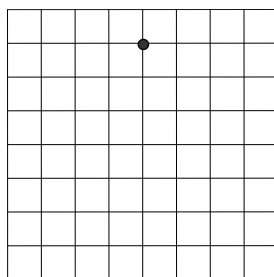
3 Beginning from the point provided, draw and label the following vectors :

- (a) \vec{AB} represents a displacement of 60 m to the right.
 (b) \vec{p} represents a velocity of 40 km/h due south.
 (c) \vec{u} represents an acceleration $5\sqrt{2} \text{ ms}^{-2}$ due northwest.

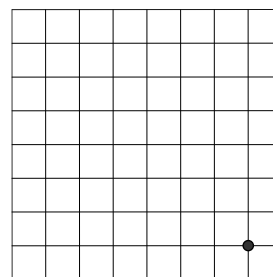
Answer :



1 unit represents 10 m



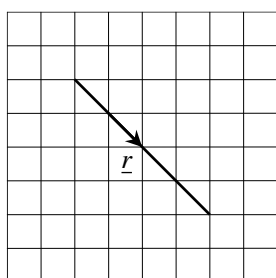
1 unit represents 8 km/h



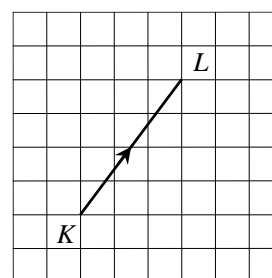
1 unit represents 1 ms^{-2}

4 Determine the magnitude and direction of each of the following vectors :

(a)



(b)



Answer :

(Ans : $N 36.87^\circ E @ 036.87^\circ$)

(a)

(b)

- 5 Two bus, P and Q are moving away from town O . Bus P moves due south while bus Q moves due west. Given that $|\vec{OP}| = 40$ km and $|\vec{OQ}| = 96$ km, after both buses travelled for one hours. Find the distance between the two buses. (Ans : 104)

[2 marks] [clon textbook form 4]

Answer :

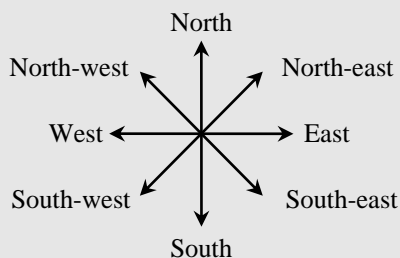
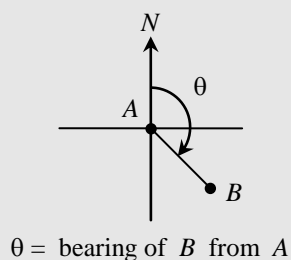
- 6 A car moves due north from A to B with a distance of 6 km, then moves due east from B to C with a distance of 8 km and finally turn back to A . Find the magnitude and displacement direction from C to A . (Ans : 10 km, S 53.13° W @ 233.13°)

[3 marks]

Answer :

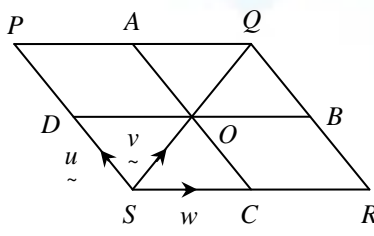
MIND think :

- the bearing of a point B from a point A is the angle measured in a **clockwise direction** from the north-line of A to the line joining A and B .
- written in a **three-digit form**, from 000° to 360°



\Rightarrow same vector / negative vector

- 7 The diagram shows a parallelogram, $PQRS$. The points A , B , C and D are the midpoints of PQ , QR , RS and ST respectively.



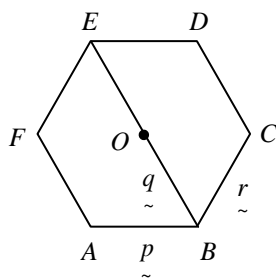
Given that $\vec{SD} = \vec{u}$, $\vec{SO} = \vec{v}$ and $\vec{SC} = \vec{w}$. State the vectors for the following in terms of \vec{u} , \vec{v} or \vec{w} .

- (a) \vec{RB} (b) \vec{QO} (c) \vec{OB} (d) \vec{AO}

[clon textbook form 4]

Answer :

- 8 The diagram shows a regular hexagon with centre O .



Given that $\vec{AB} = \vec{p}$, $\vec{OB} = \vec{q}$ and $\vec{BC} = \vec{r}$. State the vectors for the following in terms of \vec{p} , \vec{q} or \vec{r} .

- (a) \vec{ED} (b) \vec{CD} (c) \vec{EF} (d) \vec{OE}

Answer :

MIND think :

- A Zero vectors, , has magnitude, and its cannot be determine
- Two vector are the same if and only if both the vectors have the same and .
- A vector is negative if the vector consists of magnitude, but in the direction, that is $\vec{AB} = \text{}$.

- 9 Given that $(2x^2 - 7x + 3) \vec{b} = \vec{0}$, find the value of x .

(Ans : $\frac{1}{2}, 3$)

[2 marks] [Forecast]

Answer :

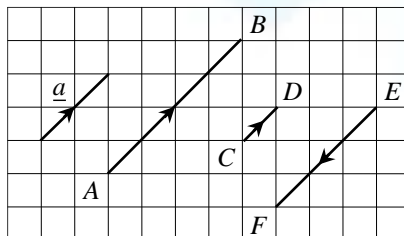
- 10 If \vec{a} and \vec{b} are non-parallel vectors and $(m + 3)\vec{a} + (2n - m + 1)\vec{b}$ is a zero vector, find the values of m and n .

(Ans : $m = -3, n = -2$)

Answer :

8.1.3 Make and verify conjectures about the properties of scalar multiplication on vectors.

- 11 The diagram shows four vectors.



State the following vectors in terms of \underline{a} .

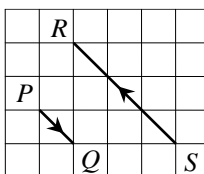
(a) \vec{AB}

(b) \vec{CD}

(c) \vec{EF}

Answer :

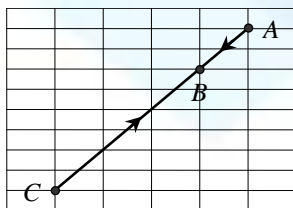
- 12 The diagram shows the vector $\vec{SR} = \underline{z}$.



State the vector \vec{QP} in term of \underline{z} .

Answer :

- 13 The diagram shows the vector $\vec{AB} = 2\vec{u}$.



State the vector \vec{CB} in term of \vec{u} .

Answer :

MIND think :

- Multiplication of scalar k with vector \vec{a} produces vector $k\vec{a}$, with the conditions :

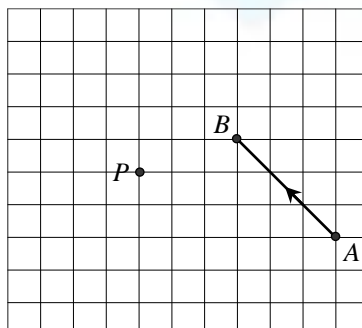
→ $|k\vec{a}| =$

→ if k 0, then $k\vec{a}$ is in the same direction with \vec{a} .

→ if k 0, then $k\vec{a}$ is in the opposite direction with \vec{a} .

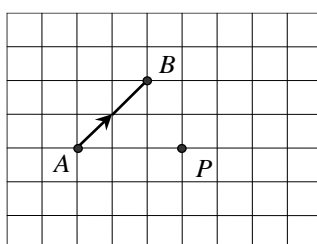
- 14 The diagram in the answer space shows the vector \vec{AB} .
On the diagram, construct the vector \vec{PQ} such that $\vec{PQ} = \frac{4}{3} \vec{BA}$.

Answer :



- 15 The diagram in the answer space shows the vector \vec{AB} .
On the diagram, construct the vector \vec{PQ} such that $\vec{PQ} = -\frac{3}{2} \vec{BA}$.

Answer :



8.1.4 Make and verify conjectures about parallel vectors.

\Rightarrow **parallel vectors**

- 16 Given $\vec{AB} = 2\vec{x}$, $\vec{CD} = 2\vec{y}$ and $\vec{EF} = 4\vec{x}$. Which pairs of vectors are parallel ?

Hence, determine the relation between the two parallel vectors.

[2 marks] [Forecast]

Answer :

MIND think :

- \vec{a} is parallel to $\vec{b} \Leftrightarrow \boxed{}$, where λ is a constant.
- A, B, C are collinear $\Leftrightarrow \boxed{}$
- \vec{a} and \vec{b} are not parallel and non-zero, and $h\vec{a} = k\vec{b} \Leftrightarrow h = k = \boxed{}$.

- 17 Given $\vec{PQ} = 3\vec{u}$, $\vec{RS} = -2\vec{u}$ and $\vec{TU} = 3\vec{v}$. Which pairs of vectors are parallel ?

Hence, determine the relation between the two parallel vectors.

[2 marks] [Forecast]

Answer :

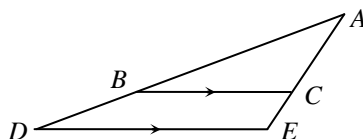
- 18 PQRS is a trapezium such that PQ parallel to SR, PQ = 4 cm, and SR = 10 cm. Express \vec{RS} in terms of \vec{PQ} .

(Ans : $\vec{RS} = -\frac{5}{2}\vec{PQ}$)

[2 marks] [Forecast]

Answer :

- 19 In the diagram, ABC is a triangle, \vec{BC} and \vec{DE} are two parallel vectors.



Given $|\vec{BC}| = 4 \text{ cm}$ and $|\vec{DE}| = 6 \text{ cm}$. Express

(b) \vec{AB} in terms of \vec{AD} ,

(Ans : $\vec{AB} = \frac{2}{3}\vec{AD}$)

(c) \vec{CE} in terms of \vec{AE} .

(Ans : $\vec{CE} = \frac{1}{3}\vec{AE}$)

)

[2 marks] [Forecast]

Answer :

(a)

(b)

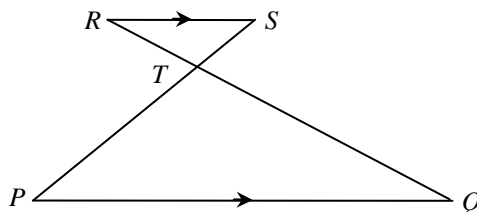
MIND think :

Diagram 1: $\frac{\text{area } \triangle ABC}{\text{area } \triangle ADE} = \left(\frac{BC}{DE}\right)^2 = \left(\frac{AB}{AD}\right)^2 = \left(\frac{AC}{AE}\right)^2$

Diagram 2: $\frac{\text{area } \triangle ABE}{\text{area } \triangle CDE} = \left(\frac{AB}{CD}\right)^2 = \left(\frac{AE}{ED}\right)^2 = \left(\frac{BE}{EC}\right)^2$

Diagram 3: $\frac{\text{area } \triangle ABC}{\text{area } \triangle ACD} = \frac{BC}{CD} / \frac{\text{luas } \triangle ABC}{\text{luas } \triangle ABD} = \frac{BC}{CD}$

- 20** In the diagram, PQ and RS are parallel.



Given that $RS : PQ = 1 : 3$, $|\vec{PQ}| = 12 \text{ cm}$, $\vec{PS} = 8\underline{p}$ and $\vec{QT} = 9\underline{q}$. Find

- (a) $|\vec{SR}|$, (Ans : 4)
- (b) \vec{PT} in terms of \underline{p} , (Ans : $6\underline{p}$)
- (c) \vec{RT} in terms of \underline{q} , (Ans : $-3\underline{q}$)

[3 marks] [clon textbook form 4]

Answer :

\Rightarrow **collinear**

- 21** Given $\vec{ST} = 12\underline{a}$ and $\vec{TU} = 8\underline{a}$, show that S , T and U are collinear.

[2 marks] [Forecast]

Answer :

- 22 Given points K , M and N are collinear with $\vec{MN} = \frac{3}{5}\vec{KN}$. Express \vec{KM} in terms of \vec{MN} .
(Ans : $\vec{KM} = \frac{2}{3}\vec{MN}$)

[2 marks] [Forecast]

Answer :

- 23 Given points A , B and C are collinear with $\vec{AB} = h\vec{a}$ and $\vec{BC} = (3k-2)\vec{a}$, where k is a constant. If $5\vec{AB} = 2\vec{AC}$, express k in terms of h .
(Ans : $k = \frac{3h+4}{6}$)

[2 marks] [Forecast]

Answer :

 \Rightarrow not parallel and non-zero

- 24 The vector \underline{a} and \underline{b} are non-zero and non-parallel. It is given that $(h+3)\underline{a} = (k-5)\underline{b}$, where h and k are constants. Find the value of

(a) h ,

(Ans : -3)

(b) k .

(Ans : 5)

[2 marks]

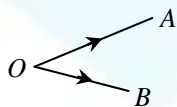
[2008, No.15]

Answer :

(a)

(b)

- 25 The diagram shows two vectors, $\vec{OA} = x$ and $\vec{OB} = y$.



Find the value of h and k such that $(h - 2)x = (3h + k)y$.

(Ans : $h = 2, k = -6$)

[3 marks]

Answer :

- 26 If \vec{a} and \vec{b} are non-zero and non-parallel vectors such that $(2m + n - 1)\vec{a} - (m - n + 7)\vec{b} = 0$ find the values of m and n .

(Ans : $m = -2, n = 5$)

[3 marks] [Forecast]

Answer :

8.2 Addition and subtraction of vectors

8.2.1 Perform addition and subtraction involving two or more vectors to obtain a resultant vector.

\Rightarrow addition and subtraction of parallel vector

- 27 Solve each of the following :

$$2\vec{u} + \frac{1}{3}\vec{u} + \frac{2}{3}\vec{u} =$$

$$8\vec{c} - 5\vec{c} - \vec{c} =$$

\Rightarrow **addition and subtraction of non-parallel vector**

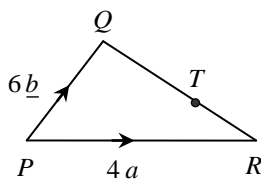
28 Solve each of the following :

$$4\vec{x} + 2\frac{1}{2}\vec{y} + \frac{1}{2}\vec{x} + \vec{y} =$$

$$6\vec{c} - 5\vec{d} - 4\vec{c} + 3\vec{d} =$$

\Rightarrow **addition and subtraction of non-parallel vector ~ triangle law**

29 The diagram shows a triangle PQR .



The point T lies on QR such that $QT : TR = 3 : 1$. Express in terms of \underline{a} and \underline{b} :

(a) \vec{QR} ,

(b) \vec{PT} .

(Ans : $3\underline{a} + \frac{3}{2}\underline{b}$)

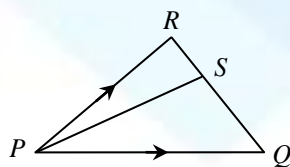
[3 marks] [2008, No.16]

Answer :

(a)

(b)

- 30 The diagram shows a triangle PQR .



Given that $\vec{PQ} = 3\underline{a}$, $\vec{PR} = 6\underline{b}$ and point S lies on QR such that $QS : SR = 2 : 1$, express in terms of \underline{a} and \underline{b} :

(a) \vec{QR} ,

(b) \vec{SP} .

(Ans : $-\underline{a} - 4\underline{b}$)

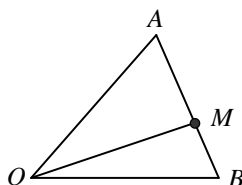
[4 marks] [2009, No.14]

Answer :

(a)

(b)

- 31 The diagram shows a triangle PQR and M is a point on AB .



Given that $\vec{OA} = 5\underline{a}$, $\vec{OB} = 4\underline{b}$ and $2AM = 3MB$, find

(a) \vec{AB} ,

(b) \vec{OM} .

(Ans : $2\underline{a} + \frac{12}{5}\underline{b}$)

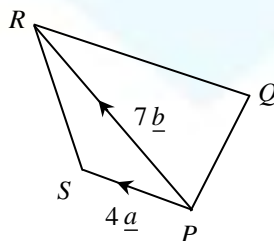
[4 marks] [2010, No.16]

Answer :

(a)

(b)

- 32 The diagram shows a trapezium $PQRS$ with $QR = 2PS$.



Express in terms of \underline{a} and / or \underline{b}

(a) $\vec{SR} - \vec{PR}$,

(b) \vec{QP} .

(Ans : $8\underline{a} - 7\underline{b}$)

[3 marks] [2014, No.15]

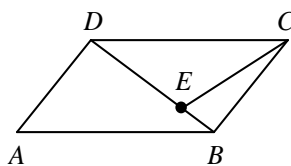
Answer :

(a)

(b)

\Rightarrow **addition and subtraction of non-parallel vector ~ triangle law @ parallelogram law 1**

- 33 The diagram shows a parallelogram $ABCD$ with BED as a straight line.



Given that $\vec{AB} = 6p$, $\vec{AD} = 4q$, and $DE = 2EB$, express, in terms of p and q :

(a) \vec{BD} .

(Ans : $-6p + 4q$)

(b) \vec{EC} .

(Ans : $2p + \frac{8}{3}q$)

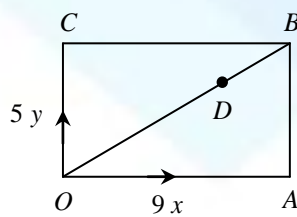
[4 marks] [2003, No.14]

Answer :

(a)

(b)

- 34 The diagram shows a rectangle $OACB$ and the point D lies on the straight line OB .



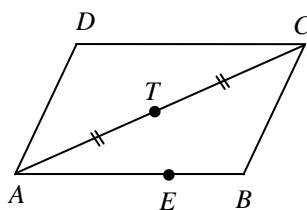
It is given that $OD = 3DB$. Express \vec{OD} in terms of \underline{x} and \underline{y} .

[Ans : $\frac{3}{4}(5\underline{y} + 9\underline{x})$]

[3 marks] [2007, No.15]

Answer :

- 35 The diagram shows a parallelogram $ABCD$.



Point E lies on AB such that $AE : EB = 2 : 1$. It is given that $\vec{EB} = 4\underline{u}$ and $\vec{AD} = 3\underline{v}$. Express in terms of \underline{u} and \underline{v}

(a) \vec{AE} ,

(b) \vec{ET} .

(Ans : $\frac{3}{2}\underline{v} - 2\underline{u}$)

[3 marks] [2019, No.15]

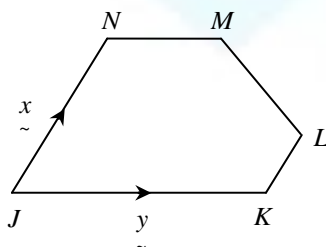
Answer :

(a)

(b)

\Rightarrow addition and subtraction of non-parallel vector ~ polygon law

- 36 The diagram shows a pentagon $JKLMN$.



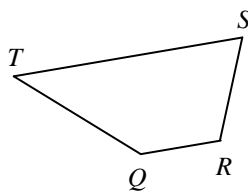
Given that $\vec{KL} = \frac{1}{3}\vec{JN}$, $2\vec{NM} = \vec{JK}$, $\vec{JN} = \underline{\underline{x}}$ and $\vec{JK} = \underline{\underline{y}}$, express \vec{LM} in terms of $\underline{\underline{x}}$ and $\underline{\underline{y}}$.

(Ans : $\frac{2}{3}\underline{\underline{x}} - \frac{1}{2}\underline{\underline{y}}$)

[2 marks] [clon textbook form 4]

Answer :

- 37 The diagram shows a quadrilateral such that $\vec{PS} = (m-1)\underline{\underline{x}}$, $\vec{SR} = n\underline{\underline{y}}$ and $\vec{QR} = n\underline{\underline{x}}$. m and n are constants.



If $\vec{PQ} = 3\underline{\underline{x}} + \left(\frac{m+1}{5}\right)\underline{\underline{y}}$, find the values of m and n .

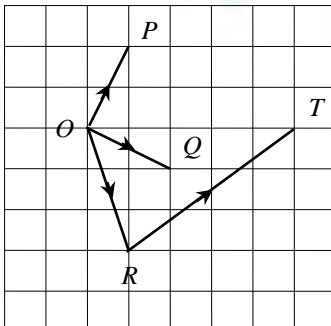
(Ans : $m = \frac{21}{4}$, $n = \frac{5}{4}$)

[3 marks] [Forecast]

Answer :

\Rightarrow addition and subtraction of non-parallel vector ~ triangle law @ parallelogram law 2(a)

38 The diagram shows two vectors, $\vec{OP} = \vec{x}$ and $\vec{OQ} = \vec{y}$.



Express in terms of \vec{x} and \vec{y} :

(a) \vec{OR} ,

(b) \vec{RT} .

(Ans : $\vec{y} + 2\vec{x}$)

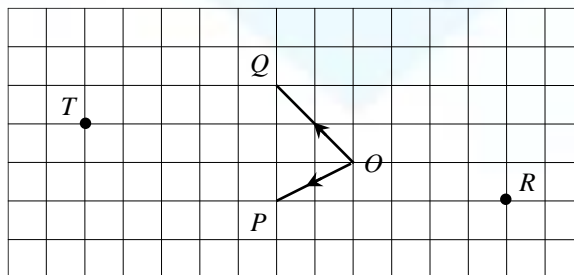
[2 marks] [Forecast]

Answer :

(a)

(b)

- 39 The diagram shows two vectors, $\vec{OP} = \underline{x}$ and $\vec{OQ} = \underline{y}$.



Express in terms of \underline{x} and \underline{y} :

(a) \vec{OR} ,

(b) \vec{PT} .

(Ans : $\underline{x} + \frac{3}{2}\underline{y}$)

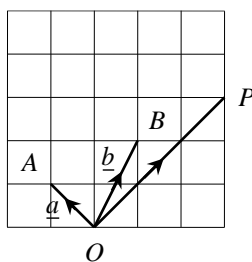
[2 marks] [Forecast]

Answer :

(a)

(b)

- 40 The diagram shows the vectors \vec{OA} , \vec{OB} and \vec{OP} drawn on a grid of equal squares with sides of 1 unit.



Determine

(a) $|\vec{OP}|$,

(Ans : $3\sqrt{2}$)

(b) \vec{OP} in terms of \underline{a} and \underline{b} .

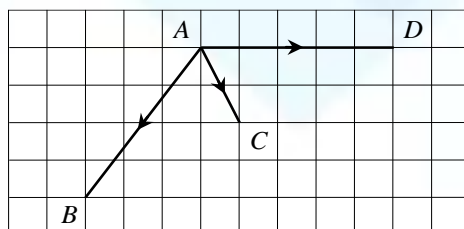
[2 marks] [2012, No.15]

Answer :

(a)

(b)

- 41 The diagram shows vectors \vec{AB} , \vec{AC} and \vec{AD} drawn on a square grid with sides of 1 unit.



(a) Find $\left| -\vec{BA} \right|$.

- (b) Given $\vec{AB} = \underline{b}$ and $\vec{AC} = \underline{c}$, express in terms of \underline{b} and \underline{c} ,

(i) \vec{BC} ,

(ii) \vec{AD} .

(Ans : $2\underline{c} - \underline{b}$)

[3 marks] [2017, No.3]

Answer :

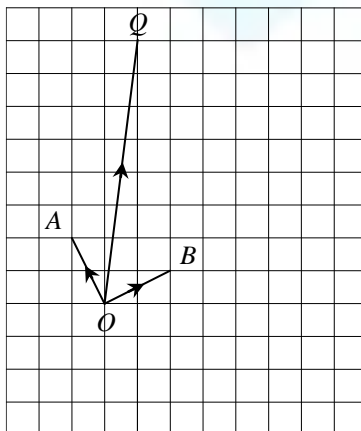
(a)

(b) (i)

(ii)

\Rightarrow addition and subtraction of non-parallel vector ~ triangle law @ parallelogram law 2(b)

- 42 The diagram shows two vectors, $\vec{OA} = \underline{a}$ and $\vec{OB} = \underline{b}$.



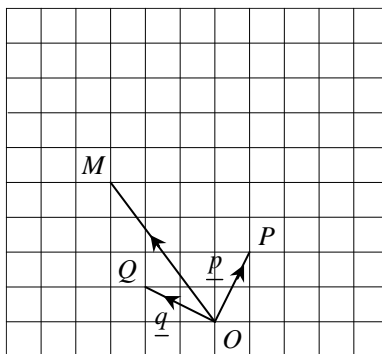
- (a) State vector \vec{OQ} in term of \underline{a} and \underline{b} .
- (b) Given that $\vec{OP} = -2\underline{a} + \underline{b}$, mark and label the point P on the above diagram.

[2 marks] [Forecast]

Answer :

- (a) (b) refer the diagram

- 43 The diagram shows the vectors \vec{OP} , \vec{OQ} dan \vec{OM} drawn on a square grid.



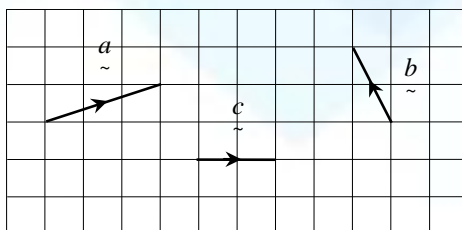
- (a) Express \vec{OM} in the form $h\underline{p} + k\underline{q}$, where h and k are constants. (Ans : $\underline{p} + 2\underline{q}$)
- (b) On the diagram, mark and label the point N such that $\vec{MN} + \vec{OQ} = 2\vec{OP}$.

[3 markah] [2018, No.8]

Answer :

- (a) (b)

- 44 The diagram shows three vectors drawn on square grids.

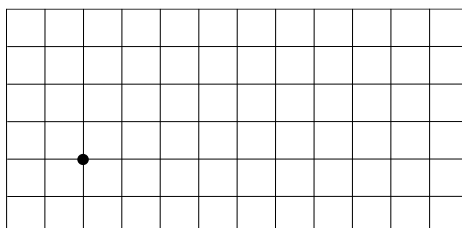


- (a) Draw the vector $2\vec{a} - \vec{b} + \frac{1}{2}\vec{c}$ on the square grids in the answer space, start from the given point.
- (b) If $|\vec{c}| = 4$, find $|2\vec{a} - \vec{b} + \frac{1}{2}\vec{c}|$.

[2 marks] [Forecast]

Answer :

(a)



(b)

8.2.2 Solve problems involving vectors.

⇒ *solve problems 1*

- 45** Janelle rows her boat from point K across the river due north with a velocity of 7 kmh^{-1} . The river stream flows due west with a with velocity of 12 kmh^{-1} .

(a) Sketch the digram which shows the movement of the boat and the river stream.

(b) After affected by the river stream, calculate for the boat :

(i) new velocity,

(Ans : 13.89)

(ii) new direction.

(Ans : 300.26°)

[3 marks] [*clon textbook form 4*]

Answer :

(a)

(b) (i)

(ii)

- 46** An aeroplane is flying to the south from airpoint A to airport B for 1000 km in 2 hours. The wind blows from east with a velocity of 250 kmh^{-1} .

(a) Sketch the digram which shows the movement of the aeroplane and the wind blow.

(b) Hence, find

(i) the velocity of the plane without the influence of the wind,

(Ans : 433.01)

(ii) the original direction of the aeroplane.

(Ans : 150°)

[3 marks] [*clon textbook form 4*]

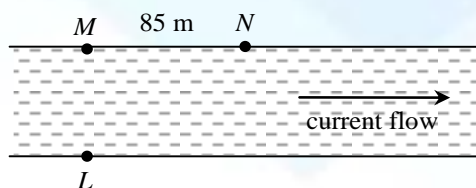
Answer :

(a)

(b) (i)

(ii)

- 47 The diagram shows the positions of L , M and N along a river.



The width of the river is 50 m, M is due north of L and the velocity of the downstream river flow is 2.5 m/s. Nathaniel wanted to row his boat from L across the river to M , but the boat was swept by the current flow and stopped at N in 15 seconds. Calculate the speed, in m/s, of Nathaniel's boat. (Ans : 6.08)

[3 marks] [clon textbook form 4]

Answer :

\Rightarrow solve problems 2

- 48 Given the position vector for three toy cars are $\vec{OP} = x + 3y$, $\vec{OQ} = 2x + 5y$ and $\vec{OR} = kx + 4y$, where k is a constant. These toy cars are placed in a straight line, find the value of k . (Ans : $\frac{3}{2}$)

[3 marks] [clon textbook form 4]

Answer :

- 49 Given that O , P , Q , and R are four points such that $\vec{OP} = \underline{p}$, $\vec{OQ} = \underline{q}$ and $\vec{OR} = 4\underline{p}$. M is the midpoint of PQ , and the line OM is extended to a point S such that $\vec{OS} = \frac{8}{5}\vec{OM}$.

(a) Express in terms of \underline{p} and \underline{q} :

(i) \vec{OS} .

[Ans : $\frac{4}{5}(\underline{p} + \underline{q})$]

(ii) \vec{QR} .

(Ans : $4\underline{p} - \underline{q}$)

[4 marks]

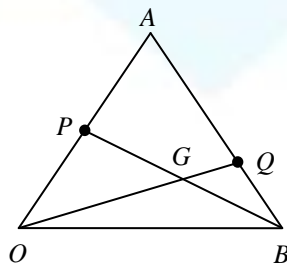
(b) Hence, show that point S lies on QR and state the ratio of $QS : SR$.

(Ans : 1 : 4) [3 marks] [Forecast]

Answer :

⇒ solve problems 3

- 50 In the diagram, P is the midpoint of OA , and Q is a point on AB such that $\vec{AQ} = 3\vec{QB}$.



- (a) Given that $\vec{OA} = 5\vec{a}$ and $\vec{OB} = 10\vec{b}$. Express in terms of \vec{a} and \vec{b} :

(i) \vec{BP} . (Ans : $\frac{5}{2}\vec{a} - 10\vec{b}$)

(ii) \vec{OQ} , (Ans : $\frac{5}{4}\vec{a} + \frac{15}{2}\vec{b}$)

[4 marks]

- (b) Given that $\vec{BG} = \lambda \vec{BP}$ and $\vec{OG} = \mu \vec{OQ}$, find

(i) the values of λ and μ , (Ans : $\lambda = \frac{2}{5}$, $\mu = \frac{4}{5}$)

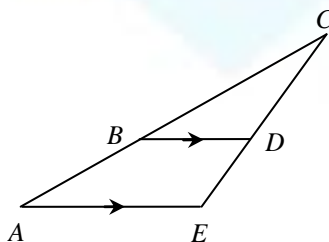
(ii) the ratio of area of triangle OGP : area of triangle QGB .
(Ans : 6 : 1)

[6 marks]

[Forecast]

Answer :

- 51 The diagram shows a triangle, ACE . It is given that $\vec{AE} = k\vec{x}$, $\vec{BD} = 3\vec{x}$ and $\vec{ED} = (h-1)\vec{y}$, where k and h are constants.



If $\vec{AB} = 2\vec{x} + 6\vec{y}$, find

- the values of h and k , (Ans : $h = 7$, $k = 5$) [4 marks]
- the area of triangle BCE , if the area of triangle ABE is 18 unit^2 .
(Ans : 27) [2 marks]
- the area of triangle BCD , if the area of triangle ABE is 15 unit^2 .
(Ans : 13.5) [2 marks]
- the area of triangle BCD , if the area of triangle ACE is 30 unit^2 .
(Ans : 10.8) [2 marks]

[Forecast]

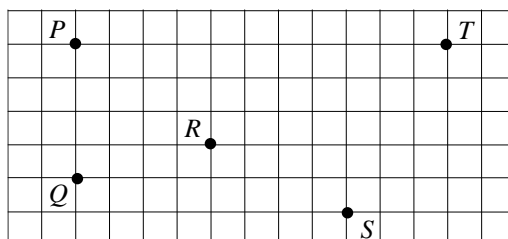
Answer :

8.3 Vectors in a cartesian plane

8.3.1 Represent vectors and determine the magnitude of the vectors in the Cartesian plane.

⇒ *vectors in the Cartesian plane ~ 1*

52 The diagram shows five points, P , Q , R , S and T on a grid.



Express

(a) \vec{PR} , \vec{QR} , \vec{TR} , \vec{SR} , \vec{PQ} and \vec{PT} in the form $\begin{pmatrix} x \\ y \end{pmatrix}$,

(b) \vec{RP} , \vec{RQ} , \vec{RT} , \vec{RS} , \vec{QP} and \vec{TP} in the form $x\mathbf{i} + y\mathbf{j}$.

Answer :

(a) $\vec{PR} =$ $\vec{QR} =$ $\vec{TR} =$

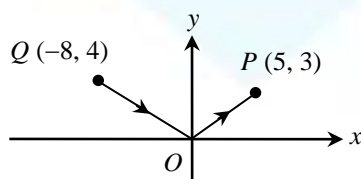
$\vec{SR} =$ $\vec{PQ} =$ $\vec{PT} =$

(b) $\vec{RP} =$ $\vec{RQ} =$ $\vec{RT} =$

$\vec{RS} =$ $\vec{QP} =$ $\vec{TP} =$

\Rightarrow **vectors in the Cartesian plane ~ 2**

- 53 The diagram shows two vectors, \vec{OP} and \vec{OQ} .



Express

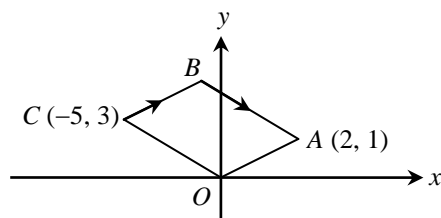
- (a) \vec{OP} in the form $\begin{pmatrix} x \\ y \end{pmatrix}$,
 (b) \vec{OQ} in the form $x\mathbf{i} + y\mathbf{j}$.

[2 marks] [2003, No.12]

Answer :

- (a) (b)

- 54 The diagram shows a parallelogram $OABC$, drawn on a Cartesian plane.



Express

- (a) \vec{CB} in the form $\begin{pmatrix} x \\ y \end{pmatrix}$,
 (b) \vec{BA} in the form $x\mathbf{i} + y\mathbf{j}$.

[2 marks]
 [Forecast]

Answer :

- (a) (b)

MIND think :

$A(-3, 2)$ — — —

in the form of component \vec{i} and \vec{j} $\vec{OA} = \boxed{} \Rightarrow \vec{AO} = \boxed{}$
 in the form column vector $\vec{OA} = \begin{pmatrix} \\ \end{pmatrix} \Rightarrow \vec{AO} = \begin{pmatrix} \\ \end{pmatrix}$

\Rightarrow **magnitude of a vector**

- 55 Given that OAB is a right-angled triangle with $\angle AOB = 90^\circ$, $\vec{OA} = 4\vec{i} + 2\vec{j}$ and $\vec{OB} = -3\vec{i} + 6\vec{j}$. Find the area of triangle AOB . (Ans : 15)

[2 marks] [Forecast]

Answer :

MIND think :

$\vec{r} = x\vec{i} + y\vec{j} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \text{magnitude } \vec{r}, \left| \vec{r} \right| = \boxed{}$

- 56 Given that $a = -2\vec{i} + h\vec{j}$. Find the values of h such that $|a| = \sqrt{20}$.

(Ans : ± 4)

[2 marks] [Forecast]

Answer :

- 57 Given that $\vec{OP} = 3\vec{i} + k\vec{j}$ and $\vec{OQ} = 4\vec{j}$. If OP and OQ are the two sides of a rhombus, find the value of k . (Ans : $\pm\sqrt{7}$)

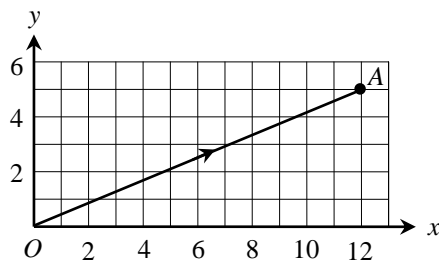
[2 marks] [Forecast]

Answer :

8.3.2 Describe and determine the unit vector in the direction of a vector.

\Rightarrow **unit vector in the direction of a vector**

- 58 The diagram shows vector \vec{OA} drawn on a Cartesian plane.



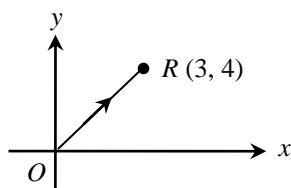
- (a) Express \vec{OA} in the form $\begin{pmatrix} x \\ y \end{pmatrix}$.
- (b) Find the unit vector in the direction of \vec{OA} .

[2 marks]
[2005, No.15]

Answer :

- (a) (b)

- 59 The diagram shows the vector \vec{OR} .



Express in the form $x\mathbf{i} + y\mathbf{j}$:

- (a) \vec{OR} ,
- (b) the unit vector in the direction of \vec{OR} .

[3 marks] [2010, No.15]

Answer :

- (a) (b)

- 60 The length of vector \underline{u} is 13 units and the direction is opposite with vector $-3\mathbf{i} + 2\mathbf{j}$.

Find the vector of \underline{u} .

[Ans : $3\sqrt{13}\mathbf{i} - 2\sqrt{13}\mathbf{j}$]

[2 marks] [clon textbook form 4]

Answer :

- 61 Given that $\underline{p} = 3\underline{i} + k\underline{j}$ and $\hat{\underline{p}} = \frac{1}{5}(3\underline{i} + k\underline{j})$, find the possible values of k . (Ans : ± 4)

[2 marks] [forecast]

Answer :

MIND think :

- $\underline{r} = x\underline{i} + y\underline{j} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow$ unit vector in the direction \underline{r} , $\hat{\underline{r}} =$
- if \underline{r} is a unit vector $\Rightarrow |\underline{r}| =$

Note : $\hat{\underline{r}} = \frac{\underline{r}}{|\underline{r}|} = \frac{\underline{r}}{1} = \underline{r}$. . .

- 62 Given that $\frac{5\underline{i} + k\underline{j}}{\sqrt{74}}$ is a unit vector, find the possible values of k . (Ans : ± 7)

[2 marks] [clon textbook form 4]

Answer :

- 63 Given that $\hat{\underline{v}} = (1-h)\underline{i} - \frac{3}{5}\underline{j}$, find the possible values of h . (Ans : $\frac{1}{5}, \frac{9}{5}$)

[2 marks] [clon textbook form 4]

Answer :

- 64 Given that $\underline{x} = \begin{pmatrix} h \\ k \end{pmatrix}$ and $\underline{y} = \begin{pmatrix} -k \\ h \end{pmatrix}$. If the unit vector in the direction of $2\underline{x}$ is $\frac{2}{3} \begin{pmatrix} h \\ k \end{pmatrix}$. Find the value of $|3\underline{y}|$.
(Ans : $\frac{9}{2}$)

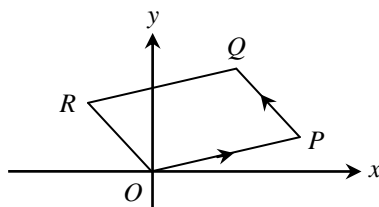
[2 marks] [Forecast]

Answer :

8.3.3 Perform arithmetic operations onto two or more vectors.

\Rightarrow *arithmetic operations / vectors in the Cartesian plane 1*

- 65 The diagram shows a parallelogram, $OPQR$, drawn on a Cartesian plane.



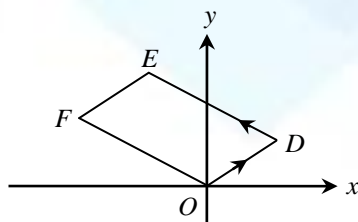
It is given that $\vec{OP} = 6\mathbf{i} + 4\mathbf{j}$ and $\vec{PQ} = -4\mathbf{i} + 5\mathbf{j}$. Find \vec{PR} .
(Ans : $-10\mathbf{i} + \mathbf{j}$)

[3 marks]

[2005, No.16]

Answer :

- 66 The diagram shows a parallelogram $ODEF$ drawn on a Cartesian plane.



It is given that $\vec{OD} = 3\vec{i} + 2\vec{j}$ and $\vec{DE} = -5\vec{i} + 3\vec{j}$. Find \vec{DF} .

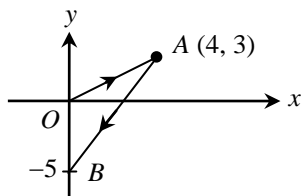
(Ans : $-8\vec{i} + \vec{j}$)

[3 marks] [2011, No.16]

Answer :

\Rightarrow arithmetic operations / vectors in the Cartesian plane 2

- 67 The diagram shows two vectors, \vec{OA} and \vec{AB} .



Express

(a) \vec{OA} in the form $\begin{pmatrix} x \\ y \end{pmatrix}$,

(b) \vec{AB} in the form $x\vec{i} + y\vec{j}$.

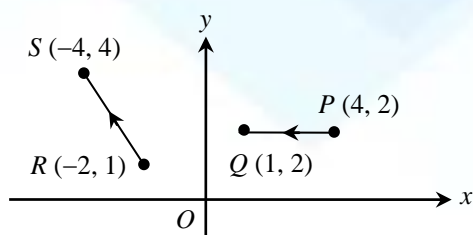
[2 marks] [2006, No.13]

Answer :

(a)

(b)

- 68 The diagram shows two vectors, \vec{PQ} and \vec{RS} .



Express

(a) \vec{PQ} in the form $\begin{pmatrix} x \\ y \end{pmatrix}$,

(b) \vec{SR} in the form $x\mathbf{i} + y\mathbf{j}$.

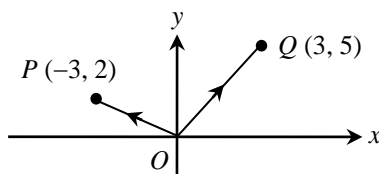
[2 marks] [Forecast]

Answer :

(a)

(b)

- 69 The diagram shows two vectors, \vec{OP} and \vec{OQ} .



If point N lies on PQ such that $\vec{PN} = \frac{1}{2}\vec{NQ}$. Find the \vec{ON} in the form $x\mathbf{i} + y\mathbf{j}$.

[Ans : : $-\mathbf{i} + 3\mathbf{j}$]

[2 marks] [Forecast]

Answer :

\Rightarrow **arithmetic operations / magnitude of a vector**

- 70** Given that $\vec{AB} = -2\hat{i} + 3\hat{j}$, $\vec{BC} = 3\hat{i} - 4\hat{j}$, and $B(-1, 5)$. Find

(a) the coordinates of point A,

[Ans : (1, 2)]

(b) the length of AC.

(Ans : $\sqrt{2}$)

[4 marks] [Forecast]

Answer :

(a)

(b)

- 71** Given that $\underline{a} = 13\hat{i} + \hat{j}$ and $\underline{b} = 7\hat{i} - k\hat{j}$, find

(a) $\underline{a} - \underline{b}$, in the form $x\hat{i} + y\hat{j}$,

[Ans : $6\hat{i} + (1+k)\hat{j}$]

(b) the values of k if $|\underline{a} - \underline{b}| = 10$.

(Ans : -9, 7)

[4 marks] [2009, No.13]

Answer :

(a)

(b)

- 72** It is given that vector $\underline{r} = \begin{pmatrix} 8 \\ -2 \end{pmatrix}$ and vector $\underline{s} = \begin{pmatrix} h \\ 7 \end{pmatrix}$, where h is a constant.

(a) Express the vector $\underline{r} + \underline{s}$, in terms of h .

(b) Given that $|\underline{r} + \underline{s}| = 13$ units, find the positive value of h .

(Ans : 4)

[4 marks] [2011, No.17]

Answer :

(a)

(b)

\Rightarrow arithmetic operations / unit vector in the direction of a vector 1

- 73** Given that $O(0, 0)$, $A(-3, 4)$, and $B(2, 16)$, find in terms of the unit vectors, i and j ,

(a) \vec{AB} ,

(b) the unit vector in the direction of \vec{AB} .
]

[Ans : $\frac{1}{13}(5i + 12j)$]

[4 marks] [2004, No.16]

Answer :

(a)

(b)

- 74** The following information refers to the vectors \underline{a} and \underline{b} .

$$\underline{a} = \begin{pmatrix} 2 \\ 8 \end{pmatrix}, \quad \underline{b} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$$

Find

(a) the vector $2\underline{a} - \underline{b}$,

[Ans : $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$]

(b) the unit vector in the direction of $2\underline{a} - \underline{b}$.

[Ans : $\frac{1}{13} \begin{pmatrix} 5 \\ 12 \end{pmatrix}$]

[4 marks] [2007, No.16]

Answer :

(a)

(b)

- 75** Given $STUV$ is a parallelogram, $\vec{TV} = 2\vec{i} + 3\vec{j}$ and $\vec{UV} = -2\vec{i} - 2\vec{j}$. Find the unit vector in the direction of \vec{TU} in terms of \vec{i} and \vec{j} .

[Ans : $\frac{1}{\sqrt{41}}(4i + 5j)$]

[3 marks] [**Forecast**]

Answer :

- 76** Given that $a = 2i - j$ and $b = 3i + j$. Find the vector in the same direction and parallel $2b - 4a$ and has a magnitude of $5\sqrt{10}$.

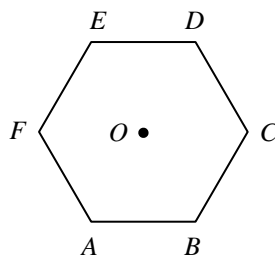
[Ans : $5(-i + 3j)$]

[3 marks] [Forecast]

Answer :

\Rightarrow **arithmetic operations / unit vector in the direction of a vector 2**

- 77** The diagram shows a regular hexagon with centre O .



- (a) Express $\vec{AC} + \vec{CE} + \vec{CB}$ as a single vector. (Ans : \vec{AF})
- (b) Given $\vec{OA} = \underline{a}$, $\vec{OB} = \underline{b}$, and the length of each side of the hexagon is 3 units, find the unit vector in the direction of \vec{AB} in terms of \underline{a} and \underline{b} . (Ans : $\frac{-a+b}{3}$)

[3 marks] [2016, No.10]

Answer :

(a)

(b)

\Rightarrow **arithmetic operations / unit vector in the direction of a vector 3**

- 78 Given that $\vec{AB} = \begin{pmatrix} q \\ q-1 \end{pmatrix}$ and $\vec{OA} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$. If \vec{OB} is a unit vector, find the possible values of q .

(Ans : -2, -1)

[4 marks] [**Forecast**]

Answer :

\Rightarrow **arithmetic operations**

- 79 $\left| \begin{array}{l} \mathbf{r} = 3\mathbf{a} + 4\mathbf{b}, \\ \mathbf{s} = 4\mathbf{a} - 2\mathbf{b}, \\ \mathbf{t} = p\mathbf{a} + (p+q)\mathbf{b}, \text{ where } p \text{ and } q \text{ are constants.} \end{array} \right.$

Use the above information to find the values of p and q when $\mathbf{t} = 2\mathbf{r} - 3\mathbf{s}$.

(Ans : $p = -6$, $q = 20$)

[3 marks] [2003, No.13]

Answer :

- 80 Given that $A(-2, 6)$, $B(4, 2)$, and $C(m, p)$, find the value of m and of p such that $\vec{AB} + 2\vec{BC} = 10\vec{i} - 12\vec{j}$.
(Ans : $m = 6$, $p = -2$)

[4 marks] [2004, No.17]

Answer :

- 81** $A(2, 3)$ and $B(-2, 5)$ lie on a Cartesian plane. It is given that $3\vec{OA} = 2\vec{OB} + \vec{OC}$. Find

(a) the coordinates of C ,

[Ans : (10, -1)]

(b) $|\vec{AC}|$.

(Ans : $4\sqrt{5}$)

[4 marks] [2018, No.9]

Answer :

(a)

(b)

- 82** It is given that $P(2, m)$, $Q(h, 6)$, $\underline{v} = 2\underline{i} - \underline{j}$, $\underline{w} = 9\underline{i} + 3\underline{j}$ and $\vec{PQ} = 2\underline{v} + k\underline{w}$, such that m , h and k are constants. Express h in terms of m .

(Ans : $h = 30 - 3m$)

[3 markah] [2019, No.16]

Answer :

\Rightarrow **parallel vectors / collinear ~ 1**

- 83** The following information refers to the vector \underline{a} and \underline{b} .

$$\underline{a} = \begin{pmatrix} 6 \\ m-4 \end{pmatrix}, \quad \underline{b} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

It is given that $\underline{a} = k\underline{b}$, where \underline{a} is parallel to \underline{b} and k is a constant. Find the value of

(a) k ,

(Ans : 3)

(b) m .

(Ans : 19)

[3 marks] [2012, No.16]

Answer :

(a)

(b)

- 84** Given $\underline{u} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\underline{v} = \begin{pmatrix} 6 \\ k-1 \end{pmatrix}$, find

(a) the unit vector in the direction of \underline{u} ,

$$[Ans : \frac{1}{5} \begin{pmatrix} 3 \\ 4 \end{pmatrix}]$$

(b) the value of k such that \underline{u} and \underline{v} are parallel.

$$(Ans : 9)$$

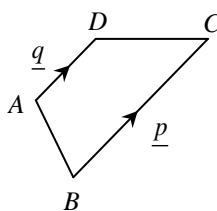
[4 marks] [2013, No.15]

Answer :

(a)

(b)

85 The diagram shows a trapezium $ABCD$.



Given $\underline{p} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\underline{q} = \begin{pmatrix} k-1 \\ 2 \end{pmatrix}$, where k is a constant, find the value of k .

$$(Ans : \frac{5}{2})$$

[3 marks] [2017, No.4]

Answer :

86 Given that $\underline{p} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$, $\underline{q} = \begin{pmatrix} -9 \\ 6 \end{pmatrix}$ and $\underline{r} = \begin{pmatrix} m \\ 4 \end{pmatrix}$. If $2\underline{p} + \underline{q}$ is parallel to \underline{r} , find the value of m .

$$(Ans : -2)$$

[4 marks] [Forecast]

Answer :

87 Vector $\begin{pmatrix} a \\ b \end{pmatrix}$ has a magnitude of $\sqrt{5}$ unit, and parallel to $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$. If $a > b$, find the value of a and of b .

$$(Ans : a = 2, b = -1)$$

[4 marks] [Forecast]

Answer :

\Rightarrow **parallel vectors / collinear ~ 2**

- 88 The points P , Q , and R are collinear. It is given that $\vec{PQ} = 4\vec{a} - 2\vec{b}$ and $\vec{QR} = 3\vec{a} + (1+k)\vec{b}$, where k is a constant. Find

(a) the value of k . (Ans : -2.5)

(b) the ratio of $PQ : QR$. (Ans : 4 : 3)

[4 marks] [2006, No.14]

Answer :

(a) (b)

- 89 Given that $\vec{AB} = (2k-1)\vec{p} + 3\vec{q}$. If \vec{AB} is extended to point C such that $\vec{BC} = k\vec{p} + 6h\vec{q}$, express k in terms of h . (Ans : $k = \frac{2h}{4h-1}$)

[2 marks] [clon textbook form 4]

Answer :

- 90 It is given $\vec{OP} = \begin{pmatrix} k \\ 4 \end{pmatrix}$, $\vec{OQ} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\vec{OR} = \begin{pmatrix} h \\ -2 \end{pmatrix}$, where h and k are constants. Express h in terms of k , if points P , Q and R lie on a straight line. (Ans : $h = 6 - 5k$)

[3 marks] [2015, No.15]

Answer :

- 91 Given $\vec{OA} = \vec{i} + \vec{j}$, $\vec{OC} = 5\vec{i} + 3\vec{j}$ and $\vec{OD} = 3\vec{i} + \lambda\vec{j}$. If point D lies on AC , find the value of λ . (Ans : 2)

[3 marks] [Forecast]

Answer :

\Rightarrow **parallel vectors / collinear ~ 3**

- 92 Given that $\vec{OP} = -\vec{i} + 10\vec{j}$, $\vec{OQ} = 3\vec{i} + 2\vec{j}$, and $\vec{OR} = 4\vec{i}$. Show that P , Q , and R are collinear.

[3 marks] [Forecast]

Answer :

- 93 Given that $\vec{OS} = 2\vec{j}$, $\vec{OK} = \frac{10}{3}\vec{i} + \vec{j}$, and $\vec{OR} = 6\vec{i}$. Determine whether S , K , and R are in a straight line. Prove your answer mathematically.

(Answer : no)

[3 marks] [Forecast]

Answer :

\Rightarrow **parallel vectors / collinear ~ 4**

- 94 Given $\underline{p} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$ and $\underline{q} = \begin{pmatrix} 2 \\ k \end{pmatrix}$, find

(a) $|\underline{p}|$,

(Ans : 5)

(b) the value of k such that $\underline{p} + \underline{q}$ is parallel to the x -axis.

(Ans : -3)

[3 marks] [2014, No.16]

Answer :

(a)

(b)

- 95 Given that $\underline{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$, and $\underline{b} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$. If $2p\underline{a} + 5\underline{b}$ is parallel to y -axis, find the value of p .

(Ans : $\frac{5}{3}$)

[3 marks] [Forecast]

Answer :

- 96 Given that $\vec{p} = \begin{pmatrix} m-1 \\ 1 \end{pmatrix}$ and $\vec{q} = \begin{pmatrix} 9 \\ 8 \end{pmatrix}$, where m is a constant. Find the value of m if \vec{p} is perpendicular / orthogonal to \vec{q} . (Ans : $\frac{1}{9}$)

[3 marks] [Forecast]

Answer :

MIND think :

• \vec{a} is parallel to x -axis \Rightarrow the constant of $\vec{j} = \boxed{}$

• \vec{a} is parallel to y -axis \Rightarrow the constant of $\vec{i} = \boxed{}$

\vec{a} perpendicular to \vec{b}

$$\downarrow$$

$$(\vec{m}_a)(\vec{m}_b) = -1 \quad @ \quad \vec{a} \cdot \vec{b} = 0$$

8.3.4 Solve problems involving vectors.

- 97** A motorboat cross a river with the engine that can move with a speed of $7\hat{j}$. The speed for the current of the river and the wind blows against the motorboat are $2\hat{i} - 3\hat{j}$ and $-4\hat{i} + 6\hat{j}$ respectively. Find the resultant vector cause to the motorboat.

(Ans : $-2\hat{i} + 10\hat{j}$)

[2 marks] [Forecast]

Answer :

- 98** A ball is throw horizontally with an acceleration of 2 ms^{-2} from the top of a tower. The ball will drop freely with an acceleration due to the gravity force, $g\text{ ms}^{-2}$. By using \hat{i} as the unit vector in the direction of horizontal acceleration, and \hat{j} as the unit vector in the drop direction due to the gravity force.

(a) Find the resultant vector of the ball in terms of \hat{i} and \hat{j} . [1 mark]

(b) Hence, by using $g = 10$, calculate the magnitude of the resultant vector of the ball. (Ans : 10.20) [2 marks]

Answer :

[Forecast]

(a) (b)

- 99** A particle moves with the velocity vector, $\vec{v} = (2\hat{i} - 3\hat{j})\text{ ms}^{-1}$. If it started from the position $\hat{i} + 4\hat{j}$. Find

(a) the speed, in ms^{-1} , of the particle, (Ans : $\sqrt{13}$)

(b) the position of the particle after 3 seconds, (Ans : $7\hat{i} - 5\hat{j}$)

(c) the duration, in second, for the particle to reach the position $11(\hat{i} - \hat{j})$. (Ans : 5)

[4 marks] [Forecast]

Answer :

(a) (c)

(b)

MIND think :

speed = of velocity vector

- 100** Car P left town P $(0, 0)$ with velocity of $\vec{v}_P = (6\vec{i} + 8\vec{j}) \text{ kmh}^{-1}$. At the same time, car Q left town Q $(100, 40)$ with velocity of $\vec{v}_Q = (-4\vec{i} + 4\vec{j}) \text{ kmh}^{-1}$. Find the time when the car P will cross car Q .

(Ans : 10)

[2 marks] [clon textbook form 4]

Answer :

- 101** A particle is moving from the point P $(7, 15)$ with the velocity vector of $(3\vec{i} - 2\vec{j}) \text{ ms}^{-1}$. After t seconds leaving P , the particle is on point S .

(a) Find

(i) the speed of the particle,

(Ans : $\sqrt{13}$)

(ii) the position of the particle from O after 4 seconds.

[Ans : $(19, 7)$]

(b) When will the particle reside on the right side of the origin ?

(Ans : 7.5)

[4 marks] [clon textbook form 4]

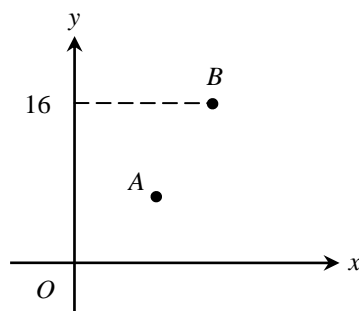
Answer :

(a) (i)

(b)

(ii)

- 102** The diagram shows the location of Pay'house, the school and the public library at point O , point A and point B respectively on the Cartesian plane. The shortest distance between Pay'house and the school is 7.5 km while the shortest distance between the school and the public library is 19.5 km.



Given the vector from Vivi's house to the school is $3\vec{i} + 4\vec{j}$, express the vector from Pay's house to the public library in the form of $x\vec{i} + y\vec{j}$.

(Ans : $8\vec{i} + 16\vec{j}$)

[3 marks] [Forecast]

Answer :

PAPER 2

⇒ **Part A ~ parallel 1 → 6 – 8 marks**

103 Given that $\vec{AB} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$, $\vec{OB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\vec{CD} = \begin{pmatrix} k \\ 5 \end{pmatrix}$, find

(a) the coordinates of A,

[Ans : (-3, -4)] [2 marks]

(b) the unit vector in the direction \vec{OA} ,

(Ans : $-\frac{3}{5}i - \frac{4}{5}j$) [2 marks]

(c) the value of k , if \vec{CD} is parallel to \vec{AB} .

(Ans : $\frac{25}{7}$) [2 marks]

[2003, No.6]

Answer :

104 It is given that $\vec{AB} = -3\vec{i} + 2\vec{j}$ and $\vec{AC} = -7\vec{i} + 5\vec{j}$

(a) Find

(i) \vec{BC} ,

(Ans : $-4\vec{i} + 3\vec{j}$)

(ii) the unit vector in the direction \vec{BC} .

(Ans : $\frac{-4\vec{i} + 3\vec{j}}{5}$)

[4 marks]

(b) Given $\vec{AD} = p\vec{i} - 15\vec{j}$, where p is a constant and \vec{AD} is parallel to \vec{BC} , find the value of p .

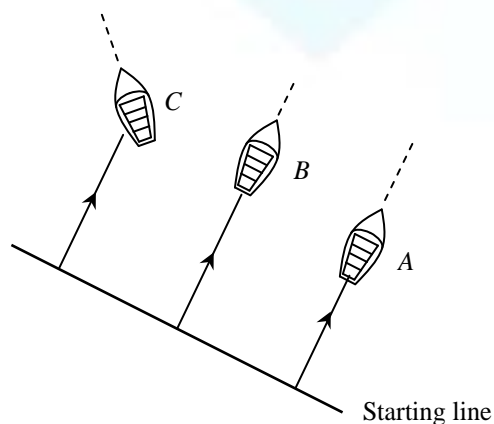
(Ans : 20) [3 marks]

[2012, No.5]

Answer :

⇒ **Part A ~ parallel 2**

105 The diagram shows the position and the direction of boats *A*, *B* and *C* in a solar boat competition.



Both boat *A* and boat *B* move in the direction of the water current. The velocity of the water current is given by $\underline{w} = \left(\underline{i} + \frac{1}{2}\underline{j} \right) \text{ ms}^{-1}$. Given the velocity of boat *A* is $\underline{a} = (2\underline{i} + \underline{j}) \text{ ms}^{-1}$ and the velocity of boat *B* is $\underline{b} = (6\underline{i} + 3\underline{j}) \text{ ms}^{-1}$.

(a) Determine how many times the resultant velocity of boat *B* compare to the resultant velocity of boat *A*. (Ans : $\frac{7}{3}$) [4 marks]

(b) On the way to the finishing line, boat *C* is facing a technical problem and off track. The velocity of boat *C* is $\underline{c} = \left(2\underline{i} - \frac{3}{2}\underline{j} \right) \text{ ms}^{-1}$. Find

(i) the resultant velocity of boat *C*, (Ans : $3\underline{i} - \underline{j}$)

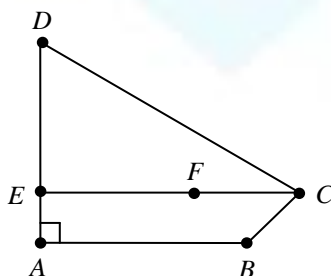
(ii) the unit vector in the direction of boat *C*, [Ans : $\frac{1}{\sqrt{10}}(3\underline{i} - \underline{j})$]
[3 marks]

[2016, No.5]

Answer :

\Rightarrow Part A ~ parallel 3

106 In the diagram, $ABCD$ is a quadrilateral. AED and EFC are straight lines.



It is given that $\vec{AB} = 20\vec{x}$, $\vec{AE} = 8\vec{y}$, $\vec{DC} = 25\vec{x} - 24\vec{y}$, $AE = \frac{1}{4}AD$ and $EF = \frac{3}{5}EC$.

(a) Express in term of \vec{x} and \vec{y} :

(i) \vec{BD} ,

(Ans : $-20\vec{x} + 32\vec{y}$)

(ii) \vec{EC} ,

(Ans : $25\vec{x}$)

[3 marks]

(b) Show that the points B , F and D are collinear.
[3 marks]

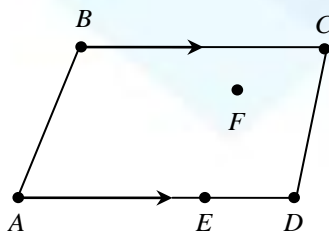
(c) If $|\vec{x}| = 2$, and $|\vec{y}| = 3$, find $|\vec{BD}|$.

(Ans : 104) [2 marks]

[2005, No.6]

Answer :

- 107 The diagram shows a trapezium $ABCD$.



It is given that $\vec{AB} = 2\vec{y}$, $\vec{AD} = 6\vec{x}$, $\vec{AE} = \frac{2}{3}\vec{AD}$ and $\vec{BC} = \frac{5}{6}\vec{AD}$

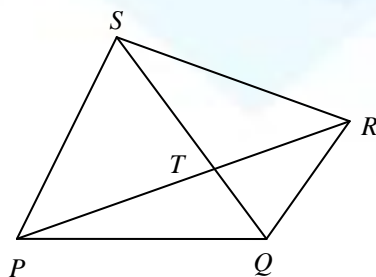
- (a) Express \vec{AC} in term of \vec{x} and \vec{y} . (Ans : $5\vec{x} + 2\vec{y}$) [2 marks]
- (b) Point F lies inside the trapezium $ABCD$ such that $2\vec{EF} = m\vec{AB}$, m is a constant.
- (i) Express \vec{AF} in term of m , \vec{x} and \vec{y} . (Ans : $4\vec{x} + m\vec{y}$)
- (ii) Hence, if the point A , F and C are collinear, find the value of m .
(Ans : $\frac{8}{5}$)

[5 marks]

[2006, No.5]

Answer :

- 108** The diagram shows a quadrilateral $PQRS$. The straight line PR intersects the straight line QS at point T .



It is given that $QT : TS = 2 : 3$, $\vec{PQ} = 10\vec{u}$, $\vec{PS} = 25\vec{v}$ and $\vec{QR} = -\vec{u} + 15\vec{v}$

- (a) Express in terms of \vec{u} and \vec{v} ,

(i) \vec{QS} ,

(Ans : $-10\vec{u} + 25\vec{v}$)

(ii) \vec{PT} ,

(Ans : $6\vec{u} + 10\vec{v}$)

[3 marks]

- (b) Find the ratio $PT : TR$.

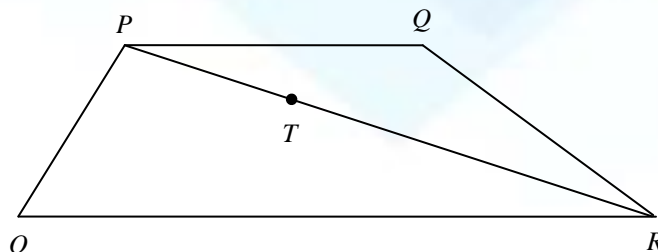
(Ans : $2 : 1$)

[5 marks]

[2013, No.3]

Answer :

- 109 The diagram shows a trapezium $OPQR$ and point T lies on PR .



It is given that $\vec{OR} = 18\vec{b}$, $\vec{OP} = 6\vec{a}$ and $\vec{OR} = 2\vec{PQ}$.

- (a) Express in terms of \vec{a} and \vec{b} ,

(i) \vec{PR} , (Ans : $-6\vec{a} + 18\vec{b}$)

(ii) \vec{OQ} , (Ans : $6\vec{a} + 9\vec{b}$)
[3 marks]

- (b) It is given that $\vec{PT} = k\vec{PR}$, where k is a constant. Find the value of k if the point O , T and Q are collinear. (Ans : $\frac{1}{3}$) [5 marks]

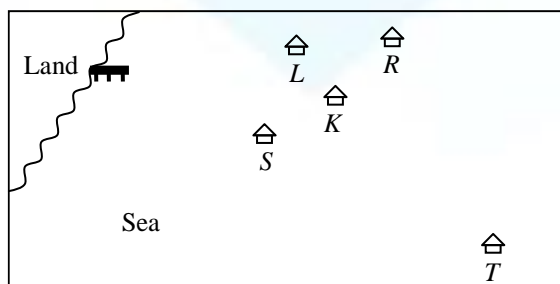
- (c) If the area of triangle $QTR = 45 \text{ unit}^2$, and the perpendicular distance from P to OR is 4 units, find $|\vec{b}|$. (Ans : 3.75) [2 marks]

[2014, No.5]

Answer :

110 Solution by scale drawing is not accepted.

The diagram shows the positions of jetty O and kelong, K , L , R , S and T in the sea.



Kelong L is situated 400 m from jetty O and kelong R is situated 600 m from jetty O in the direction of OL . Kelong S is situated 300 m from jetty O and kelong T is situated 600 m from kelong S in the direction of OS . Kelongs L , K and T are situated on a straight line such that the distance of kelong K from kelong T is 5 times its distance from kelong L .

- (a) By using \underline{p} to represent 100 m in the direction of OR and \underline{q} to represent 150 m in the direction of OT , express in terms of \underline{p} and \underline{q}

(i) \vec{OK} (Ans : $\frac{10}{3}\underline{p} + \underline{q}$)

(ii) \vec{RK} (Ans : $-\frac{8}{3}\underline{p} + \underline{q}$)

[3 marks]

- (b) If Joe uses a binocular to observe kelong R from kelong S , determine whether kelong R can be seen without being blocked by kelong K or otherwise.

Prove your answer mathematically.

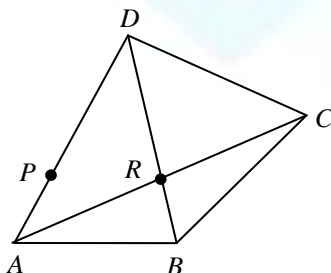
(Ans : $\vec{KS} = -\frac{10}{3}\underline{p} + \underline{q}$; can be seen) [5 marks]

[2019, No.6]

Answer :

⇒ **Part A ~ triangle law 1**

- 111** In the diagram, $ABCD$ is a quadrilateral. The diagonals BD and AC intersect at point R . Point P lies on AD .



It is given that $AP = \frac{1}{3}AD$, $BR = \frac{1}{3}BD$, $\vec{AB} = \underline{x}$ and $\vec{AP} = \underline{y}$.

- (a) Express in terms of \underline{x} and \underline{y} :

(i) \vec{DB} ,

(Ans : $\underline{x} - 3\underline{y}$)

(ii) \vec{AR} .

(Ans : $\frac{2}{3}\underline{x} + \underline{y}$)

[3 marks]

- (b) Given that $\vec{DC} = k\underline{x} - \underline{y}$ and $\vec{AR} = h\vec{AC}$, where h and k are constants, find the value of h and of k .

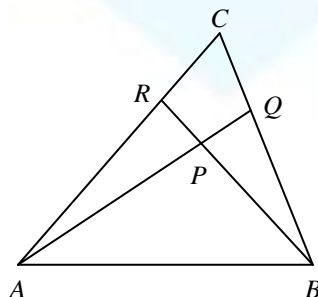
(Ans : $h = \frac{1}{2}$, $k = \frac{4}{3}$) [4 marks]

[2008, No.6]

Answer :

⇒ **Part A ~ triangle law 2**

- 112 The diagram shows triangle ABC . The straight line AQ intersect the straight line BR at P .



It is given that $AR = 3RC$, $BQ = \frac{2}{3}BC$, $\vec{AB} = 3\underline{x}$ and $\vec{AC} = 4\underline{y}$.

- (a) Express in terms of \underline{x} and \underline{y} :

(i) \vec{BC} ,

(Ans : $-3\underline{x} + 4\underline{y}$)

(ii) \vec{AQ} .

(Ans : $\underline{x} + \frac{8}{3}\underline{y}$)

[3 marks]

- (b) It is given that $\vec{AP} = h\vec{AQ}$ and $\vec{AP} = \vec{AR} + k\vec{RB}$, where h and k are constants, find the value of h and of k .

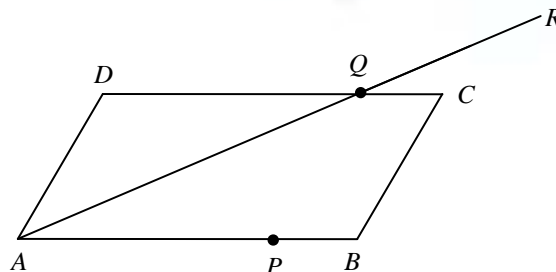
(Ans : $h = \frac{9}{11}$, $k = \frac{3}{11}$) [5 marks]

[2009, No.5]

Answer :

⇒ Part B ~ parallel 3 → 10 marks

- 113 The diagram shows a parallelogram $ABCD$. Point P lies on the straight line AB and point Q lies on the straight line DC . The straight line AQ is extended to the point R such that $AQ = 2QR$.



It is given that $AP : PB = 3 : 1$, $DQ : QC = 3 : 1$, $\vec{AP} = 6\vec{u}$ and $\vec{AD} = \vec{v}$.

- (a) Express, in terms of \vec{u} and \vec{v} :

(i) \vec{AQ} ,

(Ans : $\vec{v} + 6\vec{u}$)

(ii) \vec{PC} .

(Ans : $2\vec{u} + \vec{v}$)

Hence, show that the points, P , C and R are collinear

[6 marks]

- (b) It is given that $\vec{u} = 3\vec{i}$ and $\vec{v} = 2\vec{i} + 5\vec{j}$.

(i) Express \vec{PC} in terms of \vec{i} and \vec{j} ,

(Ans : $8\vec{i} + 5\vec{j}$)

(ii) Find the unit vector in the direction of \vec{PC} .

(Ans : $\frac{8\vec{i} + 5\vec{j}}{\sqrt{89}}$)

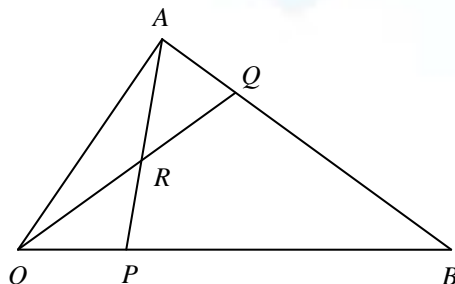
[4 marks]

[2011, No.10]

Answer :

⇒ Part B ~ triangle law 1

- 114 The diagram shows triangle OAB . The straight line AP intersects the straight line OQ at R . It is given that $OP = \frac{1}{3}OB$, $AQ = \frac{1}{4}AB$, $\vec{OP} = 6\vec{x}$, and $\vec{OA} = 2\vec{y}$.

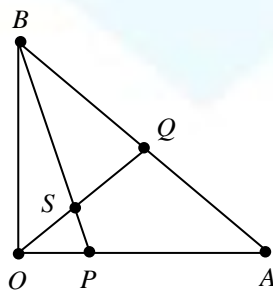


- (a) Express in terms of \vec{x} and / or \vec{y}
- (i) \vec{AP} , (Ans : $-2\vec{y} + 6\vec{x}$)
- (ii) \vec{OQ} . (Ans : $\frac{3}{2}\vec{y} + \frac{9}{2}\vec{x}$)
- [4 marks]
- (b) (i) Given that $\vec{AR} = h\vec{AP}$, state \vec{AR} in terms of h , \vec{x} and \vec{y} .
[Ans : $h(-2\vec{y} + 6\vec{x})$]
- (ii) Given that $\vec{RQ} = k\vec{OQ}$, state \vec{RQ} in terms of k , \vec{x} and \vec{y} .
[Ans : $k(\frac{3}{2}\vec{y} + \frac{9}{2}\vec{x})$]
- [2 marks]
- (c) Using \vec{AR} and \vec{RQ} from (b), find the value of h and of k .
(Ans : $h = \frac{1}{2}$, $k = \frac{1}{3}$) [4 marks]

[2004, No.8]

Answer :

- 115 The diagram shows triangle AOB . The point P lies on OA and the point Q lies on AB . The straight line BP intersects the straight line OQ at the point S .



It is given that $OA : OP = 4 : 1$, $AB : AQ = 2 : 1$, $\vec{OA} = 8\underline{x}$ and $\vec{OB} = 6\underline{y}$.

- (a) Express in terms of \underline{x} and / or \underline{y} .

(i) \vec{BP} , (Ans : $2\underline{x} - 6\underline{y}$)

(ii) \vec{OQ} . (Ans : $4\underline{x} + 3\underline{y}$)
[3 marks]

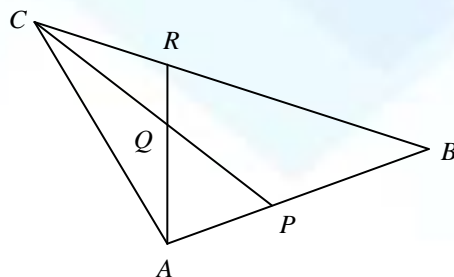
- (b) Using $\vec{OS} = h\vec{OQ}$ and $\vec{BS} = k\vec{BP}$, where h and k are constants, find the value of h and of k . (Ans : $h = \frac{2}{5}$, $k = \frac{4}{5}$)
[5 marks]

- (c) Given that $|\underline{x}| = 2$ units, $|\underline{y}| = 3$ units and $\angle AOB = 90^\circ$, find $|\vec{AB}|$.
(Ans : $\sqrt{580}$) [2 marks]

[2007, No.8]

Answer :

- 116 The diagram shows a triangle ABC .



It is given $AP : PB = 1 : 2$, $BR : RC = 2 : 1$, $\vec{AP} = 2\underline{x}$, and $\vec{AC} = 3\underline{y}$.

- (a) Express in terms of \underline{x} and \underline{y} ,

(i) \vec{CP} ,

(Ans : $-3\underline{y} + 2\underline{x}$)

(ii) \vec{CR} ,

(Ans : $-\underline{y} + 2\underline{x}$)

[3 marks]

- (b) Given $\underline{x} = 2\underline{i}$ and $\underline{y} = -\underline{i} + 4\underline{j}$, find $\left| \vec{CR} \right|$. (Ans : $\sqrt{41}$) [2 marks]

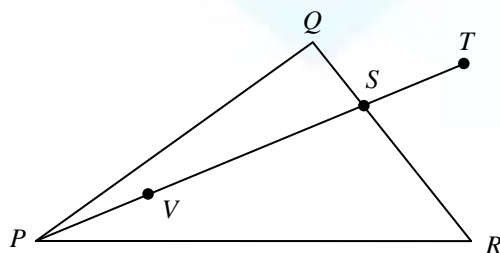
- (c) Given $\vec{CQ} = m\vec{CP}$ and $\vec{QR} = n\vec{AR}$, where m and n are constants, find the value of m and of n . (Ans : $m = \frac{3}{5}$, $n = \frac{2}{5}$) [5 marks]

[2015, No.9]

Answer :

⇒ **Part B** ~~ **triangle law 1 / parallel**

- 117** The diagram shows a triangle PQR . The straight line PT intersects with the straight line QR at point S . Point V lies on the straight line PT .



It is given that $\vec{QS} = \frac{1}{3} \vec{QR}$, $\vec{PR} = 6\vec{x}$ and $\vec{PQ} = 9\vec{y}$.

- (a) Express in terms of \vec{x} and / or \vec{y} :

(i) \vec{QR}

(ii) \vec{PS} .

(Ans : $6\vec{y} + 2\vec{x}$)

[3 marks]

- (b) It is given that $\vec{PV} = m\vec{PS}$ and $\vec{QV} = n(\vec{x} - 9\vec{y})$, where m and n are constants. Find the value of m and of n . (Ans : $m = \frac{3}{8}$, $n = \frac{3}{4}$) [5 marks]

- (c) Given $\vec{PT} = h\vec{x} + 9\vec{y}$, where h is a constant, find the value of h .

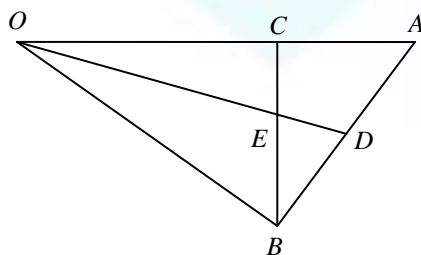
(Ans : 3) [2 marks]

[2017, No.8]

Answer :

⇒ **Part B ~ triangle law 2 → 10 marks**

- 118** The diagram shows triangle OAB . The point C lies on OA and the point D lies on AB . The straight line OD intersects the straight line BC at the point E .



It is given that $\vec{OA} = \underline{x}$, $\vec{OB} = \underline{y}$, $\vec{OC} = \frac{2}{3} \vec{OA}$ and $\vec{AB} = 2 \vec{AD}$.

- (a) Express in terms of \underline{x} and \underline{y} :

(i) \vec{BC} ,

(Ans : $-\underline{y} + \frac{2}{3}\underline{x}$)

(ii) \vec{OD} .

(Ans : $\frac{1}{2}\underline{y} + \frac{1}{2}\underline{x}$)

[4 marks]

- (b) It is given that $\vec{OE} = h \vec{OD}$ and $\vec{BE} = k \vec{BC}$, where h and k are constants. Express \vec{OE}

(i) in terms of h , \underline{x} and \underline{y} ,

[Ans : $h(\frac{1}{2}\underline{y} + \frac{1}{2}\underline{x})$]

(ii) in terms of k , \underline{x} and \underline{y} .

(Ans : $\underline{y} - k\underline{y} + \frac{2}{3}k\underline{x}$)

[3 marks]

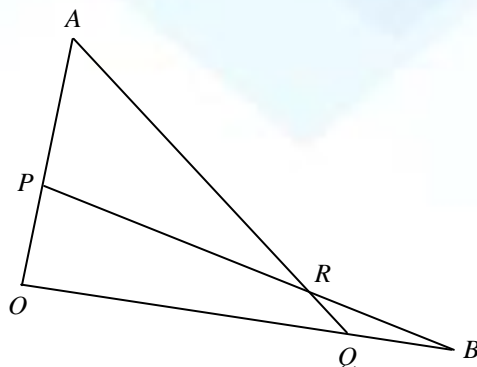
- (c) Hence, find the value of h and of k .

(Ans : $h = \frac{4}{5}$, $k = \frac{3}{5}$) [3 marks]

[2010, No.9]

Answer :

- 119 The diagram shows triangles OAQ and OPB where point P lies on OA and point Q lies on OB . The straight lines AQ and PB intersect at point R .



It is given that $\vec{OA} = 18\underline{x}$, $\vec{OB} = 16\underline{y}$, $OP : PA = 1 : 2$, $OQ : QB = 3 : 1$, $\vec{PR} = m\vec{PB}$, $\vec{QR} = n\vec{QA}$, where m and n are constants.

- (a) Express \vec{OR} in terms of :

(i) m , \underline{x} dan \underline{y} ,

(Ans : $6\underline{x} - 6m\underline{x} + 16m\underline{y}$)

(ii) n , \underline{x} dan \underline{y} .

(Ans : $12\underline{y} - 12n\underline{y} + 18n\underline{x}$)

[4 marks]

- (b) Hence, find the value of m and of n .

(Ans : $m = \frac{2}{3}$, $n = \frac{1}{9}$) [4 marks]

- (c) Given $|\underline{x}| = 2$ unit, $|\underline{y}| = 1$ unit and OA is perpendicular to OB , calculate $|\vec{PR}|$.

(Ans : $\frac{40}{3}$) [2 marks]

[2018, No.8]

Answer :

FORECAST

⇒ **Part A ~ 6 – 8 marks**

120 A boat is set on a course of $N 30^\circ E$ with a speed of 12 knots. However, the water current is flowing at 5 knots towards the east.

(a) Sketch the diagram which shows the movement of the boat and the water current.

[1 mark]

(b) Find :

(i) the magnitude, (Ans : 15.13) [2 marks]

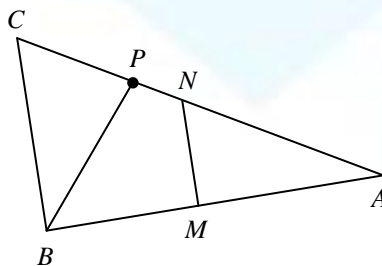
(ii) the direction, (Ans : 046.63°) [2 marks]

of the resultant velocity of the boat.

(c) If the boat intends to move northward, show that the direction at which must be steered in order to do so is $N 24.62^\circ W$. [2 marks]

Answer :

- 121** The diagram shows a right-angled triangle ABC . M is the midpoint of AB . MN is parallel to BC and P is a point on CN such that $CP = 2PN$. Given $\vec{BM} = \underline{a}$ and $\vec{BC} = 2\underline{b}$.



- (a) Express in terms of \underline{a} and / or \underline{b} :

(i) \vec{MN} ,

(ii) \vec{CA} ,

(Ans : $-2\underline{b} + 2\underline{a}$)

(iii) \vec{BP} .

(Ans : $\frac{4}{3}\underline{b} + \frac{2}{3}\underline{a}$)

[4 marks]

- (b) (i) If $|\underline{a}| = 12$ units, $|\underline{b}| = 9$ units, find the area of triangle ABC .

(Ans : 216)

- (ii) Find the shortest distance for P to BC .

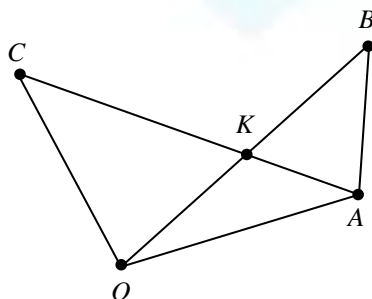
(Ans : 12)

[3 marks]

Answer :

\Rightarrow Part A ~ parallel 3

- 122 The diagram shows a triangle OAB and OAC . The straight lines OB and AC intersect at point K such that $AK:AC = 1:3$. Given $\vec{OA} = 3\underline{a}$ and $\vec{OC} = h\underline{c}$, where h is a constant.



Find

- (a) \vec{AK} in terms of h , \underline{a} and \underline{c} ,

[Ans : $\frac{1}{3}(-3\underline{a} + h\underline{c})$] [2 marks]

- (b) \vec{OK} in terms of h , \underline{a} and \underline{c} ,

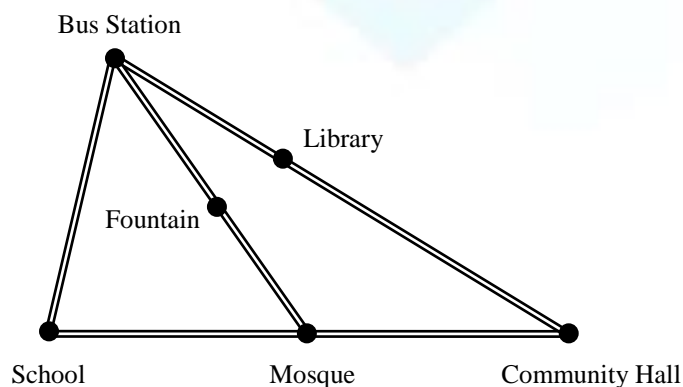
(Ans : $2\underline{a} + \frac{h}{3}\underline{c}$) [1 mark]

Hence, if $\vec{KB} = 10\underline{a} + 5\underline{c}$, find the value of h .

(Ans : 3) [4 marks]

Answer :

- 123** The diagram shows a map of part of Telipok District, with the condition that all roads are straight. The mosque is equidistance from the school and the Community Hall, while the fountain is equidistance from the bus station and the mosque.



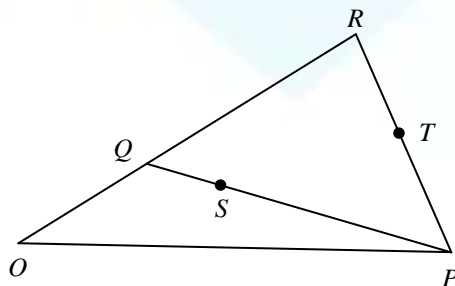
The Telipok District Council has decided to build a straight road from the school to the library through the fountain. The distance from the school to the library is k times the distance from the school to the fountain. The distance from the Community Hall to the library is twice the distance from the library to the bus station. Given that the displacement of bus stations and mosques from the school is \vec{u} and \vec{v} respectively.

- (a) Find the value of k . (Ans : $\frac{4}{3}$) [5 marks]
- (b) Given that the cost of road construction from school to fountain is RM600000. Find the cost, in RM, of the construction of the same type of road from the fountain to the library. (Ans : 200000) [2 marks]

Answer :

\Rightarrow Part A ~ triangle law 1

124 The diagram shows a triangle OPR and QSP is a straight line.



It is given that $\vec{OP} = \underline{p}$, $\vec{OQ} = \underline{q}$ and $\vec{OR} = 4\vec{OQ}$.

(a) Express in terms of \underline{p} and \underline{q} .

(i) \vec{PQ} ,

(Ans : $-\underline{p} + \underline{q}$) [1 mark]

(ii) \vec{PR} .

(Ans : $-\underline{p} + 4\underline{q}$) [1 mark]

(b) It is given that $\vec{PS} = m\vec{PQ}$ and $\vec{PT} = k\vec{PR}$.

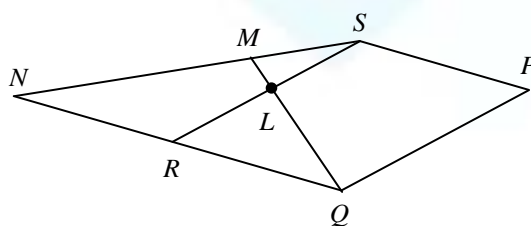
By using $\vec{OT} = 2\vec{OS}$, find the value of m and of k .

(Ans : $k = \frac{1}{3}$, $m = \frac{2}{3}$) [5 marks]

Answer :

⇒ **Part A ~ triangle law 2**

- 125** The diagram shows a parallelogram $PQRS$, where L is the midpoint of RS . QR is produced to N such that $QR = RN$ and QL is produced to meet SN at M .



It is given that $\vec{PQ} = 3\vec{a}$ and $\vec{QR} = 2\vec{b}$.

- (a) Express in terms of \vec{a} and / or \vec{b} :

(i) \vec{QL} , (Ans : $2\vec{b} - \frac{3}{2}\vec{a}$)

(ii) \vec{SN} , (Ans : $3\vec{a} + 2\vec{b}$)
[2 marks]

- (b) Given that $\vec{QM} = h\vec{QL}$ and $\vec{NM} = k\vec{NS}$, find \vec{QM} in terms of

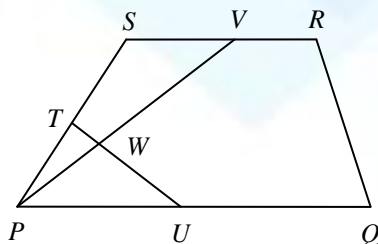
(i) h , \vec{a} and \vec{b} , (Ans : $2h\vec{b} - \frac{3}{2}h\vec{a}$)

(ii) k , \vec{a} and \vec{b} . (Ans : $4\vec{b} - 3k\vec{a} - 2k\vec{b}$)

Hence, find the value of h and of k . (Ans : $h = \frac{4}{3}$, $k = \frac{2}{3}$) [5 marks]

Answer :

- 126 The diagram shows a trapezium $PQRS$. U is the midpoint of PQ and $\vec{PU} = 2\vec{SV}$. PV and TU are two straight lines intersecting at W where $TW : WU = 1 : 3$ and $PW = WV$.



It is given that $\vec{PQ} = 12\vec{a}$, $\vec{PS} = 18\vec{b}$ and $\vec{QR} = 18\vec{b} - 5\vec{a}$

- (a) Express in terms of \vec{a} and / or \vec{b}

(i) \vec{SR} , (Ans : $7\vec{a}$)

(ii) \vec{PV} , (Ans : $3\vec{a} + 18\vec{b}$)

(iii) \vec{PW} . (Ans : $\frac{3}{2}\vec{a} + 9\vec{b}$)

[4 marks]

- (b) Using $PT : TS = h : 1$, where h is a constant, express \vec{PW} in terms of h , \vec{a} and / or \vec{b} . [Ans : $\frac{3}{2}\vec{a} + \frac{27h}{2(h+1)}\vec{b}$]

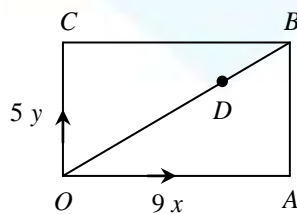
Hence, find the value of h . (Ans : 2)

[4 marks]

Answer :

CONTINUOUS EXERCISES

- 127** The diagram shows a rectangle $OABC$ and the point D lies on the straight line OB .

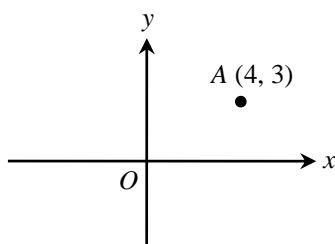


It is given that $OD = kDB$. Express \vec{OD} in terms of k , \underline{x} and \underline{y} . [Ans : $\frac{k}{k+1}(5\underline{y} + 9\underline{x})$]

[3 marks] [clon 2007, No.15]

Answer :

- 128** The diagram shows the point A on the Cartesian plane.



- (a) State \vec{OA} in the form of $\begin{pmatrix} x \\ y \end{pmatrix}$.

- (b) Point A is reflected about the y -axis to point A' . It is given $\vec{OB} = \underline{i} + m\underline{j}$ and unit vector of $\vec{A'B}$ is $n\begin{pmatrix} 10 \\ 24 \end{pmatrix}$, where m and n are constants. Find the value of m and n .

(Ans : $m = 15$, $n = \frac{1}{26}$)

[4 marks] [2020, No.14]

Answer :

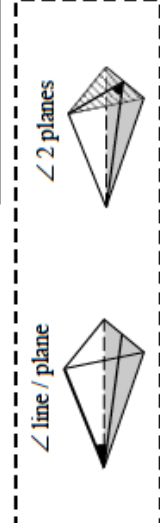
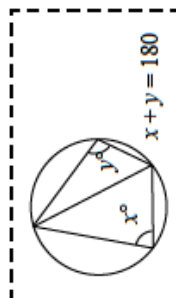
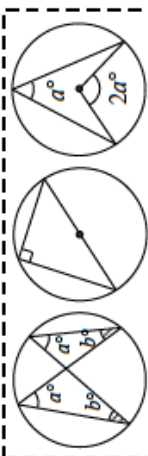
(a)

(b)

SOLUTION OF TRIANGLES

- ONE PAGE NOTE (OPN) - WORKSHEET

Encik Mohd Salleh Ambo



Area of triangle using the "Heron's formula"

$$s = \frac{9.8 + 5.2 + 12.3}{2} = 13.65$$

$$\text{area } \triangle ADC = \frac{\sqrt{13.65(13.65 - 9.8)(13.65 - 5.2)(13.65 - 12.3)}}{2} = 24.485$$

"given 3 sides"

$$\cos \angle ADC = \frac{9.8^2 + 5.2^2 - 12.3^2}{2(9.8)(5.2)}$$

$$\angle ADC = 106.07^\circ$$

"given 2 sides & 1 included angle"

$$AC^2 = 20^2 + 15^2 - 2(20)(15) \cos 65^\circ$$

$$AC = 19.27$$

"Cosine Rule"

$$\text{area } \triangle ABC = \frac{1}{2}(20)(15) \sin 65^\circ = 135.95$$

"Sine Rule"

$$\frac{BC}{\sin 73^\circ} = \frac{9.6}{\sin 85^\circ}$$

$$BC = \frac{9.6 \sin 85^\circ}{\sin 73^\circ} = 9.216$$

"given 1 side & 2 angles"

$$\frac{AB}{\sin 22^\circ} = \frac{9.6}{\sin 85^\circ}$$

$$AB = \frac{9.6 \sin 85^\circ}{\sin 22^\circ} = 3.610$$

"given 2 sides & 1 non-included angle"

$$\frac{BC}{\sin 73^\circ} = \frac{9.6}{\sin 85^\circ}$$

$$BC = \frac{9.6 \sin 85^\circ}{\sin 73^\circ} = 9.216$$

"Area of Triangle"

$$\text{area } \triangle ABC = \frac{1}{2}bh$$

"Ambiguous Case"

$$\text{area } \triangle ABC = \frac{1}{2}b \times h$$

"given 2 sides & 1 non-included angle"

$$\sin \angle LMN = \frac{\sin 68^\circ}{\frac{23.2}{21.8}}$$

$$\angle LMN = 80.65^\circ \text{ (acute)}$$

$$\angle LMN = 99.35^\circ \text{ (obtuse)}$$

$$\angle LNM = 180 - 68 - 99.35 = 12.65^\circ$$

"given 2 sides & 1 non-included angle"

$$\sin \angle LMN = \frac{\sin 68^\circ}{\frac{23.2}{21.8}}$$

$$\angle LMN = 80.65^\circ \text{ (acute)}$$

$$\angle LMN = 99.35^\circ \text{ (obtuse)}$$

$$\text{[HINT : } \angle \text{acute} + \angle \text{obtuse} = 180 \text{]}$$

"given 2 sides & 1 non-included angle"

$$\sin \angle LMN = \frac{\sin 68^\circ}{\frac{23.2}{21.8}}$$

$$\angle LMN = 80.65^\circ \text{ (acute)}$$

$$\angle LMN = 99.35^\circ \text{ (obtuse)}$$

$$\text{[HINT : } \angle \text{acute} + \angle \text{obtuse} = 180 \text{]}$$

ONE PAGE NOTES (Coding Method)

CHAPTER 9 (FORM 4)

"SOLUTION OF TRIANGLES"

WORKSHEET
TOPIC 9 : SOLUTION OF TRIANGLES
[Part C → 10 marks]

=====

9.1 Sine Rule

9.1.1 Make and verify conjectures on the relationship between the ratio of length of sides of a triangle with the sine of the opposite angles, and hence define the sine rule.

[the use of digital technology is encouraged]

9.1.2 Solve triangles involving sine rule.

9.1.3 Determine the existence of ambiguous case of a triangle, and hence identify the conditions for such cases.

9.1.4 Solve triangles involving ambiguous cases.

9.1.5 Solve problems related to triangles using the sine rule.

9.2 Cosine rule

9.2.1 Verify the cosine rule.

9.2.2 Solve triangles involving the cosine rule.

9.2.3 Solve problems involving the cosine rule.

9.3 Area of a triangle

9.3.1 Derive the formula for area of triangles, and hence determine the area of a triangle.

9.3.2 Determine the area of a triangle using the Heron's formula.

9.3.3 Solve problems involving areas of triangles .

9.4 Application of sine rule, cosine rule and area of a triangle

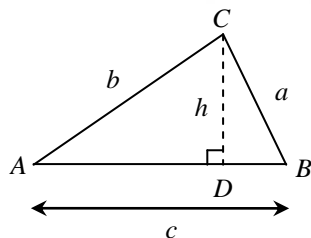
9.4.1 Solve problems involving triangles.

=====

9.1 Sine Rule

9.1.1 Make and verify conjectures on the relationship between the ratio of length of sides of a triangle with the sine of the opposite angles, and hence define the sine rule.

- 1 The diagram shows a triangle ABC .

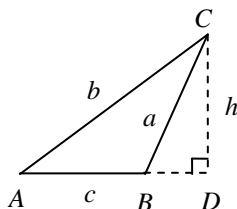


Show that $\frac{a}{\sin A} = \frac{b}{\sin B}$.

[2 marks]

Answer :

- 2 The diagram shows a triangle ABC .



Show that $\frac{\sin A}{a} = \frac{\sin B}{b}$.

[2 marks]

Answer :

MIND think :

SINE RULE

~ For any triangle ABC ,

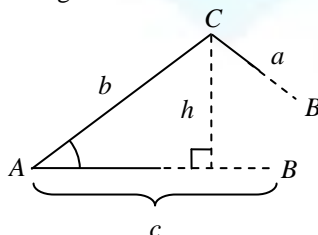
$$\frac{a}{\sin A} =$$

OR

$$\frac{\sin A}{a} =$$

9.1.3 Determine the existence of ambiguous case of a triangle, and hence identify the conditions for such cases.

- 3 The diagram shows an incomplete triangle ABC .



If $\angle A$, length of a and length of b are fixed.

For each of the following cases, state the number of possible triangle/s that can be formed.

Answer :

$a < h$	$a = h$	$a \geq b$	$h < a < b$

- 4 In the answer space, mark (✓) for the triangle which exist ambiguous case. If (✓), sketch the another difference triangle on the same diagram.

Answer :

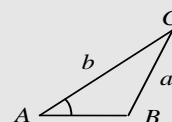
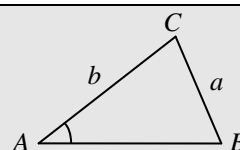
(a) <input type="checkbox"/>	(b) <input type="checkbox"/>	(c) <input type="checkbox"/>
(d) <input type="checkbox"/>	(e) <input type="checkbox"/>	(f) <input type="checkbox"/>

MIND think :

AMBIGUOUS exists if :

~ given sides and non-included angle.

~ $\angle A$ is an angle, **and** a b .



- 5 PQR is a triangle where $PQ = 12$ cm, $QR = 6.8$ cm, and $\angle RPQ = 33^\circ$. Sketch the two possible triangles PQR . Hence, find the two possible values of $\angle PQR$.
(Ans : 40.97° , 73.03°)
[5 marks]

Answer :

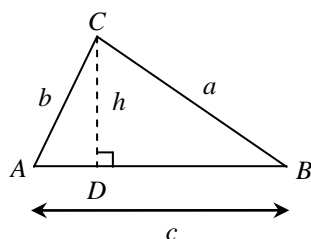
- 6 In a triangle DEF , $DE = 8$ cm, $DF = 10$ cm, and $\angle DFE = 50^\circ$. Sketch the two possible triangles DEF . Hence, find the probable values of the length of EF .
(Ans : 4.122, 8.734)
[5 marks]

Answer :

9.2 Cosine rule

9.2.1 Verify the cosine rule.

- 7 The diagram show a triangle ABC .

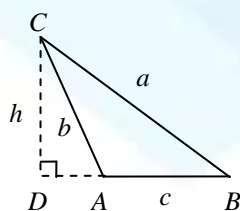


Show that $a^2 = b^2 + c^2 - 2bc \cos A$.

[3 marks]

Answer :

- 8 The diagram shows a triangle ABC .



Show that $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$.

[3 marks]

Answer :

MIND think :

COSINE RULE

~ For any triangle ABC ,

$$b^2 =$$

OR $\cos B =$

$$c^2 =$$

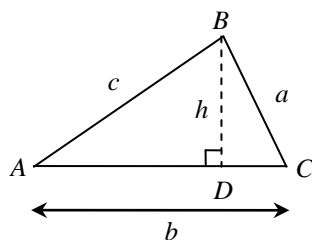
OR $\cos C =$

9.3 Area of a triangle

9.3.1 Derive the formula for area of triangles, and hence determine the area of a triangle.

9.3.2 Determine the area of a triangle using the Heron's formula.

- 9 The diagram shows a triangle ABC .

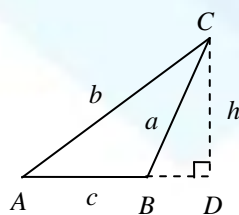


Show that the area of triangle $ABC = \frac{1}{2}ab \sin C = \frac{1}{2}bc \sin A$.

[2 marks]

Answer :

- 10 The diagram shows a triangle ABC .



Show that the area of triangle $ABC = \frac{1}{2}ab \sin C$.

[2 marks]

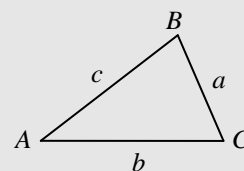
Answer :

MIND think :

HERON'S FORMULA

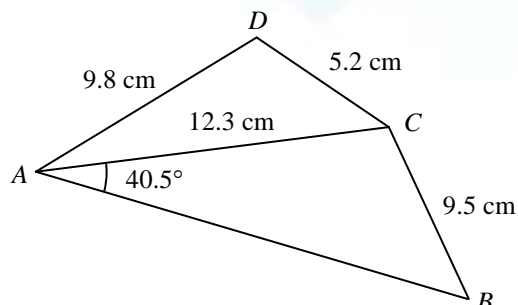
~ For any triangle ABC ,

- semi perimeter, $s =$
- area of triangle $ABC =$



PAPER 2 **\Rightarrow 2 dimensional surfaces**

- 11 The diagram shows a quadrilateral $ABCD$ such that $\angle ABC$ is acute.



- (a) Calculate

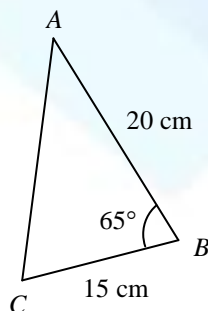
- (i) $\angle ABC$, (Ans : 57.23)
 - (ii) $\angle ADC$, (Ans : 106.07)
 - (iii) the area, in cm^2 , of quadrilateral $ABCD$. (Ans : 82.37)
- [8 marks]

- (b) A triangle $A'B'C'$ has the same measurements as those given for triangle ABC , that is, $A'C' = 12.3$ cm, $C'B' = 9.5$ cm, and $\angle B'A'C' = 40.5^\circ$, but which is different in shape to triangle ABC .

- (i) Sketch the triangle $A'B'C'$,
 - (ii) State the size of $\angle A'B'C'$. (Ans : 122.77)
- [2 marks]
[2004, No.13]

Answer :

- 12 The diagram shows a triangle ABC .



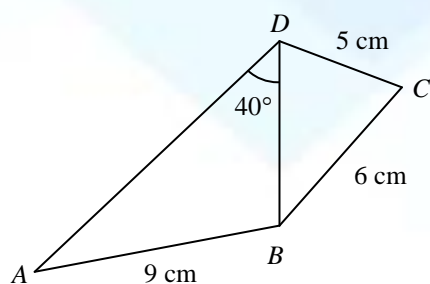
- (a) Calculate the length, in cm, of AC . (Ans : 19.27) [2 marks]
- (b) A quadrilateral $ABCD$ is now formed so that AC is a diagonal, $\angle ACD = 40^\circ$ and $AD = 16$ cm. Calculate the two possible values of $\angle ADC$.
(Ans : 50.73, 129.27) [2 marks]
- (c) By using the acute angle $\angle ADC$ from (b), calculate
- the length, in cm, of CD , (Ans : 24.89)
 - the area, in cm^2 , of the quadrilateral $ABCD$. (Ans : 290.1)

[6 marks]

[2005, No.12]

Answer :

- 13 The diagram shows a quadrilateral $ABCD$.



The area of triangle BCD is 13 cm^2 and $\angle BCD$ is acute. Calculate

- $\angle BCD$,
- the length, in cm , of BD ,
- $\angle ABD$,
- the area, in cm^2 , of quadrilateral $ABCD$.

(Ans : 60.074) [2 marks]

(Ans : 5.5738) [2 marks]

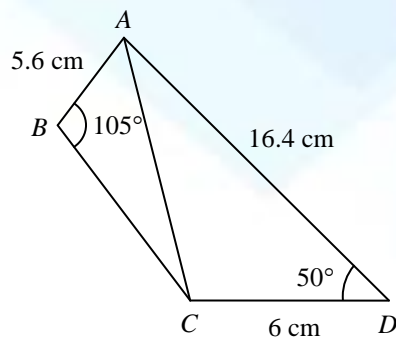
(Ans : 116.54) [3 marks]

(Ans : 35.439) [3 marks]

[2006, No.13]

Answer :

- 14 The diagram shows a quadrilateral $ABCD$.



- (a) Calculate

- (i) the length, in cm, of AC ,
 (ii) $\angle ACB$.

(Ans : 13.359)

(Ans : 23.89)
 [4 marks]

- (b) Point A' lies on AC such that $A'B = AB$.

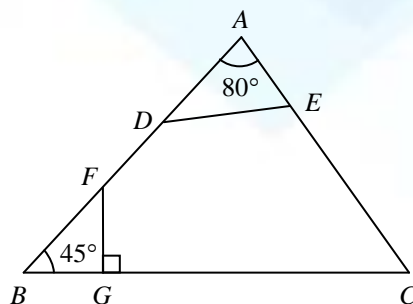
- (i) Sketch $\triangle A'BC$.
 (ii) Calculate the area, in cm^2 , of $\triangle A'BC$.

(Ans : 13.785)
 [6 marks]

[2007, No.15]

Answer :

- 15 In the diagram, ABC is a triangle. $ADFB$, AEC and BGC are straight lines. The straight line FG is perpendicular to BC .

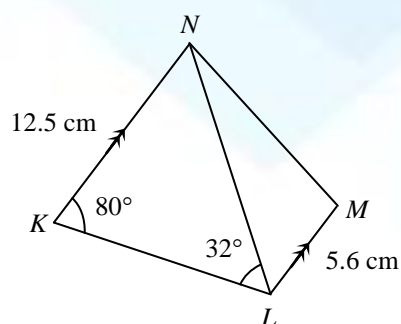


It is given that $BD = 19$ cm, $DA = 16$ cm, $AE = 14$ cm, $\angle DAE = 80^\circ$ and $\angle FBG = 45^\circ$

- (a) Calculate the length, in cm, of
- DE , (Ans : 19.34)
 - EC . (Ans : 16.21)
- [5 marks]
- (b) The area of triangle DAE is twice the area of triangle FBG . Calculate the length, in cm, of BG . (Ans : 10.502) [4 marks]
- (c) Sketch triangle $A'B'C'$ which has a different shape from triangle ABC such that $A'B' = AB$, $A'C' = AC$ and $\angle A'B'C' = \angle ABC$. [1 mark]
- [2008, No.14]**

Answer :

- 16 The diagram shows a trapezium $KLMN$. KN is parallel to LM and $\angle LMN$ is obtuse.



Find

- the length, in cm, of LN ,
- the length, in cm, of MN ,
- $\angle LMN$,
- the area, in cm^2 , of triangle LMN .

(Ans : 23.23) [2 marks]

(Ans : 21.76) [3 marks]

(Ans : 98.20) [3 marks]

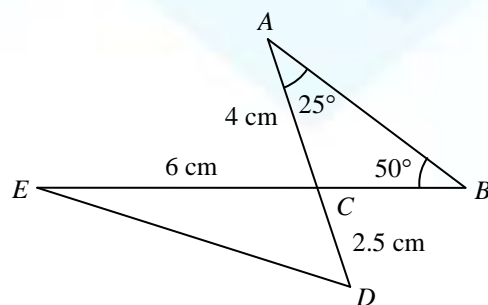
(Ans : 60.31) [2 marks]

[2009, No.12]

Answer :

17 Solutions by scale drawing will **not** be accepted.

The diagram shows triangle ABC and triangle CDE where BCE and ACD are straight lines.



(a) Calculate the length, in cm, of

(i) BC ,

(Ans : 2.207)

(ii) DE ,

(Ans : 7.072)

[5 marks]

(b) Point C' lies on BE such that $AC' = AC$.

(i) Sketch triangle $AC'B$.

(ii) Find $\angle AC'B$.

(Ans : 75°)

(iii) Calculate the area, in cm^2 , of triangle $AC'B$.

(Ans : 8.264)

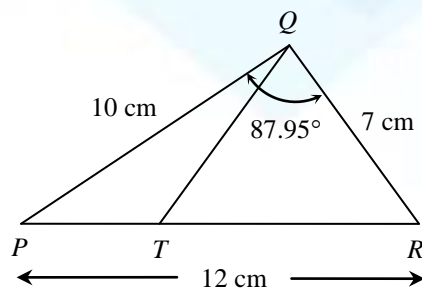
[5 marks]

[2010, No.13]

Answer :

18 Solutions by scale drawing will **not** be accepted.

The diagram shows $\triangle PQR$ and $\triangle TQR$.



It is given that $\angle PQR = 87.95^\circ$, $PQ = 10$ cm, $PR = 12$ cm and $TQ = QR = 7$ cm.

(a) Find

- $\angle PRQ$,
- the length, in cm, of TR ,
- the area, in cm^2 , of $\triangle PQT$.

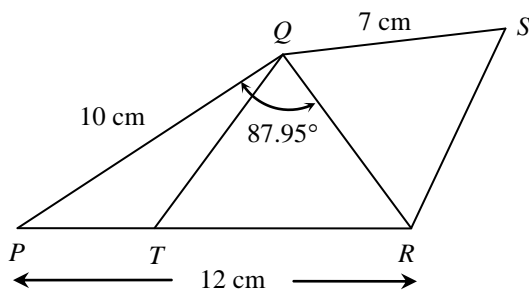
(Ans : 56.39°)

(Ans : 7.750)

(Ans : 12.39)

[7 marks]

(b) In the diagram, $\triangle SQR$ is the image of $\triangle TQR$ under the reflection in the QR .



Find the length, in cm, of PS .

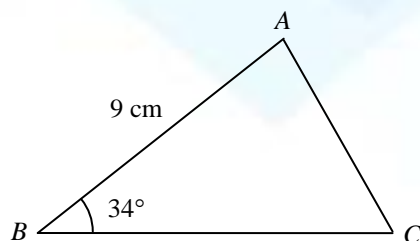
(Ans : 16.62) [3 marks]

[2011, No.14]

Answer :

19 Solutions by scale drawing will **not** be accepted.

The diagram shows triangle ABC such that $\angle ABC = 34^\circ$ and $AB = 9$ cm.



It is given that the area of triangle ABC is 28 cm^2 .

(a) Calculate

(i) the length, in cm, of BC ,

(Ans : 11.13) [2 marks]

(ii) the length, in cm, of AC ,

(Ans : 6.228) [2 marks]

(iii) $\angle ACB$.

(Ans : 53.91°) [2 marks]

(b) Point C' lies on BC such that $AC' = AC$.

(i) Sketch the triangle ABC' .

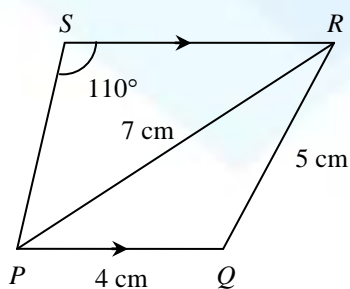
(ii) Calculate the area, in cm^2 , of the triangle ABC' .

(Ans : 9.544)
[4 marks]

[2012, No.14]

Answer :

- 20 The diagram shows trapezium $PQRS$.



- (a) Calculate

- (i) $\angle QPR$,
 (ii) the length, in cm, of PS .

(Ans : 44.42°)

(Ans : 5.214)
 [5 marks]

- (b) The straight line PQ is extended to Q' such that $QR = Q'R$.

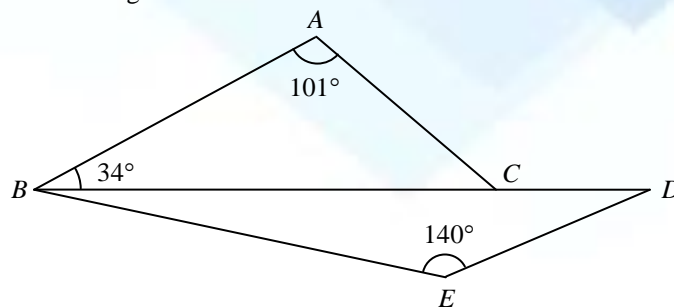
- (i) Sketch the trapezium $PQ'RS$,
 (ii) Calculate the area, in cm^2 , of $\triangle QQ'R$.

(Ans : 4.900)
 [5 marks]

[2013, No.13]

Answer :

- 21 The diagram shows two triangles ABC and BDE .



It is given that $BE = 8.5$ cm, $DE = 4.6$ cm and $AC = 5.8$ cm.

- (a) Calculate

- (i) the length, in cm, of BC ,
- (ii) the length, in cm, of CD ,
- (iii) the area, in cm^2 , of $\triangle ABC$.

(Ans : 10.18)

(Ans : 2.2)

(Ans : 20.88)

[8 marks]

- (b) (i) Sketch a $\triangle A'B'C'$ which has a different shape from $\triangle ABC$ such that $A'B' = AB$, $A'C' = AC$ and $\angle A'B'C' = \angle ABC$.

- (ii) Hence, state the size of $\angle B'A'C'$.

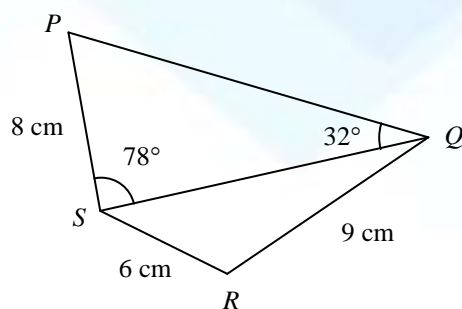
(Ans : 11)

[2 marks]

[2014, No.13]

Answer :

- 22 The diagram shows a quadrilateral $PQRS$.



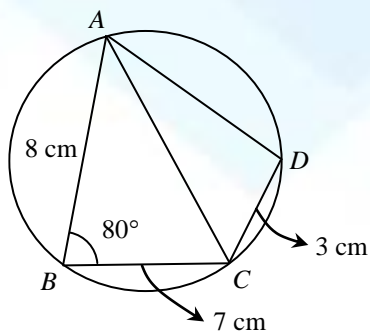
- (a) Find
- the length, in cm, of QS , (Ans : 14.19)
 - $\angle QRS$, (Ans : 141.36)
 - the area, in cm^2 , of the quadrilateral $PQRS$. (Ans : 72.38)
- [8 marks]
- (b) (i) Sketch a triangle $S'Q'R'$ which has a different shape from triangle SQR such that $S'R' = SR$, $S'Q' = SQ$ and $\angle S'Q'R' = \angle SQR$.
- (ii) Hence, state $\angle S'R'Q'$. (Ans : 38.64)

[2 marks]

[2015, No.14]

Answer :

- 23 The diagram shows a cyclic quadrilateral $ABCD$.



- (a) Calculate

- (i) the length, in cm, of AC ,
 (ii) $\angle ACD$,

(Ans : 9.672)

(Ans : 62.21)
 [6 marks]

- (b) Find

- (i) the area, in cm^2 , of $\triangle ABC$.
 (ii) the shortest distance, in cm, from point B to AC .

(Ans : 27.57)

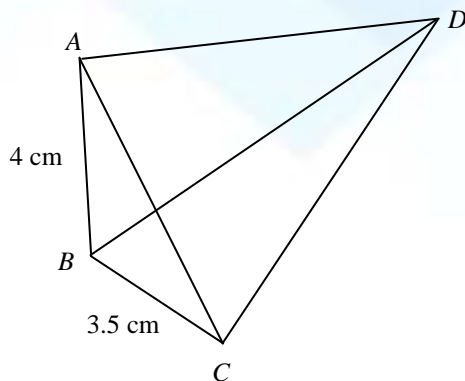
(Ans : 5.701)
 [4 marks]

[2016, No.15]

Answer :

24 Solution by scale drawing is **not** accepted.

The diagram shows a quadrilateral $ABCD$ such that AC and BD are straight lines.



It is given that the area of $\triangle ABC = 6 \text{ cm}^2$ and $\angle ABC$ is obtuse.

(a) Find

(i) $\angle ABC$,

(Ans : 121)

(ii) the length, in cm, of AC ,

(Ans : 6.532)

(iii) $\angle BAC$,

(Ans : 27.34)

[7 marks]

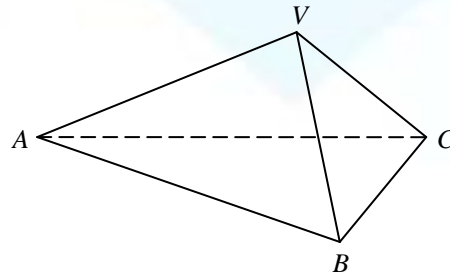
(b) Given $BD = 7.3 \text{ cm}$ and $\angle BCD = 90^\circ$, calculate the area, in cm^2 , of $\triangle ACD$.
(Ans : 17.808) [3 marks]

[2019, No.13]

Answer :

⇒ **Part C ~ 2 dimensional and 3 dimensional surfaces**

- 25 The diagram shows a tent $VABC$ in the shape of a pyramid with triangle ABC as the horizontal base. V is the vertex of the tent and the angle between the inclined plane VBC and the base is 50° .



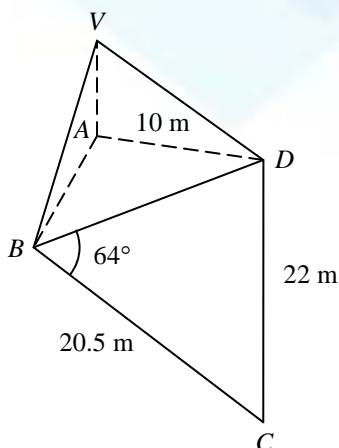
Given that $VB = VC = 2.2$ m and $AB = AC = 2.6$ m. Calculate

- the length of BC if the area of the base is 3 m^2 ,
(Ans : 2.700) [3 marks]
- the length of AV if the angle between AV and the base is 25° ,
(Ans : 3.149) [3 marks]
- the area of triangle VAB .
(Ans : 2.829) [4 marks]
[2003, No.15]

Answer :

26 Solutions by scale drawing is not accepted.

The diagram shows a quadrilateral $ABCD$ on a horizontal plane.



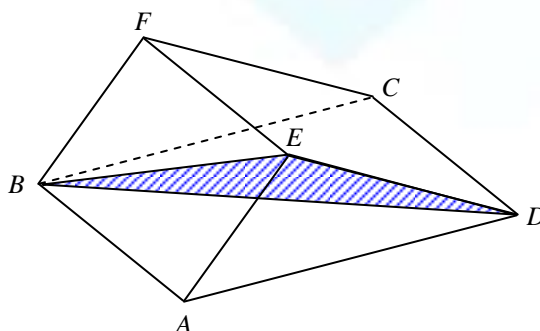
$VBDA$ is a pyramid such that $AB = 12$ m and V is 5 m vertically above A . Find

- | | |
|---|-------------------------|
| (a) $\angle BDC$, | (Ans : 56.88) [2 marks] |
| (b) the length, in cm, of BD . | (Ans : 21.01) [3 marks] |
| (c) the area, in m^2 , of inclined plane BVD . | (Ans : 62.64) [5 marks] |
- [2017, No.15]**

Answer :

27 *Solution by scale drawing is not accepted.*

The diagram shows a transparent prism with a rectangular base $ABCD$. The inclined surface $ABFE$ is a square with sides 12 cm and the inclined surface $CDEF$ is a rectangle. AED is a uniform cross section of the prism. BDE is a shaded plane in the prism.



It is given that $\angle ADE = 37^\circ$ and $\angle EAD = 45^\circ$. Find

- the length, in cm, of DE ,
- the area, in cm^2 , of the shaded plane,
- the shortest length, in cm, from point E to the straight line BD .

(Ans : 14.099) [2 marks]

(Ans : 119.06) [6 marks]

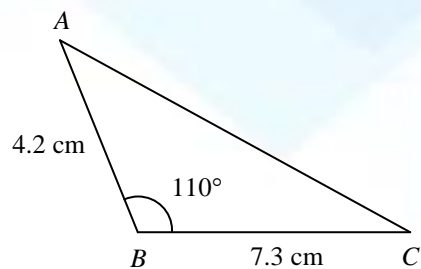
(Ans : 10.204) [2 marks]

[2018, No.14]

Answer :

FORECAST

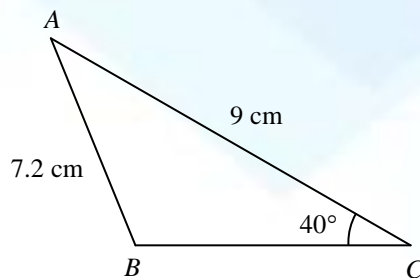
- 28 The diagram shows a triangle ABC .



- (a) Calculate the length, in cm, of AC . (Ans : 9.587) [2 marks]
- (b) If the length of AB is extended to D such that angle $ADC = 48^\circ$, find the length, in cm, of BD . (Ans : 8.673) [3 marks]
- (c) Find the area, in cm^2 , of triangle ADC . (Ans : 44.16) [3 marks]
- (d) Find the shortest distance, in, from C to AD . (Ans : 6.860) [2 marks]

Answer :

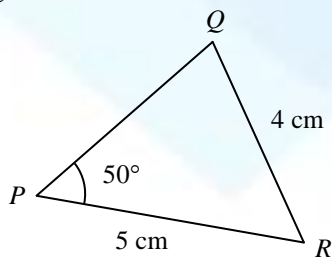
- 29 The diagram shows a triangle ABC .



- (a) Calculate obtuse angle ABC . (Ans : 126.54°) [3 marks]
- (b) Sketch and label another triangle which is different from triangle ABC , such that the lengths of AB and AC , and the $\angle ACB$ are maintained.
Hence, find the area, in cm^2 , of $\triangle AB'C$. (Ans : 32.34) [4 marks]
- (c) If the length of AB is reduced while the length of AC and $\angle ACB$ are maintained, so that only one triangle $AB'C$ can be formed.
- (i) Sketch the triangle $AB'C$. [1 mark]
- (ii) Calculate the length, in cm, of AB' . (Ans : 5.785) [2 marks]

Answer :

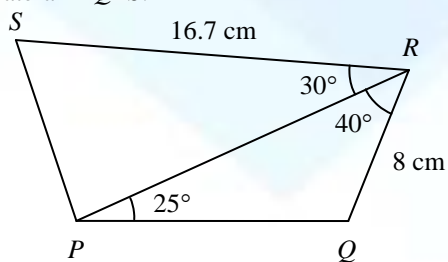
- 30 The diagram shows a triangle PQR .



- (a) Calculate $\angle PRQ$. (Ans : 56.75°) [3 marks]
- (b) Sketch and label another triangle that is different from triangle PQR in the diagram, such that the lengths of PR and RQ , and the angle RPQ remain the same.
Hence, using the cosine rule, find the length of PQ' . (Ans : 2.061) [4 marks]
- (c) If the length of RQ is shortened while the length of PR and $\angle RPQ$ are maintained, such that only one triangle $PQ'R$ can be formed
- Sketch the triangle $PQ'R$. [1 mark]
 - Find the area, in cm^2 , of the new triangle formed. (Ans : 6.155) [2 marks]

Answer :

- 31 The diagram shows a quadrilateral $PQRS$.



- (a) Calculate the length, in cm, of

(i) PR ,

(Ans : 17.16)

(ii) PS ,

(Ans : 8.775)

(iii) $\angle RPS$,

(Ans : 72.09)

[7 marks]

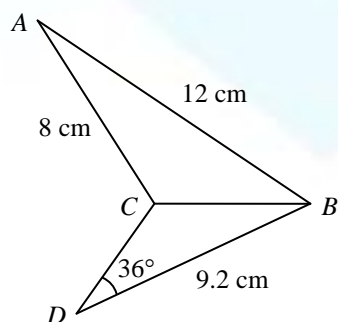
- (b) Point P' lies on PR such that $PS = P'S$.

Calculate the area, in cm^2 , of triangle $PP'S$.

(Ans : 22.53) [3 marks]

Answer :

- 32 The diagram shows two triangles ABC and BCD , where $\angle ACB$ and $\angle BCD$ are obtuse.

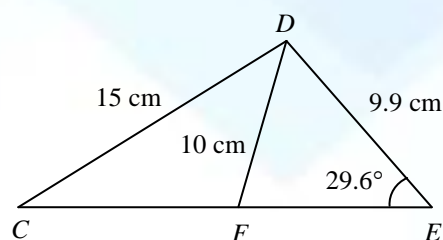


If the area of triangle ABC is 20 cm², calculate

- | | |
|----------------------------------|-------------------------|
| (a) $\angle BAC$, | (Ans : 24.62) [2marks] |
| (b) the length, in cm, of BC , | (Ans : 5.784) [2 marks] |
| (c) the length, in cm, of AD . | (Ans : 12.14) [6 marks] |

Answer :

- 33 (a) In the diagram, CFE is a straight line.

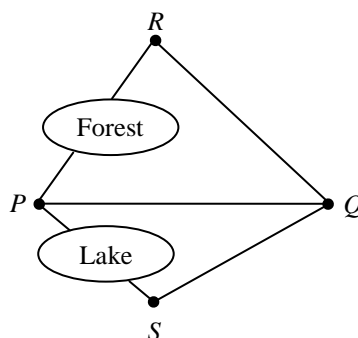


Calculate the length, in cm, of CFE .

(Ans : 22.79)

[4 marks]

- (b) In the diagram, P , Q , R , and S are four points on a horizontal ground. A surveyor want to measure the distance PR and PS . The surveyor knows that $PQ = 3.2$ km.



- (i) The distance PS cannot be measure directly because there is a lake between P and S . By measure, the surveyor found that $\angle QPS = 73^\circ 30'$ and $\angle PQS = 44^\circ 30'$.

Find the distance, in km, PS .

(Ans : 2.540) [3 marks]

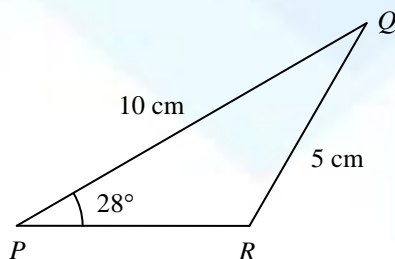
- (ii) The distance PR also cannot be measure directly because there is a forest between R and P . By measure, the surveyor found that $RQ = 4.4$ km and $\angle PQR = 62^\circ 20'$.

Find the distance, in km, PR .

(Ans : 4.065) [3 marks]

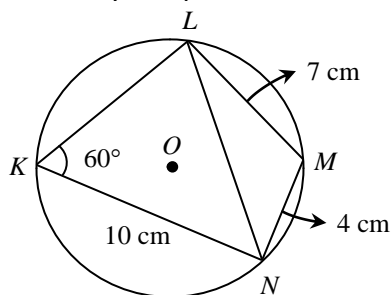
Answer :

- 34 (a) The diagram shows a triangle PQR . PR is horizontal.



Calculate

- the bearing of Q from R , (Ans : 020.13°)
 - the area of the new triangle if PR is extended, while the lengths of PQ and QR , and $\angle QPR$ are maintained. (Ans : 24.76)
- [5 markah]
- (b) In the diagram, $KLMN$ is a cyclic quadrilateral of a circle with centre O .

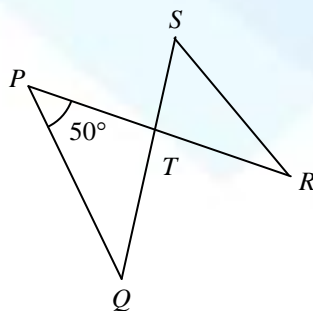


Calculate

- the length, in cm, of LN , correct to two decimal places, (Ans : 9.64)
 - $\angle KNL$. (Ans : 56.06)
- [5 marks]

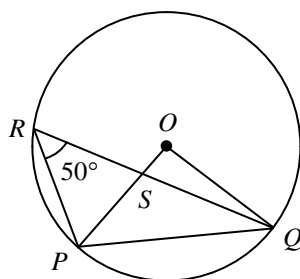
Answer :

- 35 (a) In the diagram, PTR and QTS are straight lines.



Given that $RS = 6$ cm, $RT = 4$ cm, $TS = 3$ cm, and $PQ = 14$ cm, calculate

- (i) $\angle RTS$, (Ans : 117.28)
 - (ii) the length, in cm, of QT . (Ans : 12.07)
- [4 marks]
- (b) The diagram shows a circle with centre O and a radius of 6 cm.

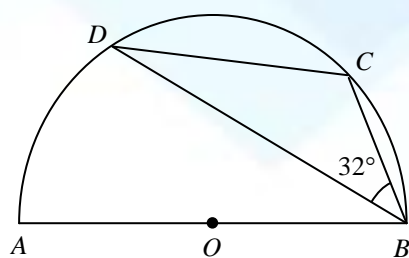


Given that $\angle PRQ = 50^\circ$ and $PR = 5$ cm, calculate

- (i) the length, in cm, of PQ , (Ans : 9.193)
 - (ii) the area, in cm^2 , of triangle PSQ . (Ans : 12.52)
- [6 marks]

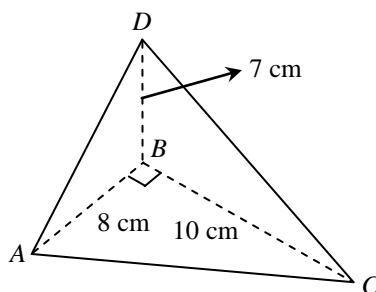
Answer :

- 36 (a) In the diagram, $ABCD$ is a semi circle with centre O and radius 10 cm.



Given that $BC = 4$ cm and $\angle CBD = 32^\circ$. Calculate

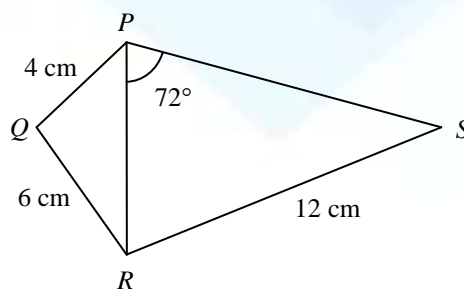
- (i) the length, in cm, of CD , (Ans : 10.596)
 (ii) the length, in cm, of AD . (Ans : 14.501)
 [6 marks]
- (b) The diagram shows a pyramid with a horizontal triangular base ABC .



Given $AB = 8$ cm, $BC = 10$ cm and $\angle ABC = 90^\circ$. Vertex D is 7 cm vertically above B . Calculate the area, in cm^2 , of the slanting surface ADC . (Ans : 60.07)
 [4 marks]

Answer :

- 37 (a) In the diagram, $\sin \angle PQR = \frac{12}{13}$, where $\angle PQR$ is obtuse.



Calculate

- (i) the length, in cm, of PR ,

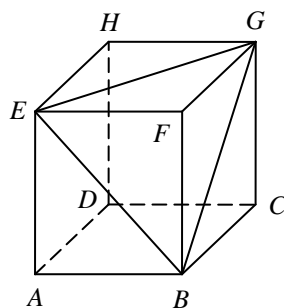
(Ans : 8.39)

- (ii) $\angle PSR$.

(Ans : 41.70)

[5 marks]

- (b) The diagram shows a cuboid with a square base of 4 cm, and a height of 6 cm.



Calculate

- (i) $\angle EGB$,

(Ans : 66.91)

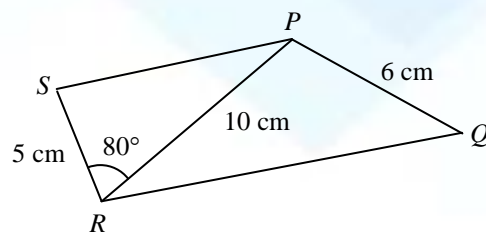
- (ii) the area, in cm^2 , of triangle EGB .

(Ans : 18.76)

[5 marks]

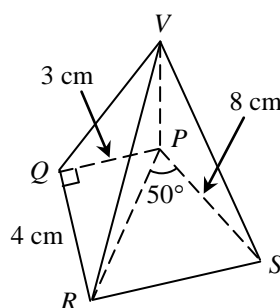
Answer :

- 38 (a) In the diagram, $\cos \angle QPR = -\frac{3}{5}$.



Calculate

- (i) the length, in cm, of RQ , (Ans : 14.422)
 (ii) $\angle PSR$. (Ans : 71.66)
[5 marks]
- (b) The diagram shows a pyramid with a horizontal quadrilateral base.

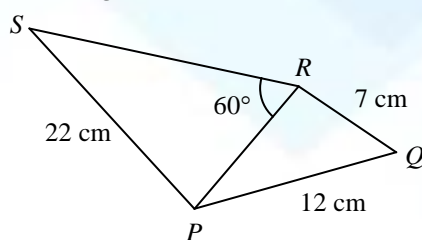


The apex V is 12 cm vertically above P and $\angle RPS$ is 50° . Calculate

- (i) the length, in cm, of RS , (Ans : 6.130)
 (ii) the area, in cm^2 , of triangle VRS . (Ans : 39.84)
[5 marks]

Answer :

- 39 (a) In the diagram, $\sin \angle PQR = \frac{3}{5}$, where $\angle SPR$ is obtuse.

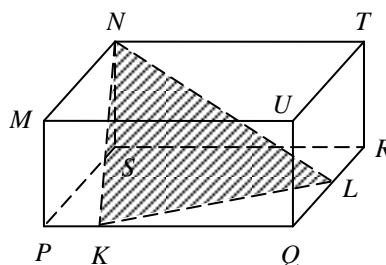


Calculate

- (i) the length of PR , correct to three significant figures, (Ans : 7.66)
 (ii) $\angle SPR$. (Ans : 102.45)

[5 marks]

- (b) The diagram shows a cuboid, where $PQ = 8$ cm, $QR = 6$ cm and $MP = 4$ cm. L is the midpoint of QR and $PK : KQ = 1 : 3$.

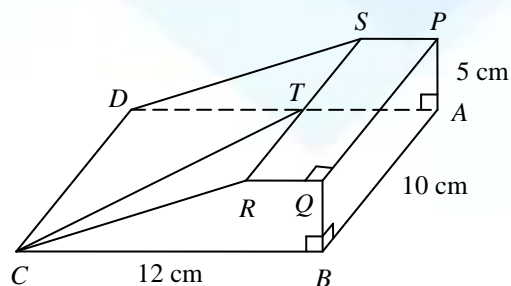


Calculate

- (i) the length, in cm, of NL , (Ans : 9.434)
 (ii) the area, in cm^2 , of triangle KNL . (Ans : 24.92)
 [5 marks]

Answer :

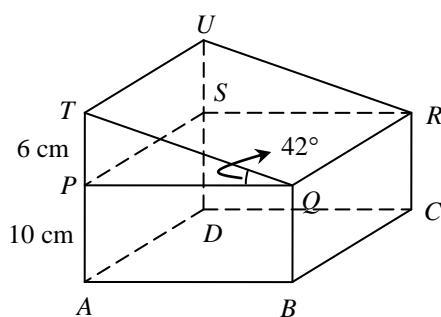
- 40 (a) The diagram shows a right prism with a horizontal rectangular base $ABCD$. Trapezium $BCRQ$ is the uniform cross section. T is the midpoint of RS .



If $RQ = \frac{1}{3}BC$, calculate the angle between the line CT and the base $ABCD$. (Ans : 27.92)

[2 marks]

- (b) The diagram shows a solid formed by a right prism and a cuboid. The rectangular surface, $QRUT$, is inclined. The angle between the line TR and the plane $PQRS$ is 30° .

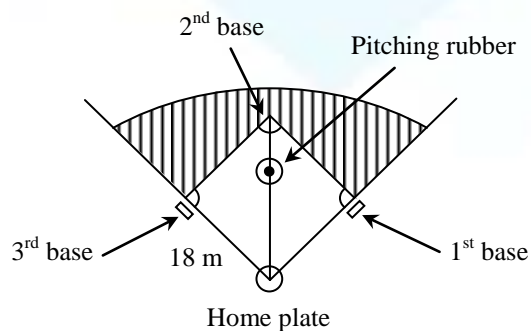


Calculate

- (i) the lengths, in cm, of PQ , TR , and AR , (Ans : 6.664, 12, 14.42) [4 marks]
 (ii) the area, in cm^2 , of triangle TAR . (Ans : 83.13) [4 marks]

Answer :

- 41 (a) The diagram shows a baseball diamond playing field in the shape of a square of side 18 m. The pitching rubber is located 13.8 m from the home plate on a line joining home plate and second base.

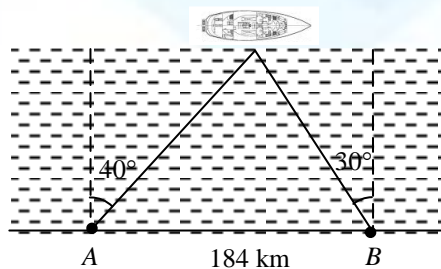


Calculate the distance, in m, from the pitching rubber to

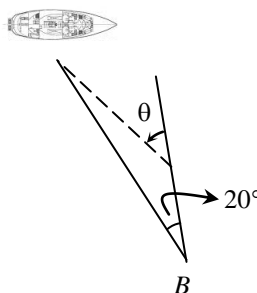
- (i) second base, (Ans : 11.66) [2 marks]
 (ii) third base. (Ans : 12.77) [2 markah]
- (b) In a triangle DEF , $DE = 8$ cm, $DF = 10$ cm, and $\angle DFE = 50^\circ$. Sketch the two possible triangles DEF . Hence, find the probable values of the length of EF . (Ans : 4.122, 8.734) [6 markah]

Answer :

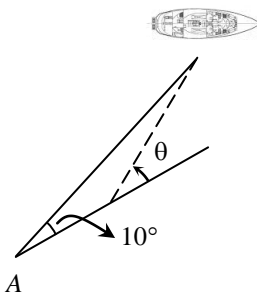
- 42 Coast guard station A is located 184 km due west of station B . A cruiser at sea sends an emergency call that is received by each station. The call to station A indicates that the location of the cruiser is 40° east of north and the call to the station B indicates that the location of the cruiser is 30° west of north.



- (a) How far is each station from the cruiser? (Ans : 150, 169.58) [3 marks]
- (b) A speedboat capable of speeding 100 km per hour is despatched from the station B to the cruiser. After travelling for $\frac{1}{2}$ an hour, the speedboat encounters heavy crosswinds and strong currents from the west. The crew finds that the speedboat is off course by 20° .



- (i) How far is the speedboat from the cruiser? (Ans : 104.43) [2 marks]
- (ii) Through what angle should the speedboat turn to correct its course? (Ans : 29.44°) [2 marks]
- (iii) How much time has been added to the trip because of this problem? (Ans : 2.658) [1 mark]
- (c) Since the cruiser needs assistance badly, a helicopter which is capable flying 200 km per hour is despatched from station A . After 15 minutes journey, the pilot discovered that he was 10° off course.



What is the average speed, in kmh^{-1} , should the pilot maintain, so that the total time to reach the cruiser is not more than 30 minutes. (Ans : 241.3) [2 marks]

Answer :

CONTINUOUS EXERCISES

- 43** Given that the lengths of the two sides of a parallelogram are 68.2 cm and 83.3 cm. One of its diagonal has a length of 42.5 cm. Calculate
- (a) the angles of the parallelogram, (Ans : 30.56, 149.44)
 - (b) the length, in cm, of another diagonal, (Ans : 146.20)
 - (c) the area, in cm^2 , of the parallelogram. (Ans : 2888.48)
- [7 marks]
- Answer :*

- 44** In a parallelogram, the adjacent angles of a diagonal of length 76.33 cm are 52.2° and 41.45° respectively. Find
- (a) the length, in cm, of the sides of the parallelogram, (Ans : 50.63, 60.44)
 - (b) the area, in cm^2 , of the parallelogram. (Ans : 3053.62)
- [7 marks]
- Answer :*

- 45** The angles of a triangle are in the ratio 5 : 10 : 21, and the shortest side is 35.64 cm. Find
- (a) the length, in cm, of the longest side, (Ans : 81.46)
 - (b) the area, in cm^2 , of the triangle. (Ans : 1112.00)
- [5 marks]

Answer :

- 46** The perimeter of $\triangle PQR$ is 40 cm and $\angle P : \angle Q : \angle R = 1 : 2 : 6$. Find
- (a) the length, in cm, of the sides p , q and r . (Ans : 7.392, 13.890, 18.717)
 - (b) the area, in cm^2 , of the $\triangle PQR$. (Ans : 44.459)
- [10 marks]

Answer :

- 47** The length of the sides of a triangle are $x - 2$, $2x + 7$ and $2x + 8$. Given the perimeter of the triangle is 63 cm.
- (a) the value of x , (Ans : 10)
 - (b) the area, in cm^2 , of the triangle. (Ans : 107.99)
- [5 marks]

Answer :

- 48** Darren want to prepare a greeting card in the shape of a triangle. The lengths of two sides of the triangle are 8 cm and 11 cm respectively, and the area of the is 30 cm^2 .

(a) Sketch the two possible triangles.

(b) Hence, find the possible lengths, in cm, of the third side.

(Ans : 7.501, 17.713)

[7 marks]

Answer :

- 49** A regular pentagon has sides of 5 cm. Find

(a) the length, in cm, of its diagonal.

(Ans : 8.090)

(b) the area, in cm^2 , of the regular pentagon.

(Ans : 43.009)

[7 marks]

Answer :

- 50** If the three sides of triangle are a , b and $\sqrt{a^2 + b^2 + ab}$.

Find the greater angle of this triangle.

(Ans : 120)

[3 marks]

Answer :

- 51 Given that the ratio of the sides of a triangle is $7 : 4\sqrt{3} : 13$, find its smallest angle. (Ans : 20.92)
[3 marks]

Answer :

- 52 Given $\triangle PQR$ in which $\sin P : \sin Q : \sin R = 3 : 5 : 7$. Find its largest angle. (Ans : 120)
[3 marks]

Answer :

- 53 Given $\triangle ABC$ in which $(b + c) : (c + a) : (a + b) = 7 : 8 : 9$. Find $\sin A : \sin B : \sin C$. (Ans : 5 : 4 : 3)
[3 marks]

Answer :

- 54 Given $\triangle ABC$ in which $(b + c) : (c + a) : (a + b) = 4 : 5 : 6$. Find $\angle A$. (Ans : 120)
[3 marks]

Answer :

INDEX NUMBERS

- ONE PAGE NOTE (OPN)

- WORKSHEET

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ONE PAGE NOTES [Coding Method]

" INDEX NUMBERS "

① " Write information in Mathematical Representation & Vice-versa " ②

- The price of an item in the year 2009 is RM 20
 $\Rightarrow Q_{09} = 20$
- The price index of an item in the year 2009 based on the year 2008 is 120
 $\Rightarrow I_{09,08} = 120$
- The price of an item has increased by 20 % from the year 2008 to the year 2009
 $\Rightarrow I_{09,08} = 120$
- The price of an item has decreased by 20 % from the year 2008 to the year 2009
 $\Rightarrow I_{09,08} = 80$
- The price of an item is unchanged from the year 2008 to the year 2009
 $\Rightarrow I_{09,08} = 100$

- h increased by 5 %
 $\Rightarrow h \times 105 \%$
 $\Rightarrow h \times 1.05$
 - k decreased by 25 %
 $\Rightarrow k \times 75 \%$
 $\Rightarrow k \times 0.75$
- " NOTES "

" Find Price "

- ① Given : $I_{b,a} = k$ & Q_a @ $Q_b \Rightarrow \frac{Q_b}{Q_a} \times 100 = k$ @ $\frac{Q_b}{Q_a} = \frac{k}{100}$
- ② Given : $I_{a,c} = k$ & Q_a @ $Q_b \Rightarrow \frac{I_{a,c}}{I_{b,c}} = \frac{Q_a}{Q_b}$ @ $\frac{I_{a,c}}{I_{b,c}} = \frac{Q_a}{Q_b}$
- ③ Given : $I_{c,a} = k$ & Q_a @ $Q_b \Rightarrow \frac{I_{c,a}}{I_{c,b}} = \frac{Q_a}{Q_b}$ @ $\frac{I_{c,a}}{I_{c,b}} = \frac{Q_a}{Q_b}$
- ④ Given : $I_{b,c} = k$ & Q_a @ $Q_b \Rightarrow \frac{I_{b,c}}{I_{c,a}} \times \frac{I_{c,a}}{100} = \frac{Q_b}{Q_a}$ @ $\frac{I_{b,c}}{I_{c,a}} \times \frac{Q_b}{100} = \frac{Q_a}{Q_b}$ continuously

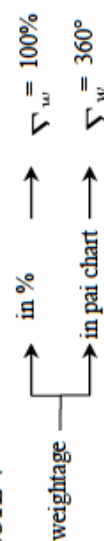
" Find Index "

- ① Given : Q_a & $Q_b \Rightarrow I_{b,a} = \frac{Q_b}{Q_a} \times 100$
- ② Given : $I_{b,a} \Rightarrow I_{a,b} = \frac{100}{I_{b,a}} \times 100$
- ③ Given : $I_{a,c}$ & $I_{b,c} \Rightarrow I_{b,a} = \frac{I_{b,c}}{I_{a,c}} \times 100$
- ④ Given : $I_{c,a}$ & $I_{c,b} \Rightarrow I_{b,a} = \frac{I_{c,a}}{I_{c,b}} \times 100$
- ⑤ Given : $I_{b,c}$ & $I_{c,a} \Rightarrow I_{b,a} = \frac{I_{b,c} \times I_{c,a}}{100}$ ~ case changes continuously
- ↳ Given : $I_{b,a}$ & the price / index increased by $k\%$ from year b to c
 $\Rightarrow I_{c,a} = h \times (100 + k) \%$ ④a
- ↳ Given : $I_{b,a}$ & the price / index increased at the same rate from year b to c
 $\Rightarrow I_{c,a} = h \times h \%$ ④b

" Find Composite Index "

- Given : $I_{b,a}$ @ Q_a & Q_b & weightage (w)
 $\Rightarrow \bar{I}_{b,a} = \frac{\sum I_{b,a} w}{\sum w}$ ① ② ③
- ↳ weightage given in the form :
 \rightarrow histogram, pai chart, ratio, %
 \rightarrow weightage is not given
 \rightarrow assuming the value of the weightage are the same for each index number (1 : 1 : 1 : ...)
- find corresponding price. ①

NOTE :



WORKSHEET
TOPIC 10 : INDEX NUMBERS
[Part C → 10 marks]

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10.1 Index numbers

10.1.1 Define index numbers and describe the use of it.

10.1.2 Determine and interpret index numbers.

10.1.3 Solve problems involving index numbers.

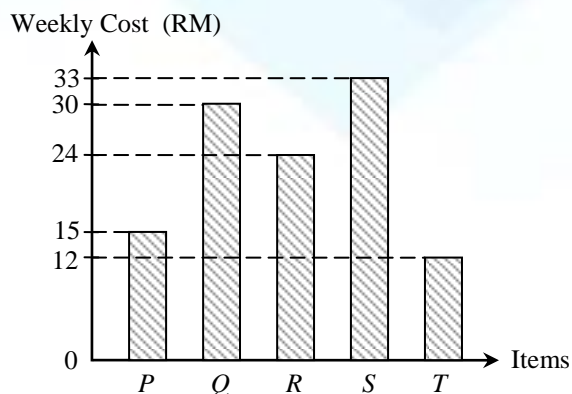
10.2 Composite index

10.2.1 Determine and interpret composite index with and without the weightage.

10.2.2 Solve problems involving index numbers and composite index.

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- 1 The diagram is a bar chart indicating the weekly cost of the items P , Q , R , S , and T for the year 1990. Table below show the prices and the price indices for the item.



Items	Price in 1990	Price in 1995	Price index in 1995 based on 1990
P	x	RM 0.70	175
Q	RM 2.00	RM 2.50	125
R	RM 4.00	RM 5.50	y
S	RM 6.00	RM 9.90	150
T	RM 2.50	z	120

- (a) Find the value of x , y , and z . (Ans : $x = 0.40$, $y = 137.5$, $z = 3.00$) [3 marks]
- (b) Calculate the composite index for the items in the year 1995 based on the year 1990. (Ans : 140.92) [2 marks]
- (c) The total monthly cost of the items in the year 1990 is RM456. Calculate the corresponding total monthly cost for the year 1995. (Ans : RM 642.60) [2 marks]
- (d) The cost of the items increase by 20% from the year 1995 to the year 2000. Find the composite index for the year 2000 based on the year 1990. (Ans : 169.104) [3 marks]

[2003, No.13]

Answer :

- 2 The table shows the price indices and percentage of usage of four items, P , Q , R , and S , which are the main ingredients in the production of a type of biscuit.

<i>Item</i>	<i>Price index for the year 1995 based on the year 1993</i>	<i>Percentage of usage (%)</i>
P	135	40
Q	x	30
R	105	10
S	130	20

- (a) Calculate

- (i) the price of S in the year 1993, if its price in the year 1995 is RM37.70. (Ans : 29)
- (ii) the price index of P in the year 1995 based on the year 1991 if its price index in the year 1993 based on the year 1991 is 120. (Ans : 162)
[5 marks]

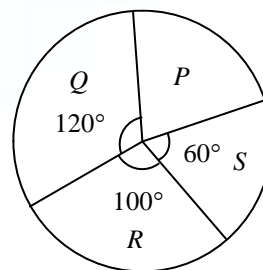
- (b) The composite index number of the cost of biscuit production for the year 1995 based on the year 1993 is 128, calculate

- (i) the value of x , (Ans : 125)
- (ii) the price of a box of biscuit in the year 1993, if the corresponding price in the year 1995 is RM32. (Ans : 25)
[5 marks]
[2004, No.12]

Answer :

- 3 The table shows the price and price indices for the four ingredients, P , Q , R , and S , used in making biscuits of a particular kind. The pie chart represents the relative amount of the ingredients P , Q , R , and S , used in making these biscuits.

<i>Ingredients</i>	<i>Price per kg (RM)</i>		<i>Price index for the year 2004 based on the year 2001</i>
	<i>Year 2001</i>	<i>Year 2004</i>	
P	0.80	1.00	x
Q	2.00	y	140
R	0.40	0.60	150
S	z	0.40	80



- (a) Find the value of x , y and z , (Ans : $x = 125$, $y = 2.80$, $z = 0.50$) [3 marks]
- (b) (i) Calculate the composite index for the cost of making these biscuits in the year 2004 based on the year 2001. (Ans : 129.44)
- (ii) Hence, calculate the corresponding cost of making these biscuits in the year 2001, if the cost in the year 2004 was RM2985. (Ans : 2306.09)
[5 marks]
- (c) The cost of making these biscuits is expected to increase by 50% from the year 2004 to the year 2007. Find the expected composite index for the year 2007 based on the year 2001. (Ans : 194.16)
[2 marks]

[2005, No.13]

Answer :

- 4 A particular kind of cake is made by using four ingredients, P , Q , R and S . The table shows the prices of the ingredients.

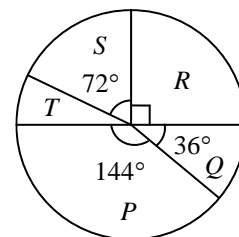
<i>Ingredient</i>	<i>Price per kilogram (RM)</i>	
	<i>Year 2004</i>	<i>Year 2005</i>
P	5.00	w
Q	2.50	4.00
R	x	y
S	4.00	4.40

- (a) The index number of ingredient P in the year 2005 based on the year 2004 is 120. Calculate the value of w .
(Ans : 6) [2 marks]
- (b) The index number of ingredient R in the year 2005 based on the year 2004 is 125. The price per kilogram of ingredient R in the year 2005 is RM 2.00 more than its corresponding price in the year 2004. Calculate the value of x and y .
(Ans : $x = 8$, $y = 10$) [3 marks]
- (c) The composite index for the cost of making the cake in the year 2005 based on the year 2004 is 127.5. Calculate
- (i) the price of a cake in the year 2004, if its corresponding price in the year 2005 is RM30.60,
(Ans : 24)
- (ii) the value of m if the quantities of ingredients P , Q , R and S used are in the ratio of 7 : 3 : m : 2.
(Ans : 4)
[5 marks]
[2006, No.15]

Answer :

- 5 The table shows the prices and the price indices of five components, P , Q , R , S and T , used to produce a kind of toy. The diagram shows a pie chart which represents the relative quantity of components used.

Component	Price (RM) for the year		Price index for the year 2006 based on the year 2004
	2004	2006	
P	1.20	1.50	125
Q	x	2.20	110
R	4.00	6.00	150
S	3.00	2.70	y
T	2.00	2.80	140

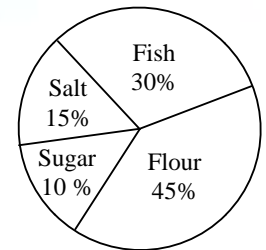


- (a) Find the value of x and of y . (Ans : $x = 2.00$, $y = 90$) [3 marks]
- (b) Calculate the composite index for the production cost of the toys in the year 2006 based on the year 2004. (Ans : 123.5) [3 marks]
- (c) The price of each component increase by 20% from the year 2006 to the year 2008. Given that the production cost of one toy in the year 2004 is RM55, calculate the corresponding cost in the year 2008. (Ans : 81.51) [4 marks]
- [2007, No.13]

Answer :

- 6 The table shows the prices and the price indices of four ingredients, fish, flour salt, and sugar, used to make a type of fish cracker. The diagram shows a pie chart which represents the relative quantity of the ingredient used.

Ingredients	Price (RM) per kg for the year		Price index for the year 2005 based on the year 2004
	2004	2005	
Fish	3.00	4.50	150
Flour	1.50	1.80	h
Salt	k	0.90	112.5
Sugar	1.40	1.47	105



- (a) Find the value of h and of k . (Ans : $h = 120$, $k = 0.80$) [3 marks]
- (b) Calculate the composite index for the cost of making these crackers in the 2005 based on the year 2004. (Ans : 126.375) [3 marks]
- (c) The composite index for the cost of making these crackers increases by 50% from the year 2005 to the year 2009. Calculate
- the composite index for the cost of making these crackers in the year 2009 based on the year 2004, (Ans : 189.5625)
 - the price of a box of these crackers in the year 2009 if its corresponding price in the year 2004 is RM 25. (Ans : 47.39)

[4 marks]

[2008, No.13]

Answer :

- 7 The table shows the prices, the price index and weightages for four types of stationery P , Q , R and S .

Stationery	Price (RM) per unit		Price index for the year 2008 based on the year 2007	Weightage
	Year 2007	Year 2008		
P	2.80	2.10	x	4
Q	4.00	4.80	120	2
R	2.00	y	130	3
S	z	5.80	116	m

- (a) Find the value of

- (i) x , (Ans : 75)
(ii) y , (Ans : 2.6)
(iii) z , (Ans : 5)
[3 marks]

- (b) The composite index for the price of the stationery in the year 2008 based on the year 2007 is 108.4. Calculate the value of m . (Ans : 6) [3 marks]
(c) The total expenditure for the stationery in the year 2007 is RM 525, Calculate the corresponding total expenditure in the year 2008, (Ans : 569.1) [2 marks]
(d) The price index for Q in the year 2009 based on the year 2007 is 132. Calculate the price index for Q in the year 2009 based on the year 2008. (Ans : 110) [2 marks]
[2009, No.13]

Answer :

- 8 The table shows the price indices for three items, P , Q and R used in the production of a type of bag.

<i>Item</i>	<i>Price index in the year 2006 based on the year 2004</i>	<i>Price index in the year 2008 based on the year 2004</i>
P	125	150
Q	116	x
R	y	120

- (a) Find the price index of item P in the year 2008 based on the year 2006.
(Ans : 120) [2 marks]
- (b) The price of item Q in the year 2004 is RM 7.50 and its price in the year 2008 is RM 10.50. Find
(i) the value of x . (Ans : 140)
(ii) the price of item Q in the year 2006. (Ans : 8.70)
[3 marks]
- (c) The composite index for the production cost of the bag in the year 2006 based on the year 2004 is 118.5. The cost of the items, P , Q and R used are in the ratio of 2 : 1 : 3. Find the value of y .
(Ans : 115) [3 marks]
- (d) Given the price of the bag in the year 2006 is RM 47.40, find the corresponding price of the bag in the year 2004.
(Ans : 40) [2 marks]
[2010, No.15]

Answer :

- 9 The table shows the price, price indices and percentage expenditure of four ingredients, P , Q , R and S , used in making of a kind of food.

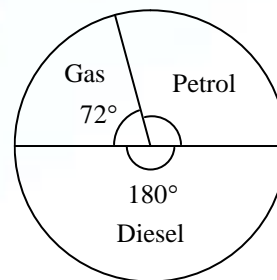
<i>Ingredient</i>	<i>Price (RM) per kg</i>		<i>Price index for the year 2007 based on the year 2005</i>	<i>Percentage expenditure (%)</i>
	2005	2007		
P	4.00	5.00	x	16
Q	3.00	y	150	12
R	8.00	10.00	125	48
S	z	3.00	120	24

- (a) Find the values of x , y and z . (Ans : $x = 125$, $y = 4.50$, $z = 2.50$) [4 marks]
- (b) Calculate the composite index for the cost of making the food in the year 2007 based on the year 2005. (Ans : 126.8) [2 marks]
- (c) The cost of making a packet of the food in the year 2005 was RM 50.00. Calculate the corresponding cost in the year 2007, (Ans : 63.40) [2 marks]
- (d) The cost of all the ingredients increased by 15 % from the year 2007 to the year 2009. Find the composite index for the year 2009 based on the year 2005. (Ans : 145.82) [2 marks]
[2011, No.13]

Answer :

- 10 The table shows the price indices of three types of fuel for the year 2008 based on the year 2006. The diagram shows a pie chart which represents the proportion of the fuel used in a factory.

<i>Fuel</i>	<i>Price index for the year 2008 based on the year 2006</i>
Diesel	150
Petrol	120
Gas	110



- (a) If the factory spends RM 9000 per week for diesel in the year 2008, find the corresponding expenditure for diesel in the year 2006. (Ans : 6000) [2 marks]
- (b) Calculate the composite index for the fuel expenditure of the factory in the year 2008 based on the year 2006. (Ans : 133) [3 marks]
- (c) The fuel expenditure used by the factory is RM 30000 per week in the year 2006. Calculate it corresponding fuel expenditure in the year 2008. (Ans : 39900) [2 marks]
- (d) The price of diesel increases by 30%, the price of petrol increases by 20% while the price of gas remains unchanged from the year 2008 to the year 2010. Calculate the composite index for the fuel expenditure of the factory in the year 2010 based on the year 2006. (Ans : 162.7) [3 marks]
- [2012, No.13]**

Answer :

- 11 The table shows the price indices, changes in price indices and weightages of four item *A*, *B*, *C* and *D*, which are the main items used to make a tin of biscuit.

<i>Item</i>	<i>Price index for the year 2012 based on the year 2010</i>	<i>Changes in price index from the year 2012 to the year 2014</i>	<i>Weightage</i>
<i>A</i>	112	No change	1
<i>B</i>	140	10% decrease	4
<i>C</i>	<i>x</i>	No change	2
<i>D</i>	130	5% increase	3

- (a) Calculate
- the price of item *B* in the year 2010 if its price in the year 2012 is RM8.40, (*Ans* : 6.00)
 - the price of item *D* in the year 2012 if its price in the year 2010 is RM4.50. (*Ans* : 5.85) [3 marks]
- (b) The composite index for the cost of making a tin of biscuit in the year 2012 based on the year 2010 is 132. Calculate the value of *x*. (*Ans* : 129) [2 marks]
- (c) Hence, calculate the composite index for the cost of making a tin of biscuit in the year 2014 based on the year 2010. (*Ans* : 128.35) [3 marks]
- (d) Calculate the cost of making a tin of biscuit in the year 2014 if the corresponding cost in the year 2010 is RM20. (*Ans* : 25.67) [2 marks]
- [2013, No.14]**

Answer :

- 12 The table shows the price indices and the weightages of four ingredients, P , Q , R and S , used in the making of a cake. The composite index for the cost of making the cake in the year 2014 based on the year 2013 is 106.

<i>Ingredient</i>	<i>Price index in the year 2014 based on the year 2013</i>	<i>Weightage</i>
P	115	3
Q	95	1
R	100	4
S	m	2

- (a) Calculate the price of ingredient Q in the year 2014 if its price in the year 2013 is RM 20.
(Ans : 19) [2 marks]
- (b) Find the percentage of price change from the year 2013 to the year 2014 for ingredient S .
(Ans : 10) [4 marks]
- (c) The composite index for the cost of making the cake increased at the same rate from the year 2014 to the year 2015, calculate **
- (i) the composite index for the expenses in the year 2015 based on the year 2013,
(Ans : 112.36) [2 marks]
- (ii) the price of the cake in the year 2015 if its corresponding price in the year 2013 is RM75
(Ans : 84.27) [2 marks]
[2014, No.15]

Answer :

- 13 The table shows the price indices for the year 2013 and 2015 based on the year 2011 of the three materials A , B , and C used in making a type of shoe.

<i>Material</i>	<i>Price index in the year 2013 based on the year 2011</i>	<i>Price index in the year 2015 based on the year 2011</i>
A	106	120
B	105	125
C	110	m

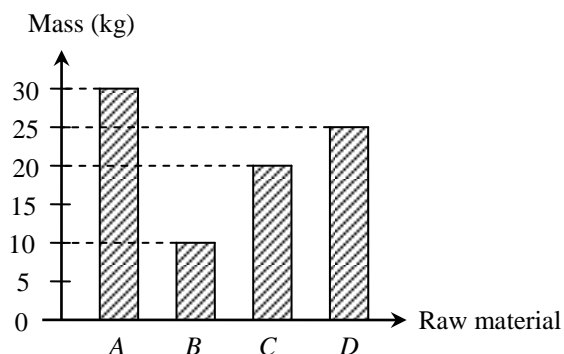
- (a) The price of material C in the year 2011 is RM12.00 and its price in the year 2015 is RM15.60. Find
- the value of m , (Ans : 130)
 - the price of the material C in the year 2013. (Ans : 13.20)
[3 marks]
- (b) The composite index for the production cost of the shoe in the year 2013 based on the year 2011 is 106.7. The ratio of the materials A , B and C used are $2 : h : 3$. Find
- the value of h , (Ans : 5)
 - the corresponding price of the shoe in the year 2011 if the price of the shoe in the year 2013 is RM58.20. (Ans : 54.55)
[5 marks]
- (c) Find the price index of material B in the year 2015 based on the year 2013. (Ans : 119.05) [2 marks]
[2015, No.15]

Answer :

- 14 The table shows the price indices and change in price indices of four raw materials A , B , C and D , used to produce a type of biscuits in a factory.

Raw material	Price index in 2011 based on 2008	Change in price index from 2011 to 2015
A	140	15 % increase
B	120	5 % increase
C	160	Unchange
D	150	10 % decrease

The diagram is a bar chart which represents the mass of the raw materials used to make the biscuits in 2008.



- (a) The price of raw material A in 2011 is RM70. Find the corresponding price in 2008. (Ans : 50) [2 marks]
- (b) Find the price indices of all the four raw materials in 2015 based on 2008. [3 marks]
- (c) (i) Calculate the composite index for the cost of producing the biscuits in 2015 based on 2008. (Ans : 149)
- (ii) Hence, find the cost of producing the biscuits in 2008 if the corresponding cost in 2015 is RM268.20. (Ans : 180)

[5 marks]

[2016, No.12]

Answer :

- 15 The table shows the prices and the price indices of three types of ingredients A , B and C , used in the production of a type of fish ball.

<i>Ingredinet</i>	<i>Price (RM) per kg for the year</i>		<i>Price index for the year 2016 based on the year 2014</i>	<i>Weightage</i>
	<i>2014</i>	<i>2016</i>		
A	5.00	6.64	132.8	50
B	y	3.00	x	20
C	0.50	0.95	190	1

- (a) The price of ingredinet B is increased by 20% from the year 2014 to the year 2016.
- State the value of x .
 - Find the value of y . (Ans : 2.50)
[3 marks]
- (b) Calculate the composite index for the cost of making the fish balls for the year 2016 based on the year 2014. (Ans : 130) [2 marks]
- (c) It is given that the composite index for the cost of making the fish balls increased by 40% from the year 2012 to the year 2016.
- Calculate the composite index for the cost of making the fish balls in the year 2014 baded on the year 2012. (Ans : $107\frac{9}{13}$)
 - The cost of making a fish ball is 10 sen in the year 2012. Find the maximum number of fish balls that can be produced using an allocation of RM80 in the year 2016. (Ans : 571)
[5 marks]
[2017, No.13]

Answer :

- 16 The table shows the information related to four ingredients, P , Q , R and S , used in the production of a type of noodle.

<i>Ingredient</i>	<i>Change in price from the year 2013 to the year 2017</i>	<i>Percentage of usage (%)</i>
P	40% increase	10
Q	20% increase	10
R	60% increase	
S	10% decrease	50

The production cost for this noodle is RM47600 in the year 2017

- (a) If the price of ingredient Q in the year 2013 is RM4.20, find its price in the year 2017.
(Ans : 5.04) [2 marks]
- (b) Percentage of usage for several ingredients were given in the table. Calculate the corresponding production cost in the year 2013.
(Ans : 40000) [5 marks]
- (c) The production cost is expected to increase by 50% from the year 2017 to the year 2019. Calculate the percentage of changes in production cost from the year 2013 to the year 2019.
(Ans : 78.5) [3 marks]
[2018, No.13]

Answer :

- 17 The table shows information related to five cake ingredients, J , K , L , M and N used by a baker in his business.

<i>Ingredient</i>	<i>Price Index for the year 2018 based on the year 2016</i>	<i>Change in the price index from the year 2018 to the year 2020</i>	<i>Price index for the year 2020 based on the year 2016</i>	<i>Weight</i>
J	124	No change	124	5
K	115	40% increase	x	6
L	130	No change	130	p
M	140	10% decrease	y	4
N	120	No change	120	2

The composite index for the cost of making the cakes in the year 2020 based on the year 2016 is 136.

- (a) (i) Find the value of x and of y .
 (ii) Calculate the price for ingredient M in the year 2016 if the price in the year 2020 is RM6.30. (Ans : 5)
[4 marks]
- (b) Calculate the p . (Ans : 3) [3 marks]
- (c) The cost of baking a cake in the year 2016 is RM25. Find the selling price of a cake in the year 2020, if the baker intends to make a profit of 80%. (Ans : 61.20) [3 marks]
[2019, No.15]

Answer :

FORECAST

- 18** The table shows the prices and the price indices of three types of ingredients A , B and C , used in the production of a type of fish ball.

<i>Ingredinet</i>	<i>Price index for the year 2016 based on the year 2014</i>
A	132.6
B	x
C	125
D	y

- (a) The price of ingredinet B is decreased by 5% from the year 2014 to the year 2016 and the price of ingredinet D is unchnaged from the year 2014 to the year 2016. State the value of :
- (i) x [1 mark]
- (ii) y [1 mark]
- (b) Calculate the composite index for the cost of making the fish balls for the year 2016 based on the year 2014. (Ans : 113.15) [2 marks]
- (c) It is given that the composite index for the cost of making the fish balls increased by 35% from the year 2012 to the year 2016.
- (i) Calculate the composite index for the cost of making the fish balls in the year 2014 baded on the year 2012. (Ans : 119.31)
- (ii) The cost of making a fish ball is 20 sen in the year 2012. Find the maximum number of fish balls that can be produced using an allocation of RM150 in the year 2016. (Ans : 555) [6 marks]

Answer :

- 19 (a) The price indices of an item for the year 2005 based on the year 2000 and the year 1995 are 120 and 135 respectively. Given that the price of the item is RM 45 in 2000, find the price of the item in 1995.
(Ans : 40) [2 marks]
- (b) The table shows the prices of three items, *A*, *B*, and *C*, in the year 1996 and 1998, together with their weightages.

<i>Item</i>	<i>Price (RM) in 1996</i>	<i>Price (RM) in 1998</i>	<i>Weightaget (%)</i>
A	70.00	105.00	<i>y</i>
B	80.00	100.00	<i>x</i>
C	60.00	67.50	$2x$

- (i) Using the year 1996 as the base year, calculate the price index of items *A*, *B*, and *C*.
(Ans : 150, 125, 112.5) [3 marks]
- (ii) Given the composite price index of these items in the year 1998 based on the year 1996 is 140, find the values of *x* and *y*.
(Ans : $x = 10$, $y = 70$) [5 marks]

Answer :

- 20 The table shows the price indices and weightages of five items, A , B , C , D , and E in the year 2002 based on the year 2000.

<i>Item</i>	<i>Price index</i>	<i>Weightage</i>
A	132	23
B	130	13
C	p	q
D	136	9
E	106	5

The total weightages is 63 and the composite index in the year 2002 based on the year 2000 is 126.

- (a) Find
- the values of q , [1 mark]
 - the percentage of price change from the year 2000 to the year 2002 for item C .
(Ans : 12.15) [3 marks]
- (b) Find the value of price index of item D in the year 2000, using the year 2002 as the base year.
(Ans : 73.529) [2 marks]
- (c) From the year 2002 to the year 2003, the cost of item B increases by 10%, item D decreases 5%, item E increases 15%, and the remaining items remain unchanged. Calculate the composite index of the items in the year 2003 based on the year 2000.
(Ans : 128.97) [4 marks]

Answer :

- 21 (a) The table shows the prices and price indices of an item from the year 1999 to 2002.

<i>Year</i>	1999	2000	2001	2002
<i>Price (RM)</i>	160	180	<i>a</i>	<i>b</i>
<i>Price index (1999 = 100)</i>	100	<i>c</i>	90	<i>d</i>
<i>Price index (2000 = 100)</i>	<i>e</i>	100	<i>f</i>	120

Find the value of *a*, *b*, *c*, *d*, *e*, and *f*.

(Ans : $a = 144$ $b = 216$, $c = 112.5$, $d = 135$, $e = 88.89$, $f = 80$)

[6 marks]

- (b) The table shows the prices and mass of four items, using in making a type of cake, in the year 1999 and 2001.

<i>Item</i>	<i>Price in 1999</i>	<i>Price in 2001</i>	<i>Mass (kg)</i>
<i>A</i>	RM2.00	RM2.50	10
<i>B</i>	RM12.00	RM14.40	5
<i>C</i>	RM1.00	RM1.30	20
<i>D</i>	RM5.00	RM6.00	15

Calculate the composite index for the cost of making these cake in the year 2001 based on the year 1999.

(Ans : 125) [4 marks]

Answer :

- 22 The table shows the cost of maintaining three type of machines by a factory in the year 2002 and 2003, price indices in the year 2003 based on the year 2002, with their respective weightages.

<i>Machine</i>	<i>2002</i>	<i>2003</i>	<i>Price index</i>	<i>Weightage</i>
<i>P</i>	RM 12,000	RM 15,000	125	2
<i>Q</i>	RM 7,000	<i>y</i>	150	3
<i>R</i>	RM 5,000	RM 5,500	<i>z</i>	10

- (a) Calculate
- the value of *y* and *z*, (Ans : $y = 10500$, $z = 110$)
 - the composite index of the cost of maintaining the machines in the year 2003 based on the year 2002. (Ans : 120)
[5 marks]
- (b) The composite index continues to increase at the same rate from the year 2003 to the year 2004.
- Calculate the composite indices of the cost of maintaining the machines in the year 2004, using the year 2002 as the base year. (Ans : 144)
 - If the cost of maintaining the machines in the year 2002 is RM 450,000, calculate the mean of the cost of maintaining the machines from the year 2002 to 2004. (Ans : 546000)
[5 marks]

Answer :

- 23 (a) The table shows the information related to four types of transportations' fares between two towns.

<i>Transport</i>	<i>Change in price from the year 2013 to the year 2017</i>	<i>Weightage</i>
Aeroplane	40% increase	2
Taxi	30% increase	3
Bus	10% increase	4
Train	22% increase	1

Calculate

- (i) the taxi's fare in the year 2013, if the taxi's fare in the year 2017 is RM80. (*Ans* : 61.54)
 - (ii) the price index of transportation's fares in the year 2017 based on the year 2013. (*Ans* : 123.2)
 - (iii) the price index of aeroplane's fare in the year 2017 based on the year 2015, if the price index in the year 2015 based on the year 2013 is 120. (*Ans* : 116.67)
[7 marks]
- (b) The price index of a certain item for the year 2000 based on the year 1997 is 130. Based on the year 2000, the price index of the item in the year 2002 is 108. Calculate the price index of the item in the year 2002 with the year 1997 taken as the base year.
(*Ans* : 140.4) [3 marks]

Answer :

- 24 The table shows the prices for four types of books in a bookstore for three consecutive years.

<i>Book</i>	<i>Price for the year (RM)</i>			<i>Price Index</i>		<i>Weightage</i>
	<i>2000</i>	<i>2001</i>	<i>2002</i>	<i>2001 (2000 = 100)</i>	<i>2002 (2000 = 100)</i>	
<i>P</i>	<i>w</i>	20	30	150	225	6
<i>Q</i>	50	<i>x</i>	65	115	130	5
<i>R</i>	40	50	56	125	140	3
<i>S</i>	80	<i>z</i>	150	<i>y</i>	<i>y</i>	2

- (a) Find the values of w , x , y and z . (*Ans* : $w = 13.3$, $x = 57.5$, $y = 187.5$, $z = 150$) [4 marks]
- (b) State the price index for the year 2002 based on the year 2001 for book *R*. [*Ans* : 112] [1 mark]
- (c) Calculate the composite index for the price of the book for the year 2002 based on the year 2001. (*Ans* : 125.075) [3 marks]
- (d) A school spent RM4865 buying the books for school library in 2002. Find the estimated allocation for the books in 2003 if the composite index for the year 2003 based on the year 2002 is equal to the composite index for the year 2002 based on the year 2001. (*Ans* : 6084.90) [2 marks]

Answer :

- 25 The table shows the price index of the cost of few items needed to run a company, with their respectively weightages.

<i>Item</i>	<i>Price index</i>		<i>Weightage</i>
	2004 (2002 = 100)	2005 (2002 = 100)	
<i>A</i>	101	102	3
<i>B</i>	118	121	6
<i>C</i>	105	107	<i>k</i>
<i>D</i>	103	110	4
<i>E</i>	105	120	5

Given the composite index in the year 2004 based on the year 2002 is 107.9.

- If the expenses of item *D* in the year 2004 is RM2678, find its expenses in the year 2005.
(Ans : 2860) [2 marks]
- If the expenses of item *B* in the year 2005 is RM3993, find the increase in its expenses compare to the year 2004.
(Ans : 99) [2 marks]
- Calculate the value of *k*.
(Ans : 2) [3 marks]
- Based on the year 2004, calculate the composite index for the year 2005.
(Ans : 106.033) [3 marks]

Answer :

CONTINUOUS EXERCISES

- 26** The table shows the price for an item in 2000 and 2015.

<i>Year</i>	<i>Price</i>
2000	RM12
2015	RM15

- (a) If the rate of price increase from 2015 to 2020 is twice the rate of price increase from 2000 to 2015, determine the price of that item in 2020. (*Ans* : 22.50)
- (b) Calculate the price index in the year 2020 based on the year 2000. (*Ans* : 187.5)
[5 marks]

Answer :

- 27** The table shows the number of visitors who visit Mount Kinabalu National Park in 2010 and 2015.

<i>Year</i>	<i>Number of visitors</i>
2010	2.54 million
2015	3.86 million

- (a) Determine the number of visitors in 2020 if the increase for the number of visitors in 2015 to 2020 is twice the increase from 2010 to 2015. (*Ans* : 6.5)
- (b) Calculate the index for the number of visitors in the year 2020 based on the year 2015. State the interpretation based on the index number obtained. (*Ans* : $168\frac{76}{193}$)
[5 marks]

Answer :

- 28 The table shows the price indices and weightage of four items P , Q , R and S .

<i>Item</i>	<i>Price index for the year 2013 based on the year 2011</i>	<i>Price index for the year 2015 based on the year 2011</i>	<i>Weightage</i>
P	105	10% increase	5
Q	120	No change	k
R	160	No change	5
S	130	5% decrease	1

Calculate :

- (a) (i) The price of item P in the year 2013 if its price in the year 2011 is RM10.20. (Ans : 10.71)
- (ii) The price of item S in the year 2011 if its price in the year 2013 is RM8.60. (Ans : 6.62) [3 marks]
- (b) The composite index for the four items in the year 2013 based on the year 2011 is 129. Calculate the value of k . (Ans : 4) [3 marks]
- (c) If the price of the four items P , Q , R and S is RM20.80 in the year 2011, calculate the price of the four items in the year 2015. (Ans : 21.42) [4 marks]

Answer :

- 29 The table shows the price indices and weightages of four items, A , B , C and D used in a production of a type of food.

<i>Ingredient</i>	<i>Price index in the year 2020 based on the year 2018</i>	<i>Weightage</i>
A	100	4
B	120	3
C	m	2
D	106	1

The composite index for the cost of making the food for the year 2020 based on the year 2018 is 105.6

- (a) Find the value of m and hence, give your comment on the change in price of item C .
(Ans : 95) [4 mark]
- (b) Calculate the price of item B in the year 2020 if its price in the year 2018 was RM7.80.
(Ans : 9.36) [2 marks]
- (c) The composite index for the cost of making food is expected to increase by 15% from the year 2020 to the year 2022. Calculate
- (i) the composite index for the year 2022 based on the year 2018, (Ans : 121.44)
- (ii) the price of 1kg of the food in the year 2022 if its price in the year 2020 is RM85.00.
(Ans : 97.75)
[4 marks]
[YIK2020, No.13]

Answer :

- 30 The table shows the prices and the price indices of four types of material R , S , T and U used in the production of a type of shoes.

Material	Price (RM) per unit		Price index for the year 2020 based on the year 2015	Percentage of usage (%)
	Year 2015	Year 2020		
R	4.00	6.00	150	20
S	q	2.23	p	30
T	0.80	1.00	125	10
U	0.50	0.67	134	m

- (a) The price of ingredient S is increased by 20% from the year 2015 to the year 2020.
- State the value of m ,
 - State the value of p ,
 - Find the value of q , (Ans : 1.83)
[4 marks]
- (b) Calculate the composite index for the production of the shoes in the year 2020 based on the year 2015. (Ans : 132.1) [2 marks]
- (c) It is given that the composite index for the cost of the production of shoes increased by 35% from the year 2013 to the year 2020.
- Calculate the composite index for the cost of the production of shoes in the year 2015 based on the year 2013. (Ans : 120.20)
 - The cost of the production was RM7.50 in the year 2013. Find the maximum number of shoes that can be produced using allocation of RM488.00 in the year 2020. (Ans : 48)
[4 marks]
- [Pulau Pinang 2020, No.12]**

Answer :

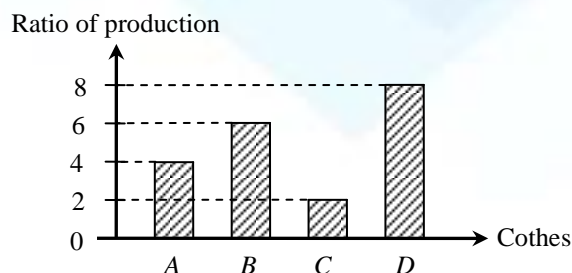
- 31 A hawker sells “Yong Tau Fu” with the selective ingredients of brown squids, chicken balls, fish balls, tofu and water spinach.

<i>Ingredient</i>	<i>Price (RM) per unit</i>		<i>Price index for the year 2020 based on the year 2017</i>	<i>Percentage of usage (%)</i>
	<i>Year 2017</i>	<i>Year 2020</i>		
Brown Squids	25.00	28.50	x	10
Chickne ball	7.20	9.00	125	w
Fish ball	8.00	y	140	25
Tofu	12.00	13.50	112.5	15
Water spinach	z	4.80	120	20

- (a) Find the value of w , x , y and z . (Ans : $x = 114$, $y = 11.20$, $z = 4$) [4 marks]
- (b) Calculate the composite index for cost of preparing the “Yong Tau Fu” in the year 2020 based on the year 2017. (Ans : 124.775) [2 marks]
- (c) If the hawker spends RM250 daily to buy the ingredient in the year 2017, find the total cost to buy the ingredients in March 2020. (Ans : 9670.14) [2 marks]
- (d) The cost of all the ingredient increases by 14% from the year 2020 to the year 2021. Find the composite index for the year 2021 based on the year 2017. (Ans : 142.24) [2 marks]
[Terengganu2020, No.12]

Answer :

- 32 The diagram is a bar chart which represents the ratio of production of clothes for the factory in the year 2020. The table shows the price indices for four types of clothes A , B , C and D produced by a factory.



Material	Price index in 2018 based on 2016	Price index in 2020 based on 2018
A	105	107
B	110	x
C	95	110
D	115	125

- (a) (i) The price of clothes B in the year 2016 is RM35, and its price in the year 2018 is RM42. Find the value of x . (Ans : 109.09)
- (ii) Find the price of clothes in the year 2018. (Ans : 38.50) [4 marks]
- (b) Find the price index of clothes A in the year 2020 based on the year 2016. (Ans : 112.35) [2 marks]
- (c) (i) Calculate the composite index for the production cost of the clothes in the year 2020 based on 2018. (Ans : 115.127)
- (ii) Hence, given the total price of the four types is RM150 in the year 2018, find the corresponding price of the clothes in the year 2020. (Ans : 172.69) [4 marks]

[Kedah2020, No.15]

Answer :

WORKSHEET ANSWERS

1) Can be viewed or downloaded using the following link.

bit.ly/MTWORKSHEETANSWER

or

2) Scan this QR code.



ACKNOWLEDGEMENT

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*"Success is not final; failure is not fatal:
It is the courage to continue that counts"*

